1.9 Test Inputs

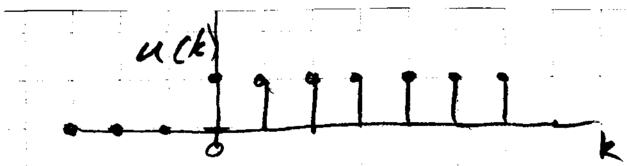
- How to characterize a system?

  Specify or measure its response to certain test inputs.

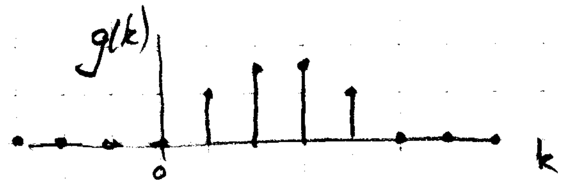
  Typical ones:
  - step
  - impulse
  - sine waves of various frequencies
  - pseudo random noise

read H & vV 1.5 except "precedence rule" 1.6 except "complex exponential"

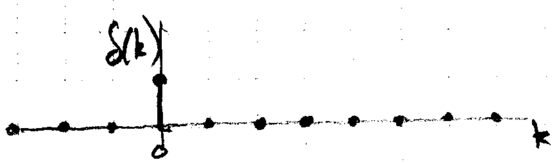
- Discrete time test signals



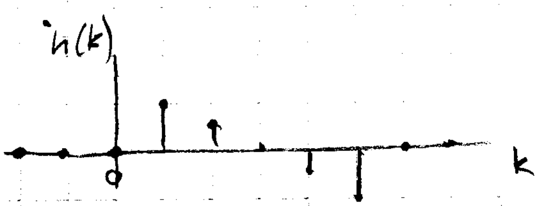
unit step $u(k) = \begin{cases} 1 & k \geq 0 \\ 0 & \text{else} \end{cases}$



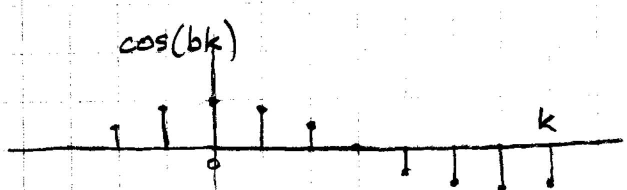
example unit step response



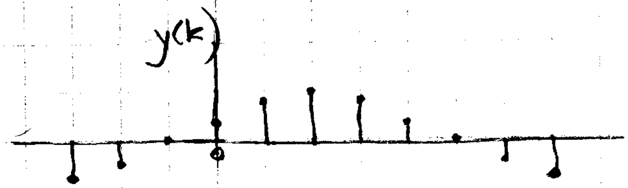
unit pulse (impulse) $\delta(k) = \begin{cases} 1 & k = 0 \\ 0 & \text{else} \end{cases}$



example unit pulse response



sampled cosine $x(k) = \cos(b\,k)$
(this one not causal)

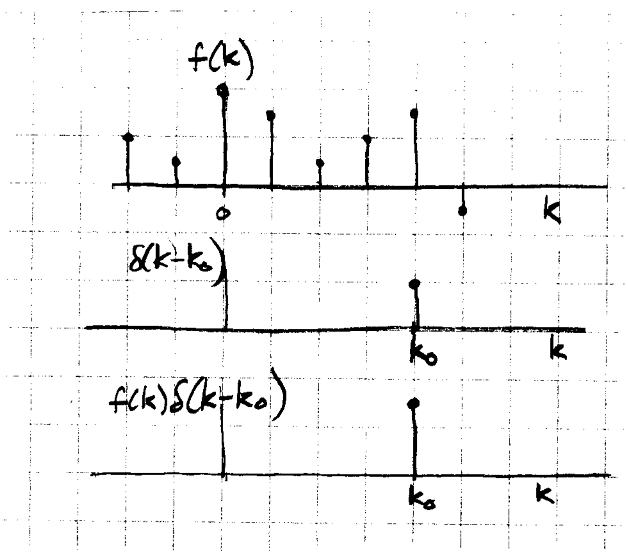


example response to the cosine

- About the unit pulse...
  - it is the first difference of the unit step

$$\delta(k) = u(k) - u(k-1)$$

  - properties



$$f(k)\,\delta(k-k_0) = f(k_0)\,\delta(k-k_0)$$

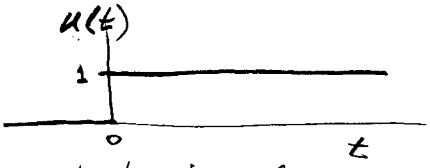
and $\displaystyle\sum_{k=-\infty}^{\infty} f(k)\,\delta(k-k_0) = f(k_0)$ (a scalar, not a time function)

- About the sampled cosine...

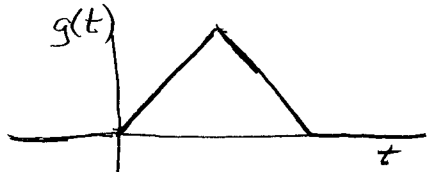
$$\cos(bk) = Re\left[e^{jbk}\right] = Re\left[\left(e^{jb}\right)^k\right]$$

We'll use all of these representations in the course

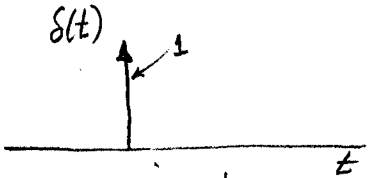
- Continuous time test inputs are similar



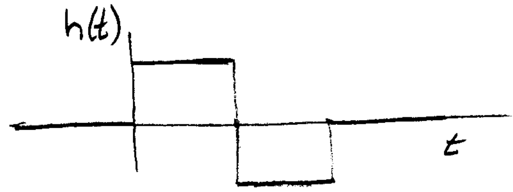
unit step is 1 for $t \geq 0$



a possible step response

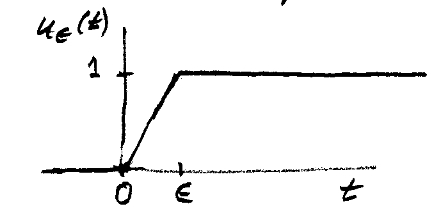


unit impulse "derivative" of unit step

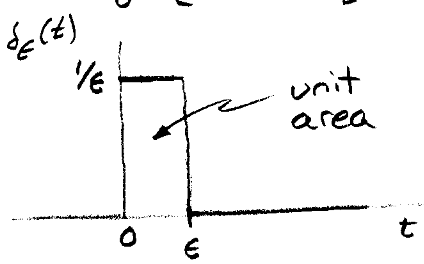


matching impulse response

- The impulse is best considered as shorthand for a limit, not a physical signal.

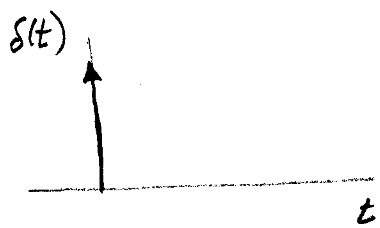


$u_\epsilon(t)$, a continuous function



approximate impulse

$$\delta_\epsilon(t) = \frac{d}{dt} u_\epsilon(t)$$



define unit impulse

$$\delta(t) = \lim_{\epsilon \to 0} \delta_\epsilon(t)$$

zero width, infinite height, unit area

- Properties of unit impulse:

$\rightarrow \int_{-\infty}^{\infty} \delta(t)\,dt = 1$ since $\lim_{\epsilon \to 0} \int_{-\infty}^{\infty} u_0(t,\epsilon)\,dt = 1$

$\rightarrow \int_{-\infty}^{\infty} \delta(t) f(t)\,dt = f(0)$ since $\lim_{\epsilon \to 0} \int_{-\infty}^{\infty} u_\epsilon(t) f(t)\,dt = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \int_0^{\epsilon} f(t)\,dt$

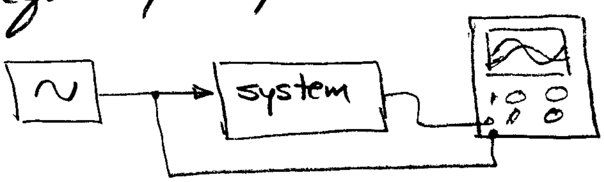
$$= \lim_{\epsilon \to 0} \frac{F(\epsilon) - F(0)}{\epsilon} = f(0)$$

$\rightarrow \int_{-\infty}^{\infty} \delta(t-\tau) f(t)\,dt = f(\tau)$

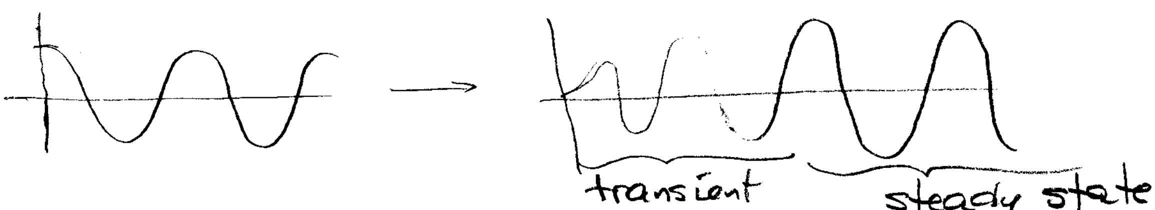
$\rightarrow \int_{-\infty}^{t} \delta(\alpha)\,d\alpha = u(t)$

$\rightarrow f(t)\,\delta(t-\tau) = f(\tau)\,\delta(t-\tau)$

- Frequency response is an alternative to step, impulse resp.



For an LTI system, we'll see that steady state resp to constant freq cosine input is cosine output of same frequency but possibly different amplitude and phase.



Measure $\underbrace{\text{gain}(f),\ \text{phase shift}(f)}_{\text{"frequency response"}}$ both functions of frequency

Not just one frequency — all frequencies

$$H(f) = A(f)\, e^{j\varphi(f)}$$

$A(f)$ amplitude gain

$\varphi(f)$ phase shift

- Generality (for LTI systems)
  - Step, impulse and frequency responses can be obtained from each other, as we will see:

$$\begin{tikzcd} g(t) \arrow[r, bend left, "\frac{d}{dt}"] & h(t) \arrow[l, bend left, "\int_{-\infty}^{t}"] \arrow[r, bend left, "\mathcal{F}[\;]"] & H(f) \arrow[l, bend left, "\mathcal{F}^{-1}[\;]"] \end{tikzcd}$$

  - Response to any input can be calculated if we know impulse, step or frequency resp.

  - Many other test inputs are possible — these ones are just convenient.

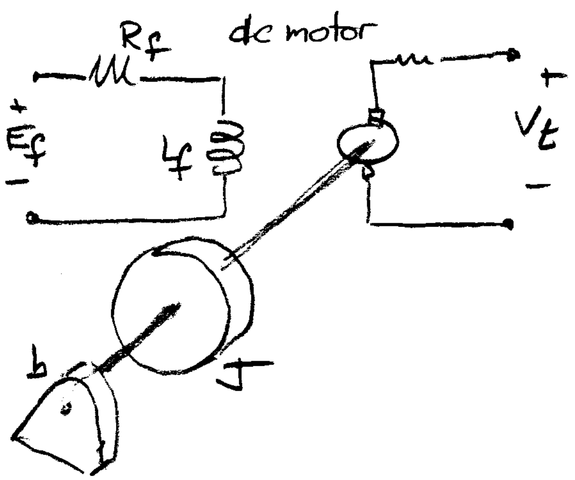
1.10 Some Comments on Modelling

- A model is a set of equations that describes the system of interest.

- Trade level of detail and accuracy against insight

less detail	⟷	more detail
– reasonably simple equations		– computer simulations
– sometimes approx results		– good accuracy
– intuition, insight good for design concepts		– just numbers

- Detail? Include only those phenomena that make a difference to results at a level you care about

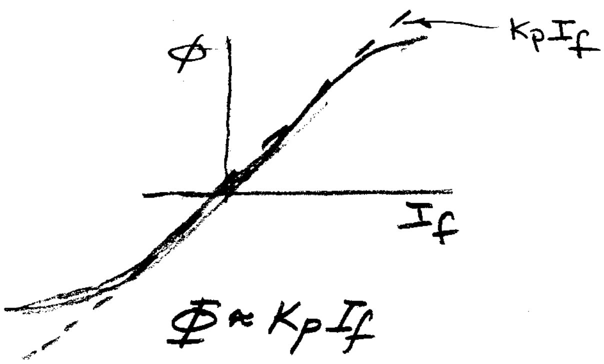


If mechanical time constant is 20 sec, then we don't worry (much) about 250 ms field time constant.

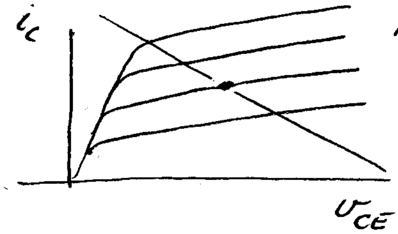
And ignore changes in viscosity of air with temperature.

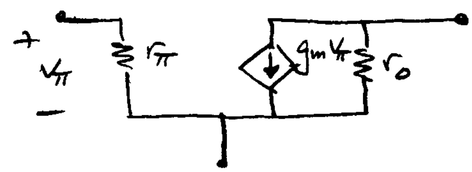
But be careful: If you can design a way to eliminate the most significant problem, what's left is not zero problem — it's the worst of the effects you ignored!

- Approximation as a virtue
  - You may have an exact equation to describe a phenomenon, yet still want to use an approximation



$\Phi \approx K_p I_f$

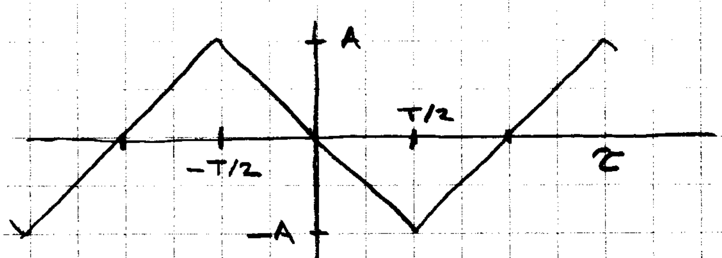


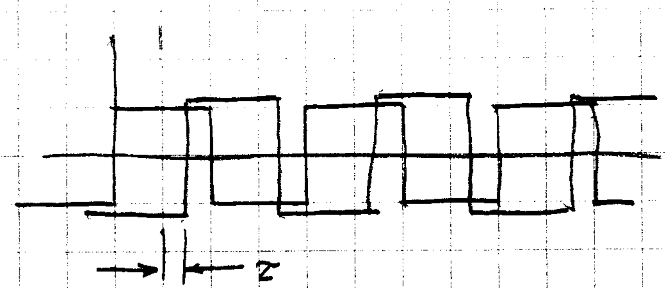


small signal

- A good approximate model lets you make design choices, then check to see if assumptions were violated. You can always resort to detailed computer model later, if nec.

- Assumptions? Here's an example. The phase detector in a square wave PLL (phase locked loop) has an output that varies with time shift τ like this





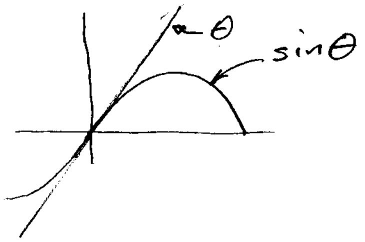
Assuming that $|\tau|$ does not exceed $T/2$, then approx as

$$-\frac{2A}{T}\tau$$

If assumption violated, think again!

- Series expansion gives useful approximations any time you have small perturbations about a point or if you have subtraction of nearly equal quantities.

Example $f(\theta) = \theta - \sin\theta$



$$f(\theta) = \theta - \sin\theta = \theta - \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)$$

$$\approx \frac{\theta^3}{3!}\left(1 - \frac{\theta^2}{20}\right) \approx \frac{\theta^3}{6}$$

- Linearized models
  - For a natural system, try to approx behaviour by a simple linear system: measure response to test inputs, find linear dynamic model which behaves roughly the same way
  - Use the model to predict behaviour with other inputs.
  - Some examples:
    - Design control systems for aerospace, robotics, industrial automation, etc
    - Design filters for better audio processing
    - Specify arithmetic processing to enhance edges or deblur an image
    - ETC!