

2. TIME DOMAIN ANALYSIS OF SYSTEMS

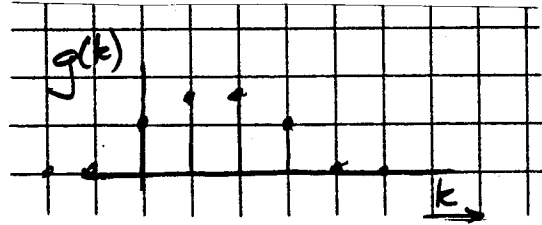
- In this section, you will learn some universal properties and representations of LTI systems:
 - superposition
 - derivative and integral properties
 - impulse, step, ramp... responses
 - convolution

We will work with both continuous time and discrete time systems.

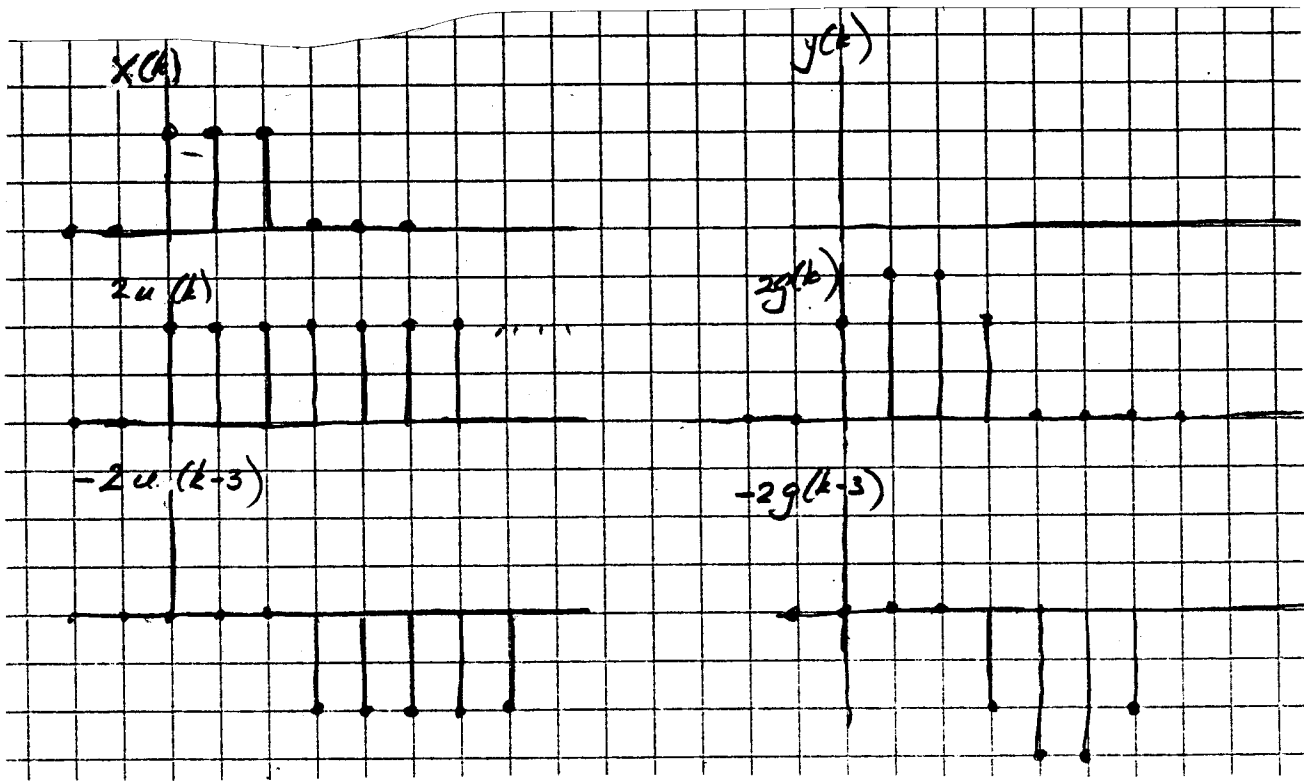
- The special case of finite order (lumped) systems is reserved to Section 3.
- Everything in this section follows from two basic properties:
 - linearity
 - time invariance

2.1 System Response by Superposition - Discrete Time

- Consider a discrete time system for which we know the step response $g(k)$.



To find the response to the input below, decompose the input to a sum of weighted, delayed steps:

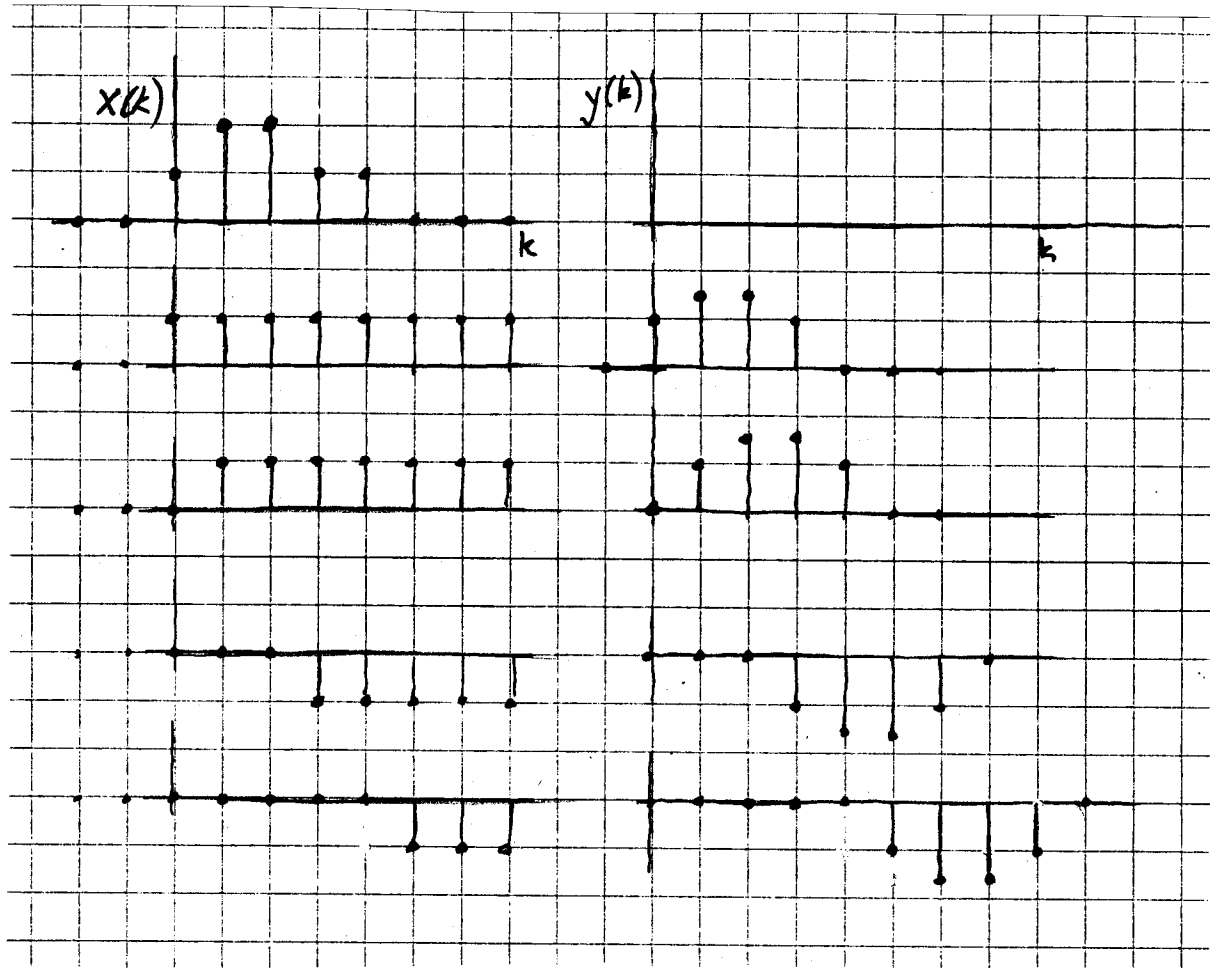


$$x(k) = 2u(k) - 2u(k-3)$$

$$y(k) = 2g(k) - 2g(k-3)$$

- In fact, any signal can be considered a superposition of steps, so we can calculate the response to any input if we know the step response.

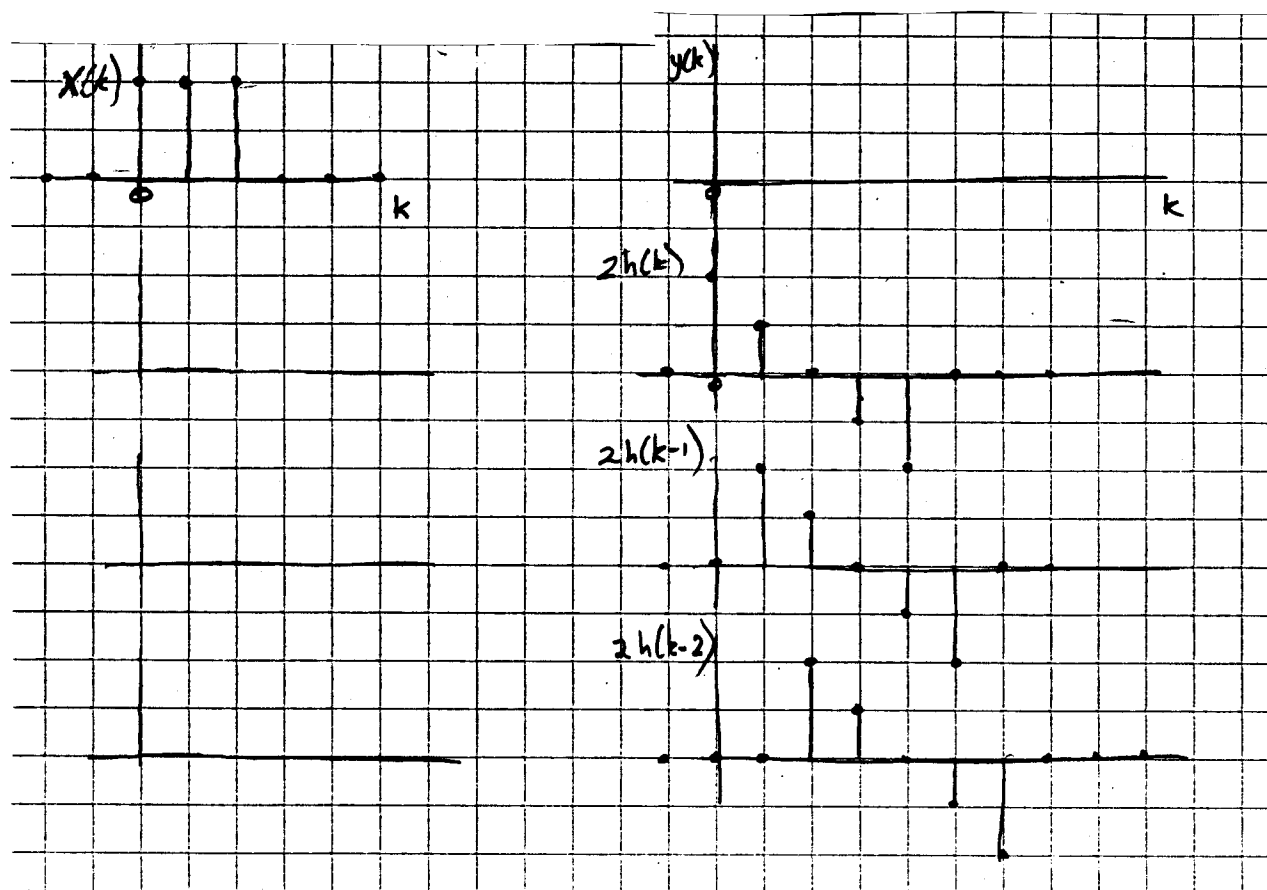
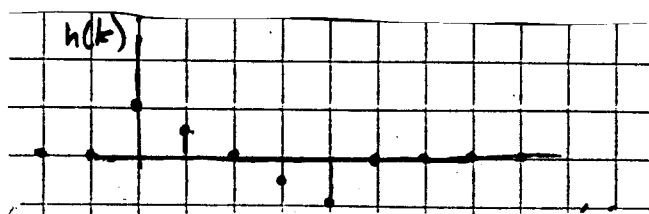
Example



$$y(k) =$$

- It is more common to perform this decomposition and resynthesis with the pulse response $h(k)$.

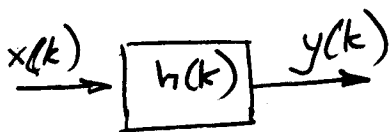
Example:

$$x(k) =$$


- An easy generalization gives the response to arbitrary inputs as

$$y(k) = x(0)h(k) + x(1)h(k-1) + x(2)h(k-2) + \dots$$

- Conclusion: the impulse response or the step response are sufficient to calculate the response to any input



In particular, the impulse response and step response can be obtained from each other. More on this below.

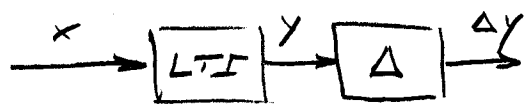
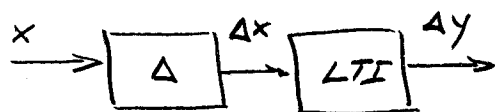
- The difference property of LTI systems

- For any sequence $x(k)$, define

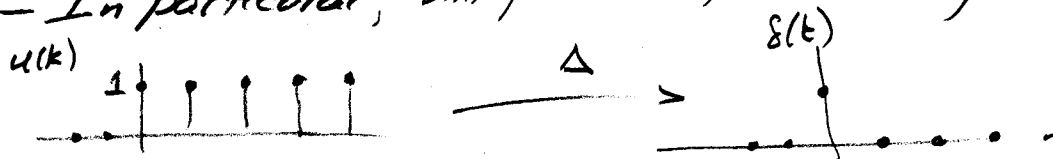
$$\Delta x(k) = x(k) - x(k-1)$$

$$\begin{aligned}\Delta^2 x(k) &= \Delta(\Delta x(k)) = (x(k) - x(k-1)) - (x(k-1) - x(k-2)) \\ &= x(k) - 2x(k-1) + x(k-2)\end{aligned}$$

- For any LTI system, if $x(k)$ produces $y(k)$ then, by superposition, $\Delta x(k)$ produces $\Delta y(k)$
 $x(k) - x(k-1)$ produces $y(k) - y(k-1)$



- In particular, unit pulse is first diff of unit step



so unit pulse resp is first diff of unit step resp.

$$h(k) = \Delta g(k) = g(k) - g(k-1)$$

• The summation property of LTI systems.

- For any sequence $x(k)$ define the summation to time k :

$$\sum_{i=0}^k x(i) = x(0) + x(1) + \dots + x(k)$$

The summation sequence $Sx(k)$ takes these values, so

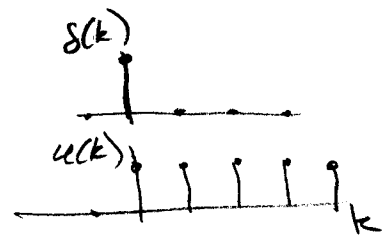
$$Sx(k) = x(k) + x(k-1) + \dots + x(0)$$

- By superposition, if $x(k) \rightarrow y(k)$

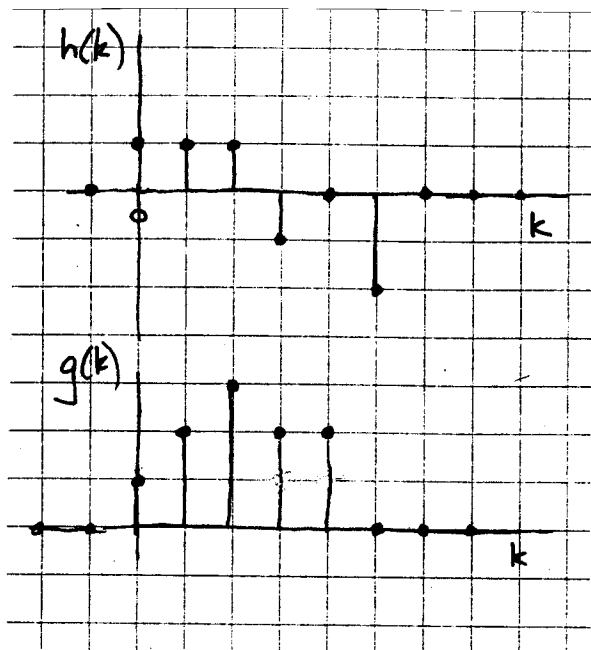
then $Sx(k) \rightarrow Sy(k)$

In particular, $u(k) = S\delta(k)$

so $g(k) = Sh(k)$



- Example

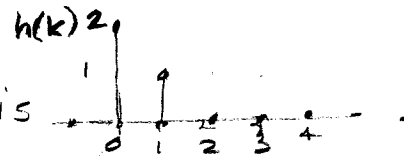


$$h(k) = 1, 1, 1, -1, 0, -2, 0, 0, \dots$$

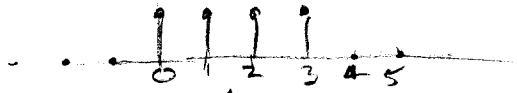
$$g(k) = 1, 2, 3, 2, 2, 0, 0, 0, \dots$$

Check: is $h(k) = \Delta g(k)$ here?

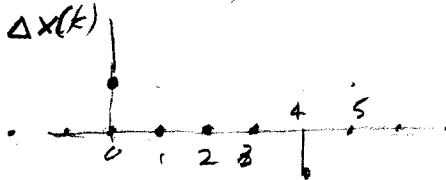
• Example use of diff & sum properties
system pulse resp



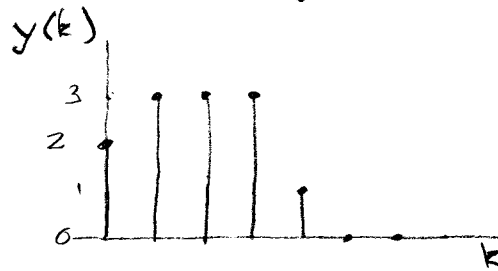
what is resp to $x(k)$



take first diff of input

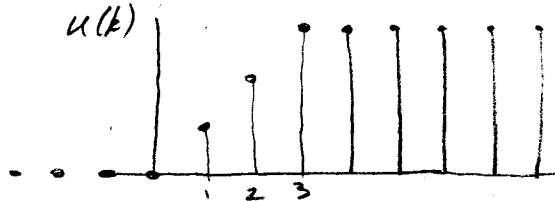


and sum to get response

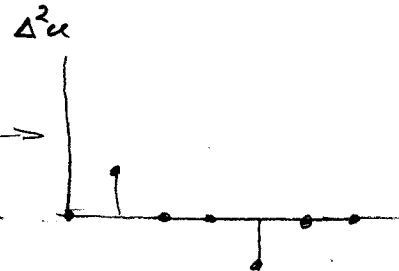
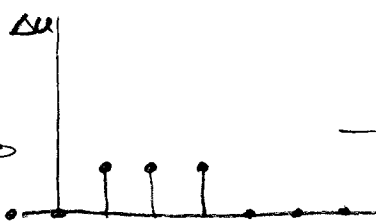
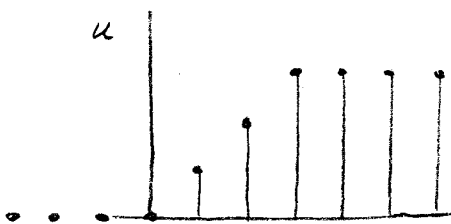


example of sketch solutions. Suppose $h(k) = 2 \cdot 0.5^k$

what is response to



Take 2nd diff:



Note
scale
change
($\frac{1}{2}$)

