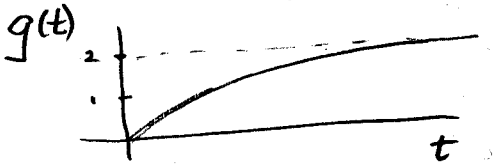
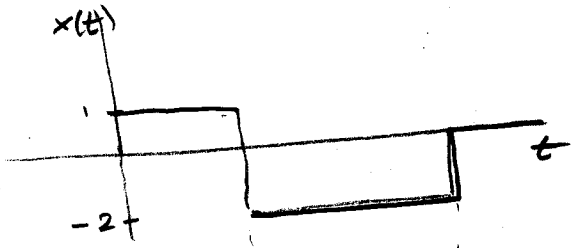
2.3 Response by Superposition — Continuous Time

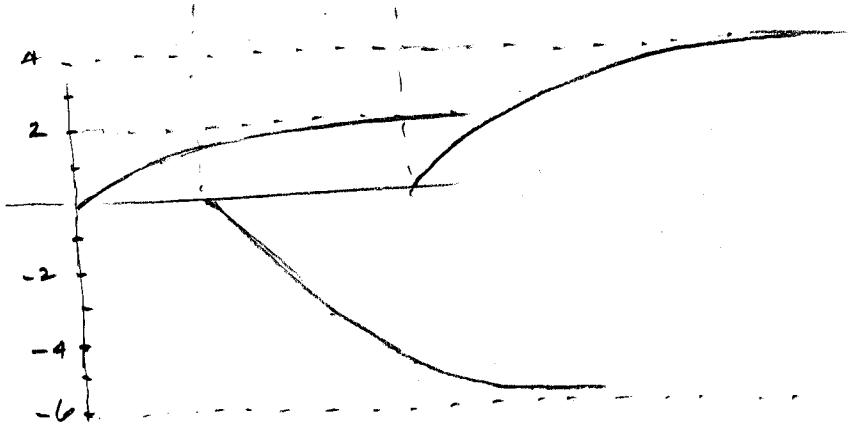
- By obvious analogy, if step response is



then response to



is $y(t)$



- Derivative property:

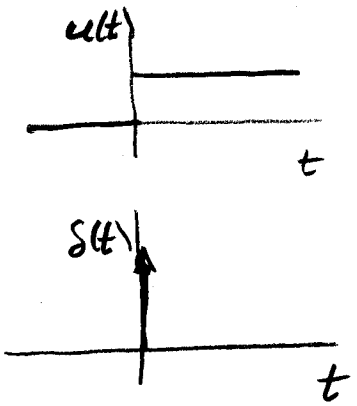
If $x(t)$ produces $y(t)$

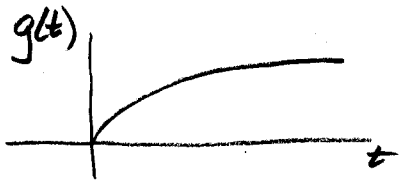
then $$x'(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t/2) - x(t-\Delta t/2)}{\Delta t}$$

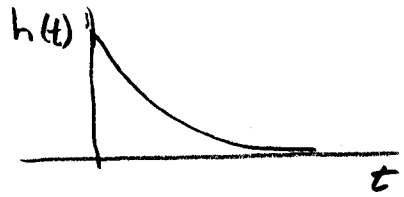
$$= \lim_{\Delta t \to 0} \frac{y(t+\Delta t/2) - y(t-\Delta t/2)}{\Delta t} = y'(t)$$

... that is, when we can represent the derivatives.

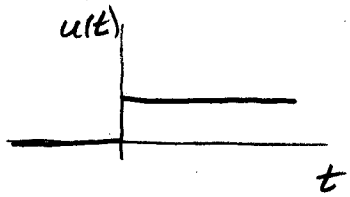
Step discontinuities produce impulses

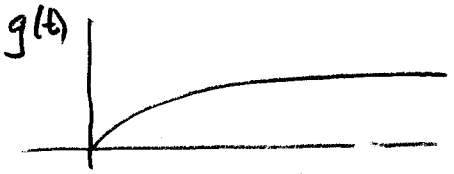


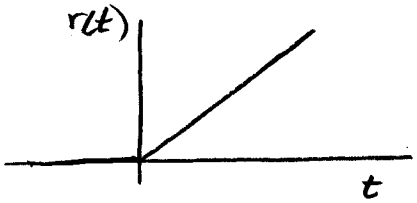


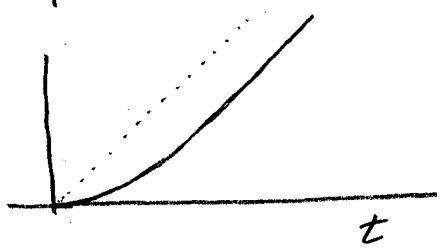


- Integral property $\int_0^t x(\alpha)\,d\alpha$ produces $\int_0^t y(\alpha)\,d\alpha$

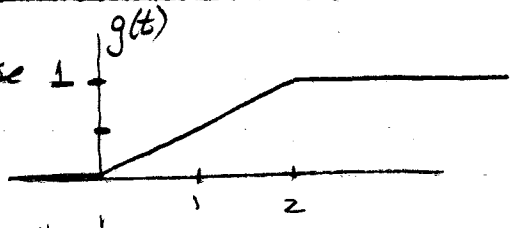




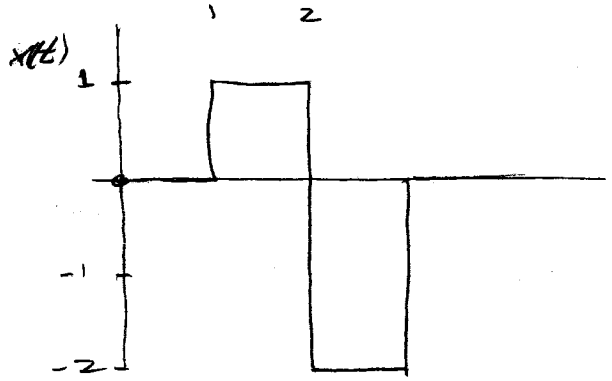




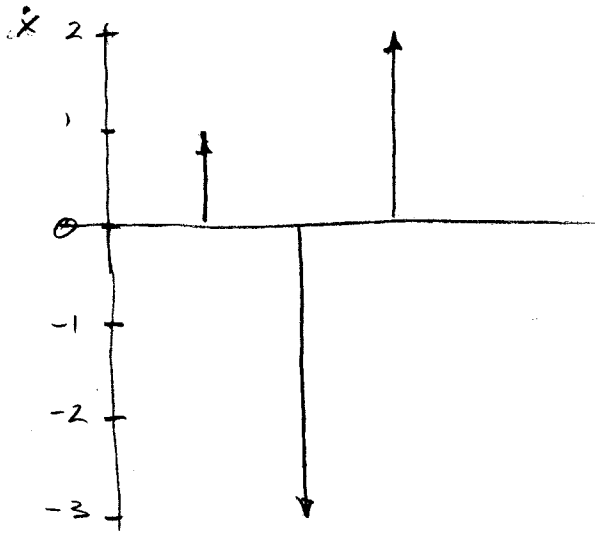
example suppose step resp



what is response to

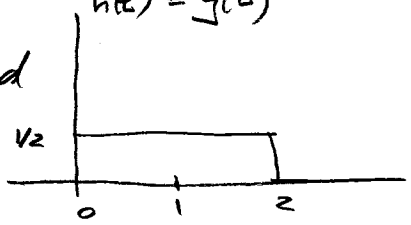


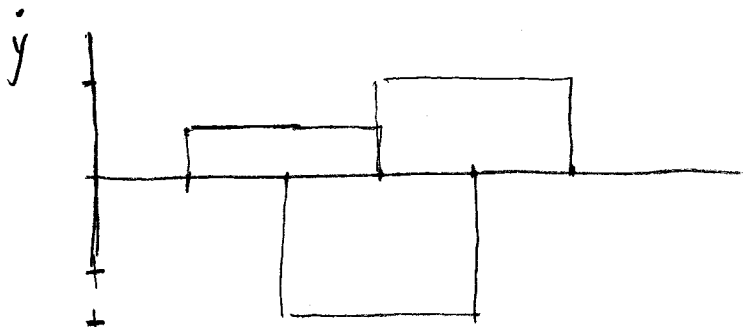
use deriv, integral properties

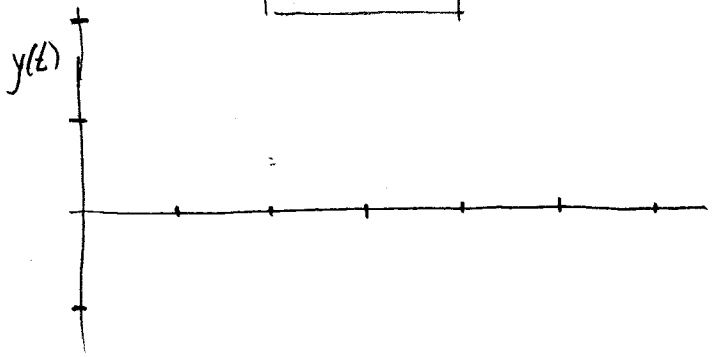


$h(t) = \dot{g}(t)$

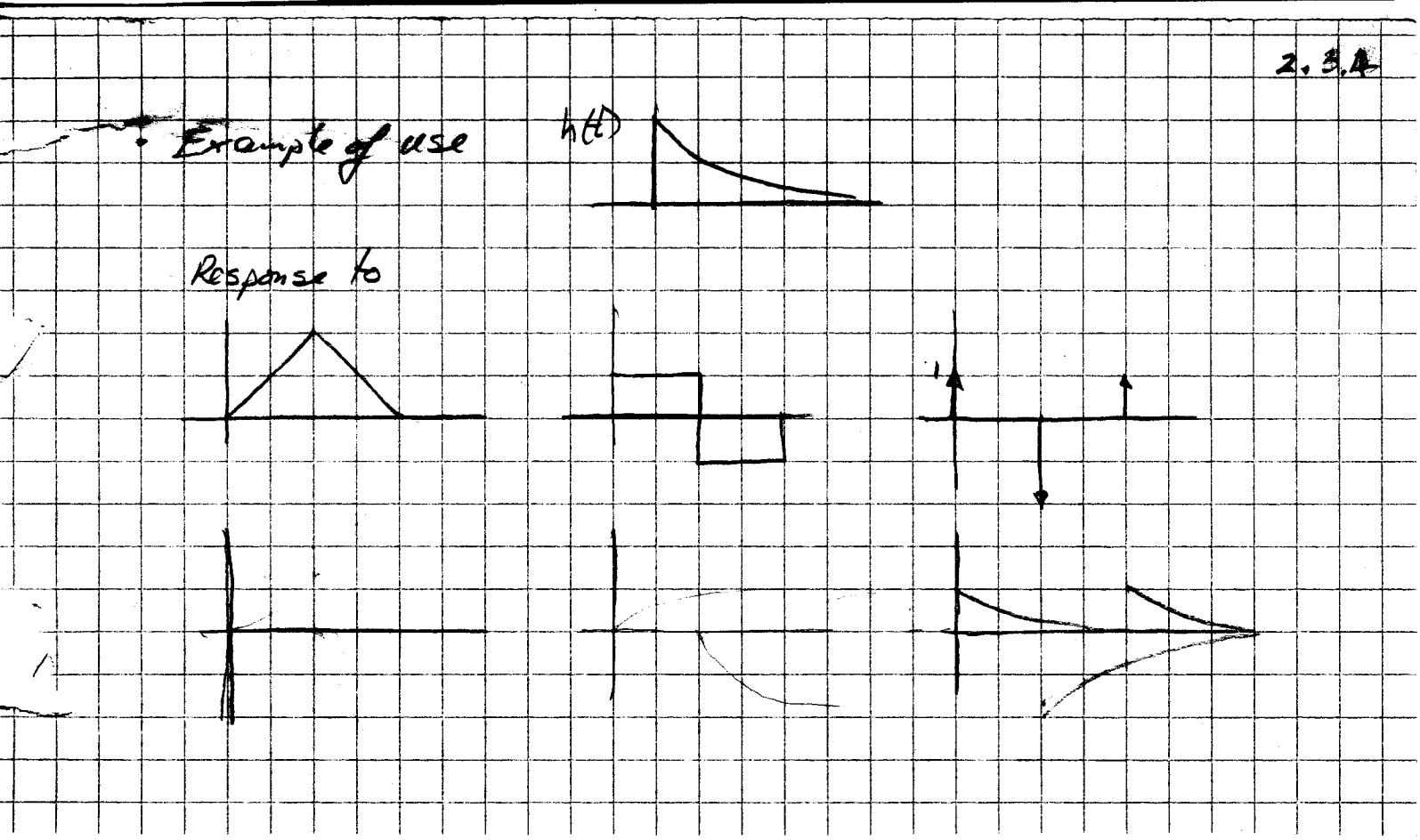
and







• Example of use



• Some simple systems

gain → [A] →

$h(t) = A\,\delta(t)$

$g(t) = A\,u(t)$

integrator → $\int_0^t (\cdot)\,d\tau$ →

$h(t) = u(t)$

$g(t) = r(t)$

differentiator → $\frac{d}{dt}(\cdot)$ →

$g(t) = \delta(t)$

delay → [T] →

$h(t) = \delta(t-T)$

$g(t) = u(t-T)$