

4 FREQUENCY DOMAIN ANALYSIS OF LTI SYSTEMS

Read H&V pp 34-37

"Complex Exponentials and Orthogonality"

H&V 3.1

- Describing and analyzing LTI systems is often easier in the frequency domain, with Fourier analysis:
 - represent signals as sum (integral) of sinusoids at different frequencies
 - system transforms each frequency differently.
- In this section, you will learn:
 - how to calculate frequency response;
 - what a Fourier series is and how to calculate its terms;
 - the Fourier transform and how it is related to Fourier series;
 - the multiplication-convolution theorem;
 - how properties of a signal in one domain are related to its properties in the other domain.

4.1 Consider the Complex Exponential...

- This is a review of complex exponentials and phasors. They are about to play a large role in your life again.
- A tone, or sinusoid, at a single frequency can be written

$$A \cos(2\pi ft + \phi) \quad \text{or} \quad V_r \cos \omega t - V_i \sin \omega t$$

$$\omega = 2\pi f$$

A more compact representation is

$$\operatorname{Re}[V e^{j2\pi ft}]$$

=

=

or

=

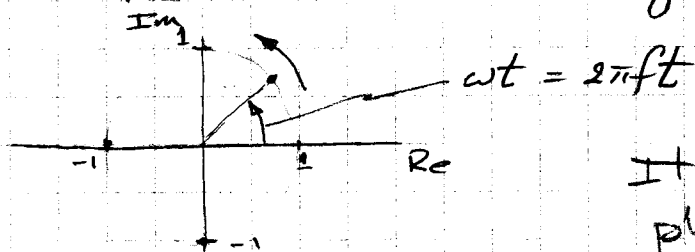
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$$\text{polar} \\ V = |V| e^{j\phi}$$

$$\text{rect} \\ V = V_r + j V_i$$

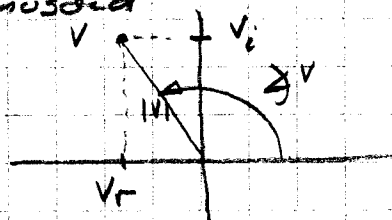
$$\text{Note: } e^{j\theta} = \cos \theta + j \sin \theta \quad e^{j\omega t} = \cos \omega t + j \sin \omega t$$

The complex exponential signal $e^{j\omega t} = e^{j2\pi ft}$ rotates on the unit circle with frequency f rotations/s

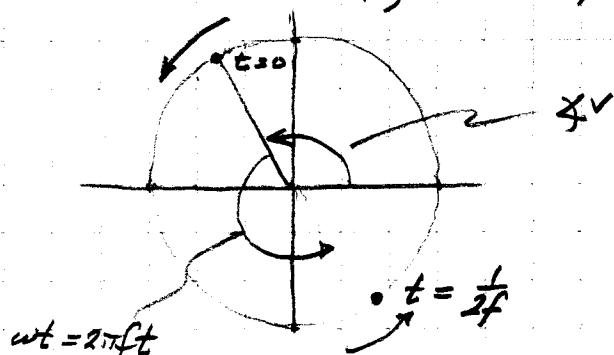


It has amplitude 1, initial phase ($t=0$) 0.

The complex constant V is the phasor describing the sinusoid



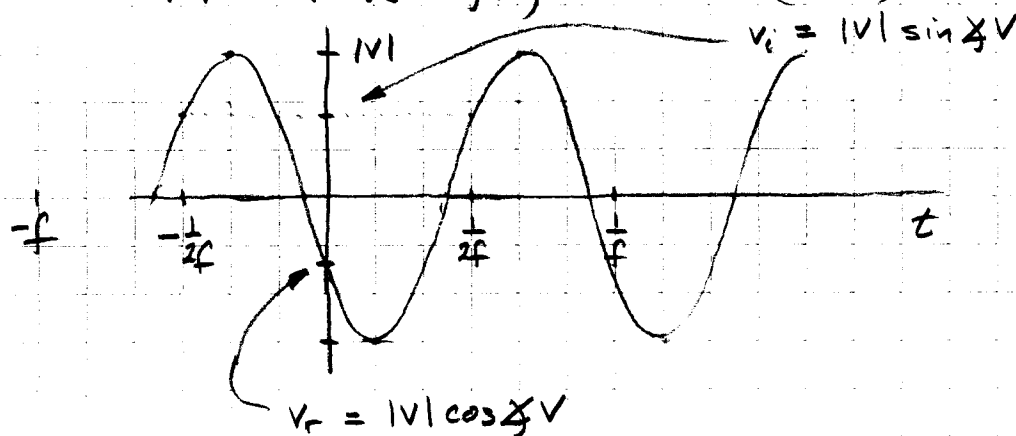
The product $V e^{j\omega t}$ follows a circular path, radius $|V|$, initial phase $\angle V$.



The phase increases at ω rad/s or f rotations/s

Finally, $\text{Re}[V e^{j2\pi f t}]$ is the projection on the Re axis, a real signal:

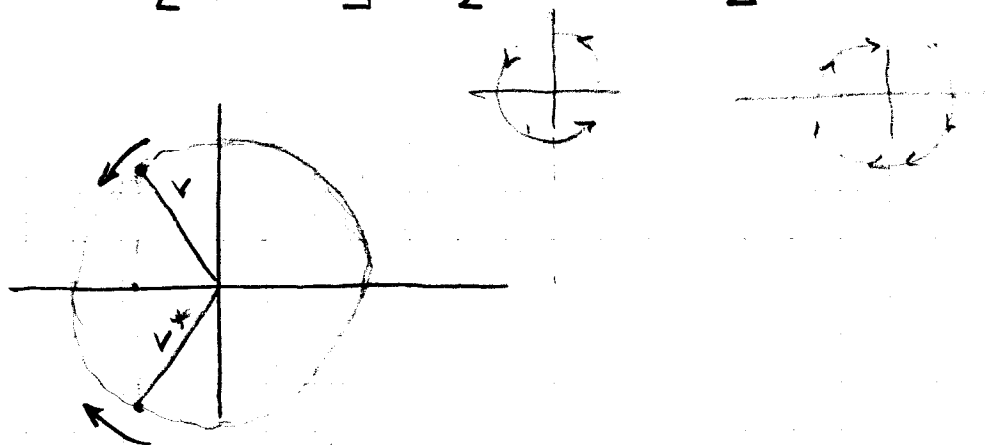
$$|V| \cos(2\pi f t + \angle V) = V_r \cos(2\pi f t) - V_i \sin(2\pi f t)$$



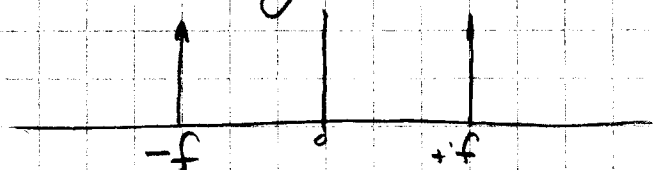
We'll use the complex representation extensively.

- Negative frequency ?? Yes - expand the complex rep'n

$$\text{Re}[V e^{j\omega t}] = \frac{1}{2} V e^{j\omega t} + \frac{1}{2} V^* e^{-j\omega t}$$



So concentration of power in frequency is

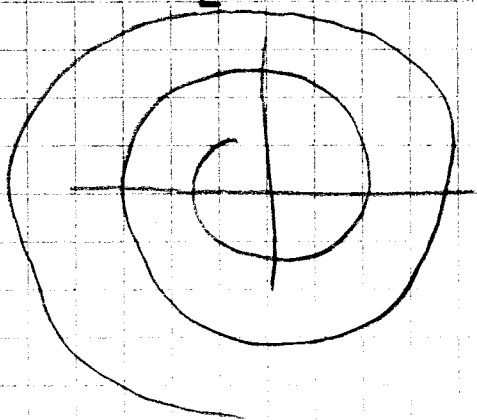


Any real sinusoid can be considered as a sum of counter-rotating complex exponentials; the phasor on the negative freq is the conj of the phasor on the pos freq.

- Decaying or growing sinusoids can be expressed similarly.

$$Ae^{at} \cos(\omega t + \phi) = \text{Re}[V e^{at} e^{j\omega t}] \quad V = A e^{j\phi}$$

$$= \text{Re}[V e^{(a+j\omega)t}]$$



"decay" constant is the real part of exponent

rot'n freq is the imag part of exponent