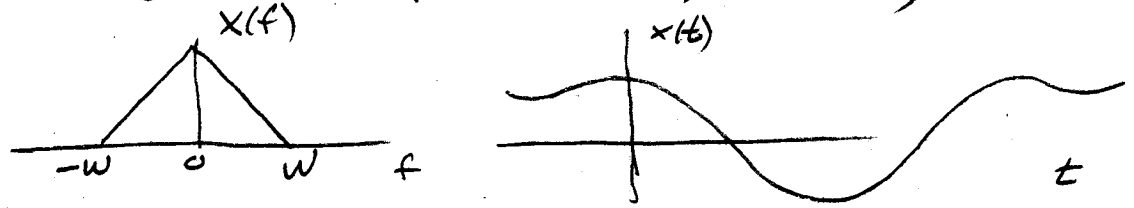
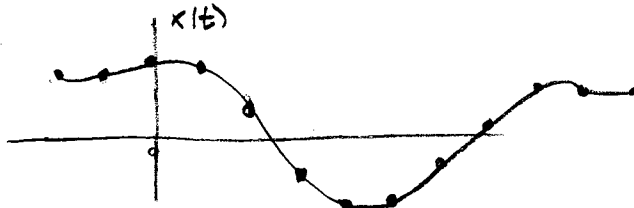


4.8 Sampling

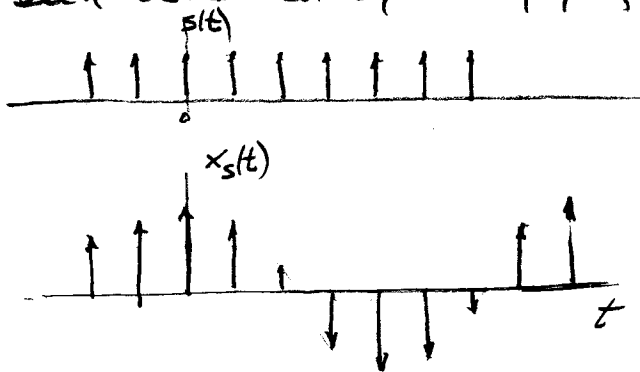
- Consider a banded function $x(t) \leftrightarrow X(f)$



- Now sample it - measure its voltage periodically



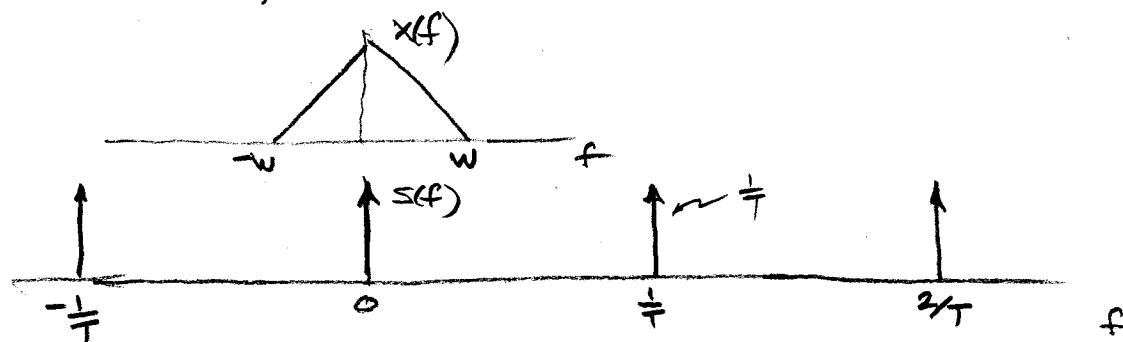
We can consider the retained information as having been obtained by multiplying $x(t)$ with impulse train $s(t)$



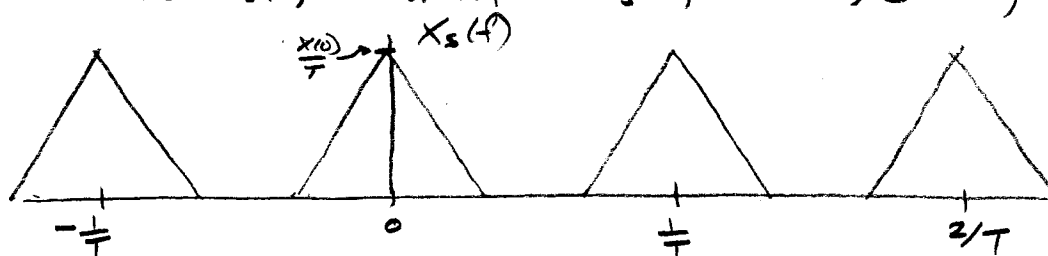
- It's clear that the sampled values give you a rough idea of the original function $x(t)$.

Actually, they give you an exact replica of $x(t)$ (if you do it right)!

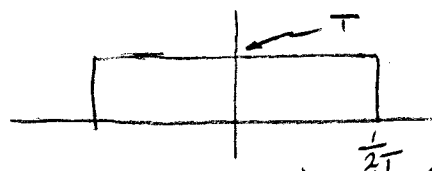
- To start, note that the two transforms are



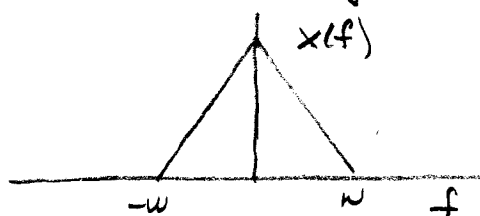
Since $x_s(t) = x(t)s(t)$ $X_s(f) = X(f) \otimes S(f)$



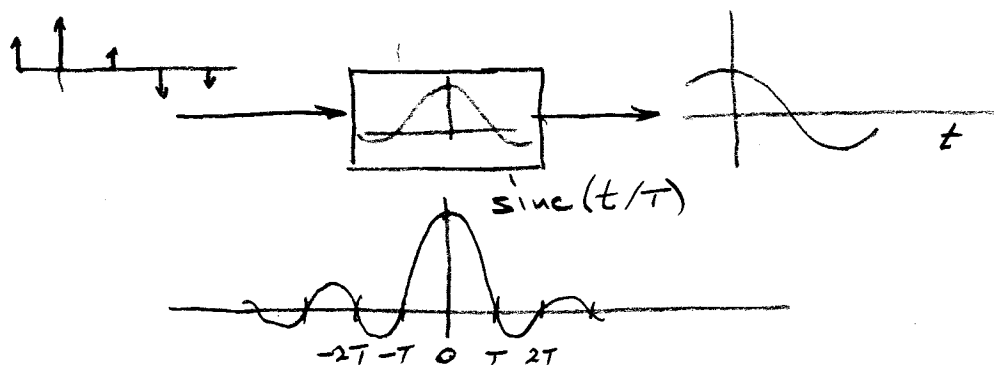
and if we apply an ideal LPF,



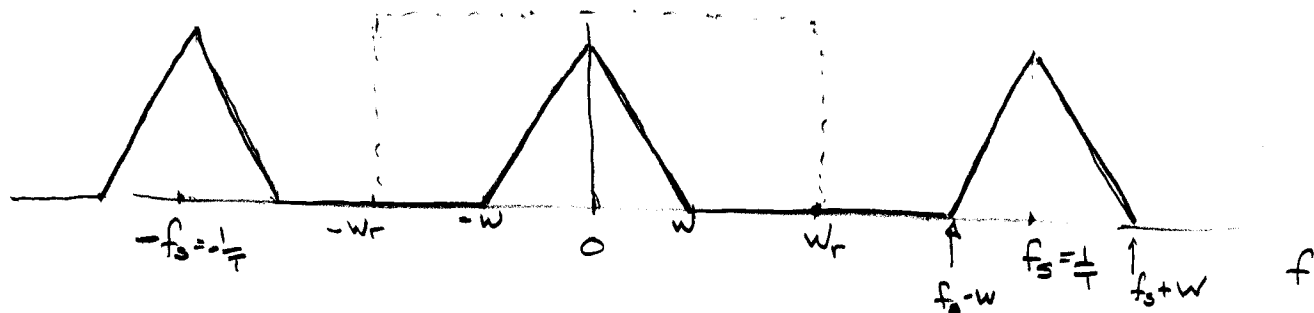
we recover the original function:



- Time domain? The weighted impulse train $x_s(t)$ drives a LPF, which interpolates between samples



- The time domain view is interesting when the "reconstruction filter" bandwidth $W_r = f_s/2 = 1/2T$

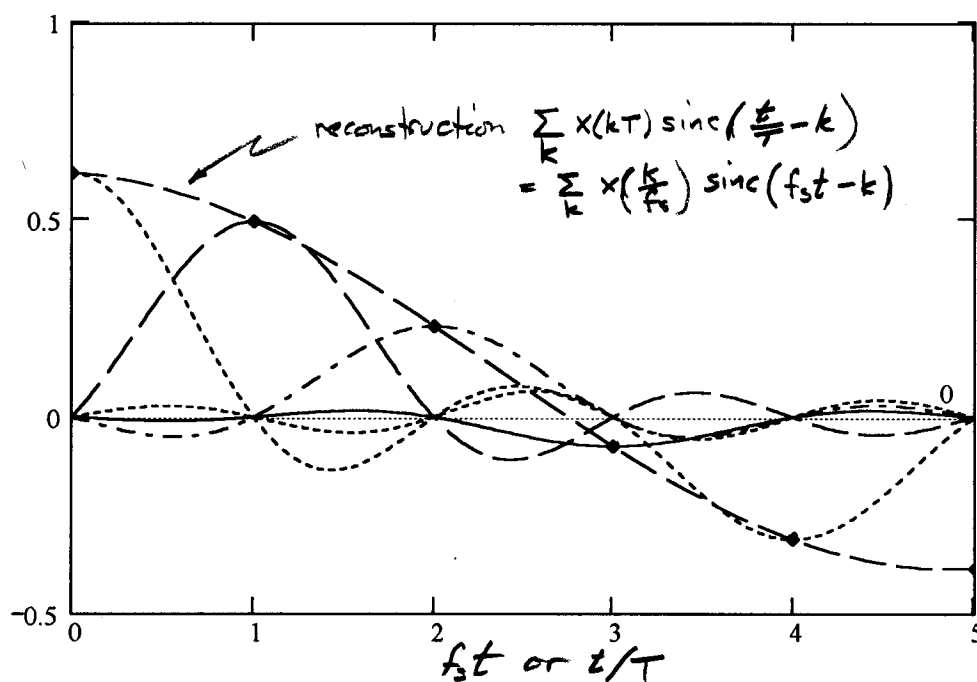


The reconstruction filter impulse response is then

$$h_r(t) = 2W_r \operatorname{sinc}(2W_r t) = f_s \operatorname{sinc}(f_s t)$$

zero crossings every $1/f_s$ sec.

Convolution of $h_r(t)$ with the data samples is shown below

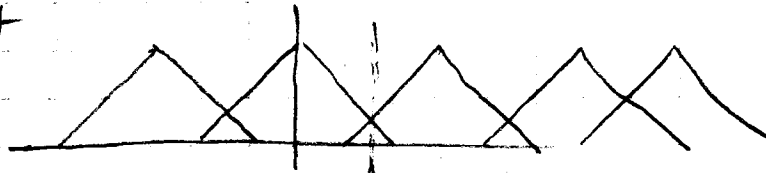


At each sample time kT , all the sinc functions are zero, except the one weighted by $x(kT)$

- It works, provided $\frac{1}{T} > 2W$

"sampling rate at least twice the highest frequency."

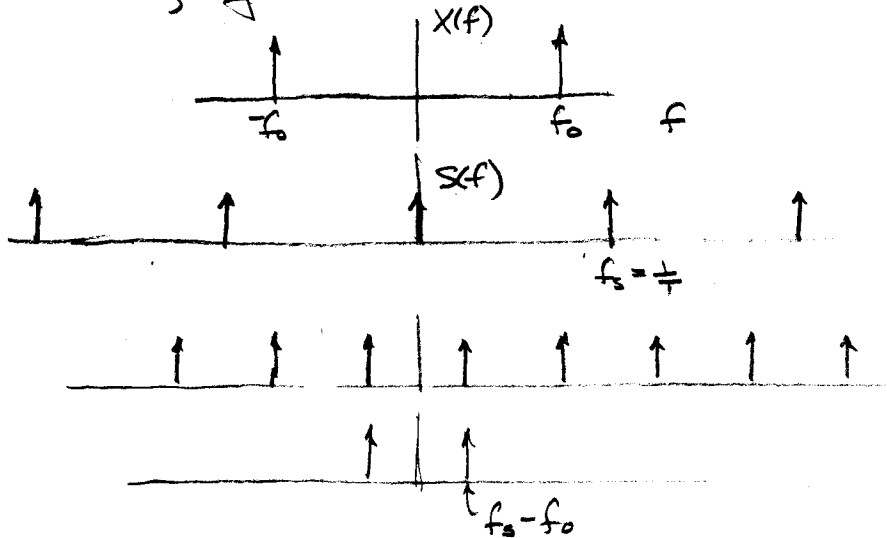
If not



"folding frequency"

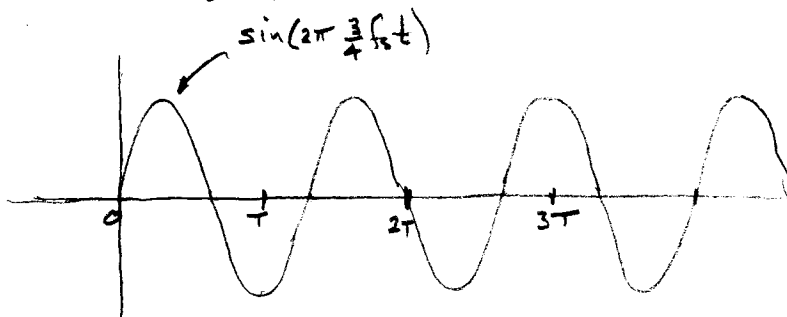
This is aliasing, in which a high frequency component is reproduced as low frequency.

Aliasing of sine wave



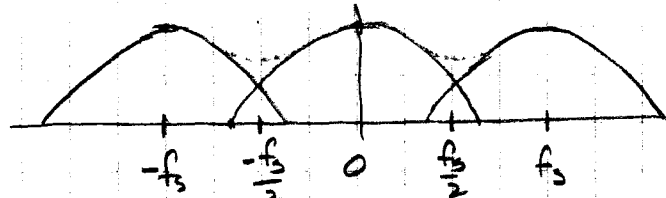
see animation
of aliasing
on the web
site

Time domain:



Antialiasing filters:

- What if the sampling frequency is fixed, but input bandwidth exceeds Nyquist limit?

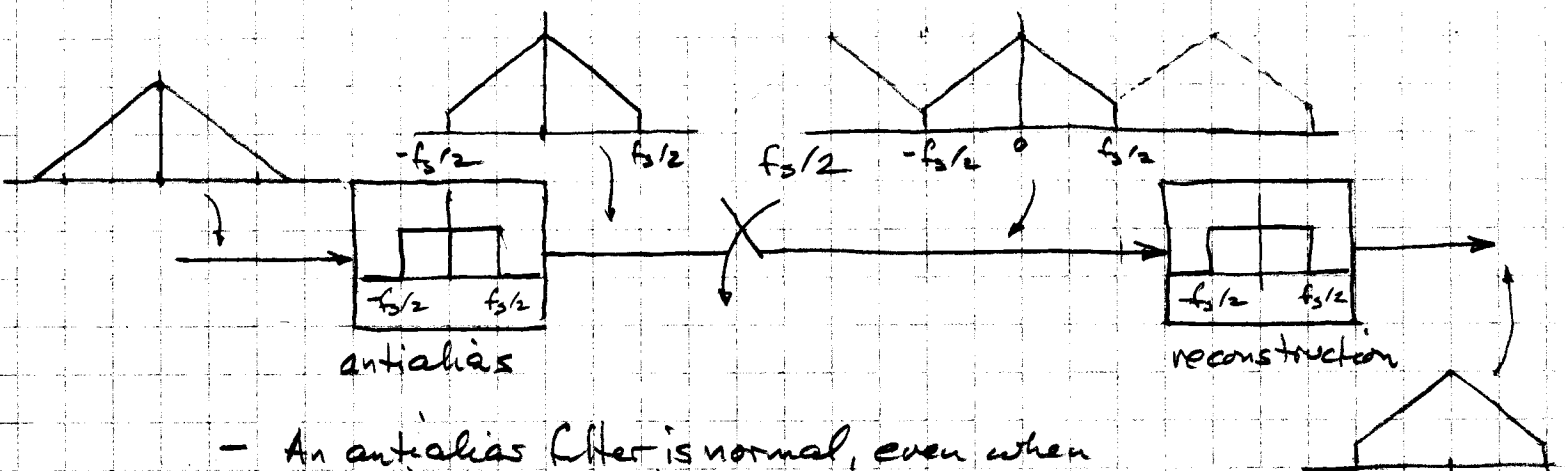


images overlap and sum, producing aliasing.



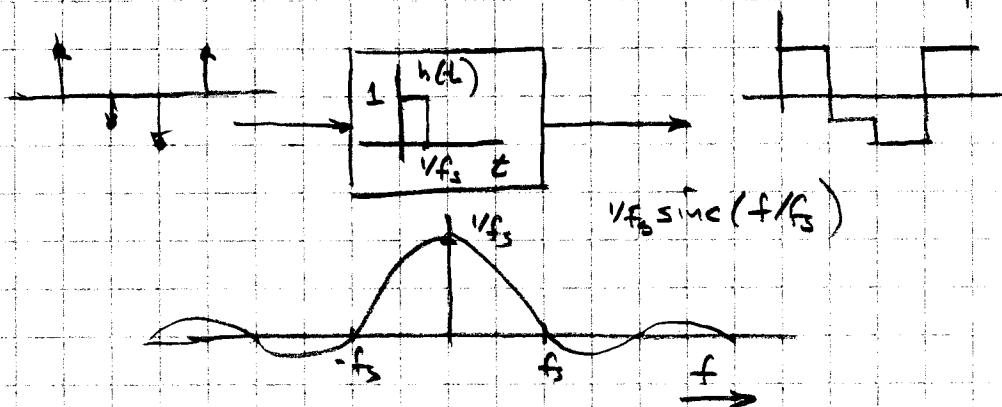
these frequency ranges are badly distorted

- The usual solution is to lowpass the input with an "antialiasing filter" before sampling. Lose the frequencies above $f_s/2$, but at least you don't poison freq's below $f_s/2$

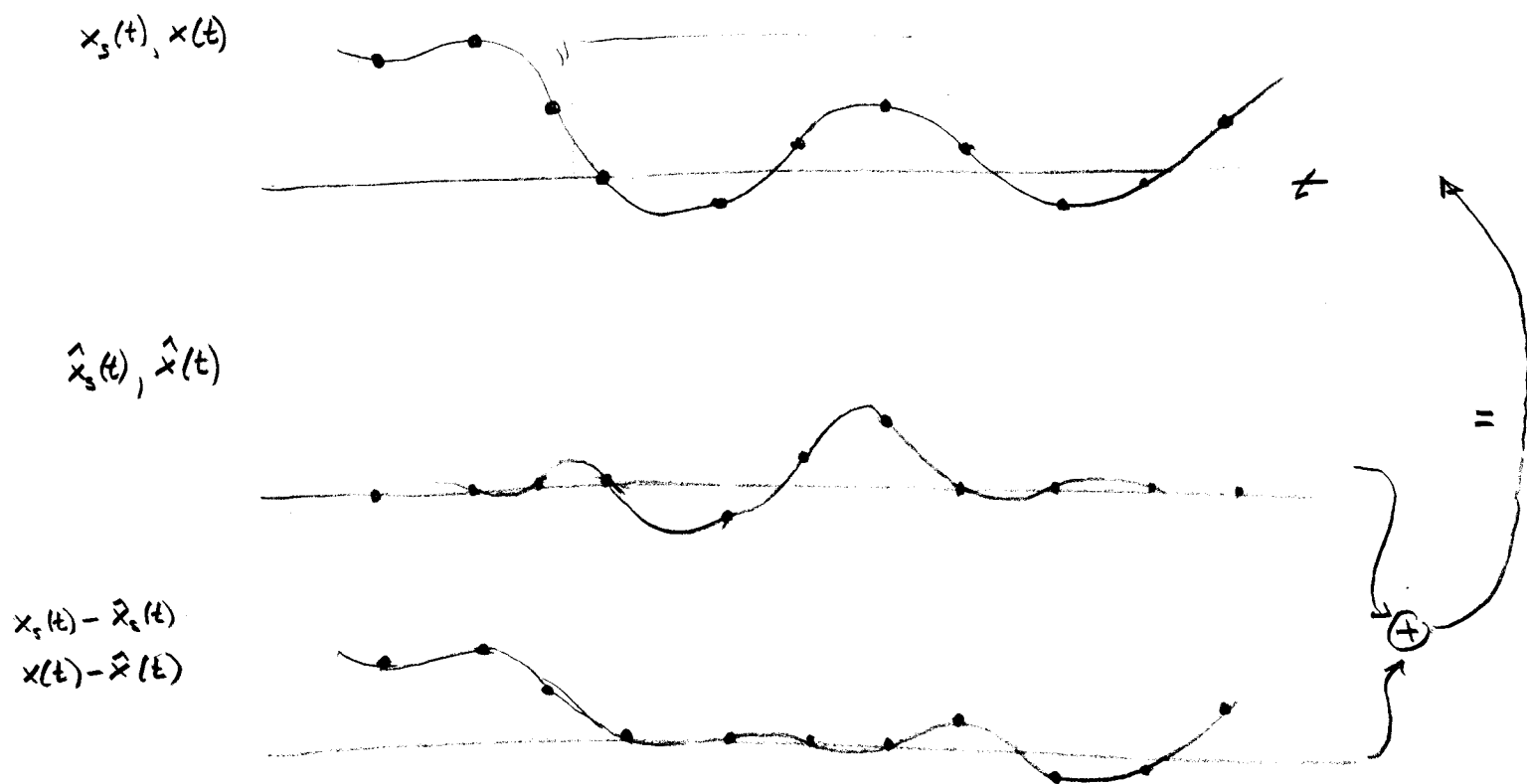


- An antialias filter is normal, even when the signal bandwidth is small enough, in order to remove any high frequency noise

- A D/A converter itself acts as a low pass (Lab 2)



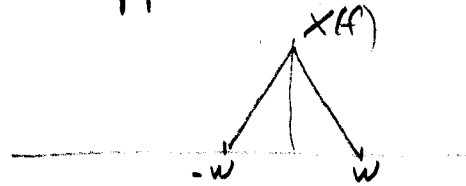
- Caveat: sampling theorem applies to infinite length sequence of samples.
- Time domain explanation.



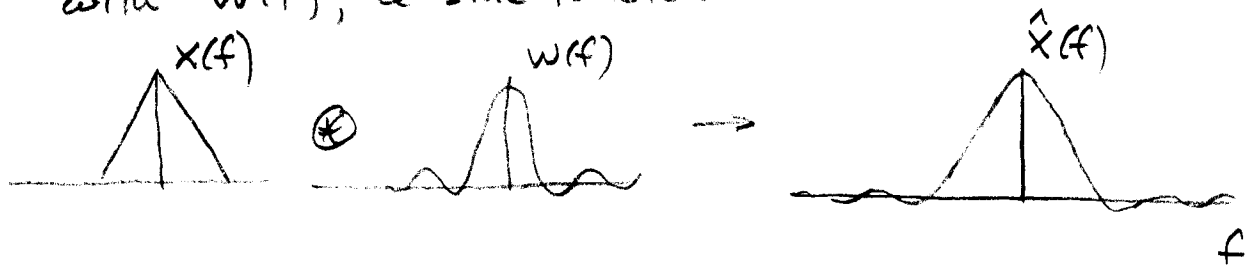
Reconstruction of the windowed portion of the signal by convolution with $\text{sinc}(f_s t)$ is distorted because we have implicitly subtracted the reconstructed version of all the omitted samples.

— Frequency domain explanation:

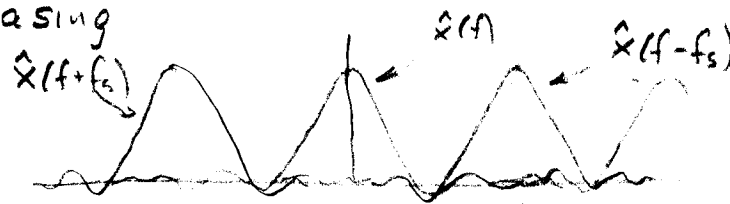
The original $X(f)$ was bandlimited, so the sampling theorem applied



Truncation (windowing) of $x(t)$ so that the record is of finite length corresponds to convolution with $W(f)$, a sinc function



Note $\hat{X}(f)$ is not bandlimited, so sampling causes aliasing



In practice, if the record is long enough ($W(f)$ is narrow enough), there is very little damage.

The distortion shows up primarily at the ends of the record, as the time domain view illustrates