6. COMPLEX SYSTEMS AND FEEDBACK CONTROL

- Many useful systems are constructed from interacting subsystems. Usually we want to know
  - overall response
  - stability
  - selected interface variables

- An important case is that of systems with feedback, either naturally or by design.

- Block diagrams with transforms provide a straightforward way to describe and analyze complex linear systems.

- We will use as examples the usual set of electrical, mechanical and thermal systems – plus a new component, the dc machine (motor, generator)

Read "Notes on Block Diagram Simplification" (website)
H&V 9.1

6.1 DC Machines

- A DC machine can act as motor or generator, depending on the way it's connected.

- It's a basic component of many servomechanisms because:
  - it is linear over a wide range, positive and negative;
  - it has reasonable ($\approx 70\%$) efficiency in electrical/mechanical power conversion.

- These notes:
  - introduce a simplified model of the DC machine;
  - illustrate and analyze some representative feedback control systems containing DC machines.

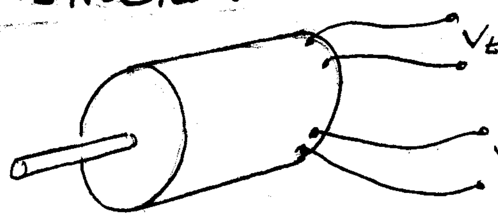
For more info read

Basic Electrical Engineering   Fitzgerald Higinbotham and Grabel

Basic Electrical Power Engineering   O I Elgard

Electric Machines   M S Sarma

- Construction



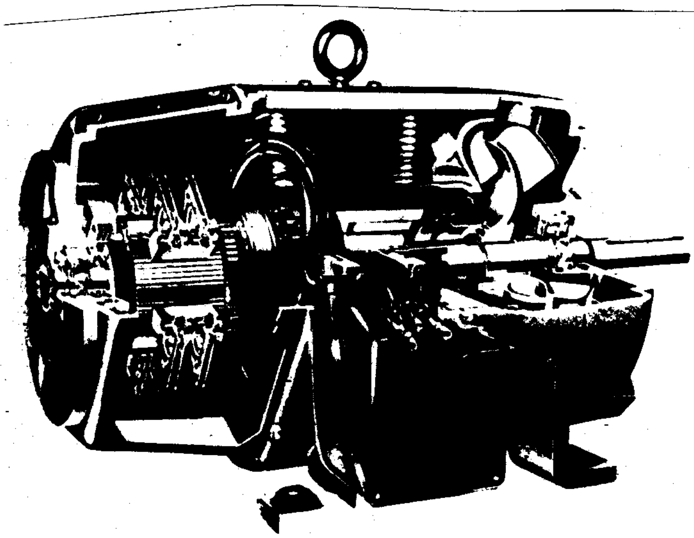


FIGURE 14-22 Cutaway view of typical dc motor. (*Westinghouse Electric Corp.*)

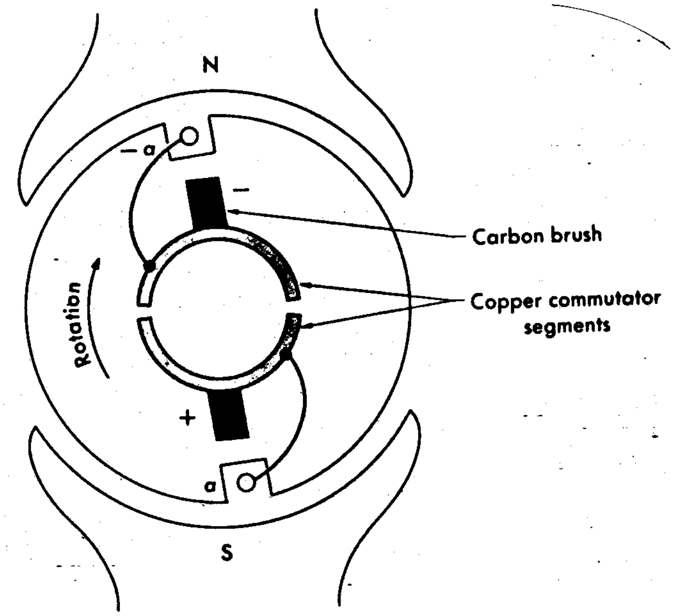


FIGURE 14-14 Elementary dc machine with commutator.

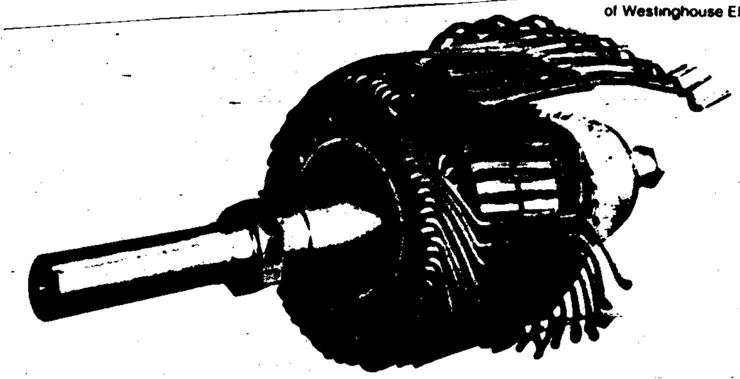
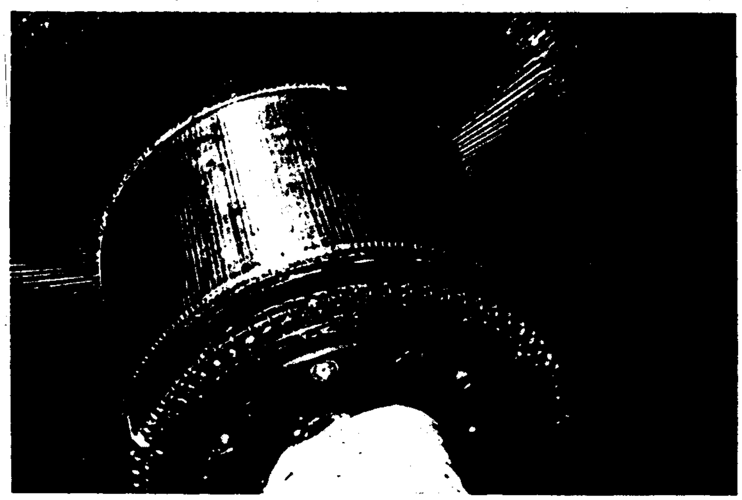
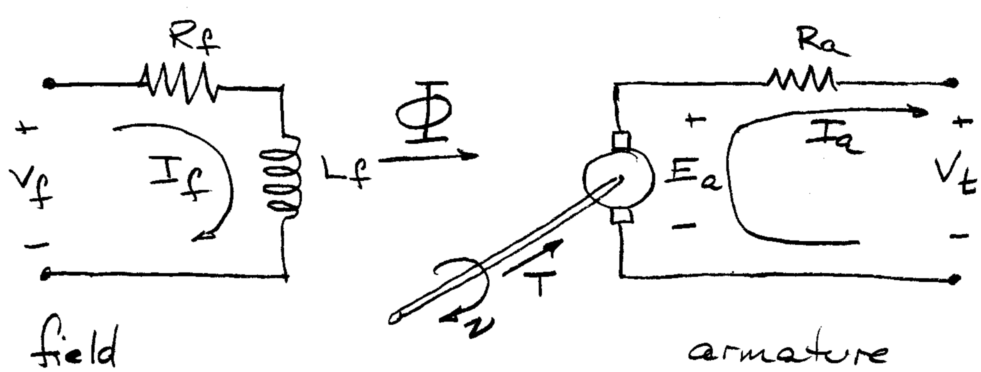


FIGURE 14-1 Direct current generator or motor armature in process of being wound. One side of each coil is placed in the bottom of a slot; the other side is placed in the top of a slot. (*General Electric Co.*)

PHOTO 5.5 Commutator of a dc machine on the rotor shaft. Photo courtesy of Westinghouse Electric Corporation.





<u>armature</u> (on the rotor for dc machine):

- windings cut lines of force (magnetic field), resulting in induced voltage E_a
- commutator ensures that E_a does not fluctuate or alternate
- windings carry current in a magnetic field, producing torque T
- conversion between electrical & mechanical power takes place here. A machine acts as motor or generator depending on terminal conditions.
- windings designed for potentially heavy current.
- losses in commutator (brush contact), and armature windings are represented by R_a.

<u>field</u> (on the stator for dc machine):

- windings just set up working field, don't contribute to power conversion
- field sometimes created by permanent magnet on small machines
- want low field current to minimize power loss in R_f, so use many turns. Lower current also means lighter wire than armature.

- Basic relationships
  - armature (the high power, rotating part)

$$E_a = K_a \Phi \nu \qquad T = K_a \Phi I_a$$

where

E_a generated emf ("back emf" if it's a motor)

ν shaft speed (rad/s)

T electromagnetic torque (i.e. before friction loss) (nt·m)

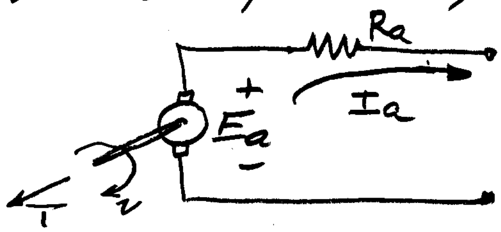
I_a armature current

Φ flux per pole (working field created by stator windings) weber

K_a constant of motor construction

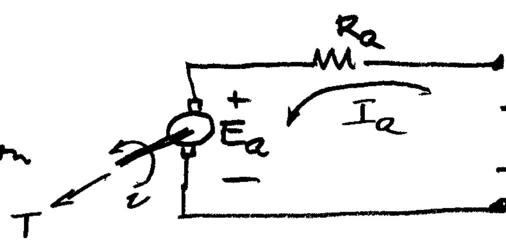
To keep most variables positive, a sign convention

generator convention



$+ \; V_t = E_a - I_a R_a \; -$

motor convention



$+ \; V_t = E_a + I_a R_a \; -$

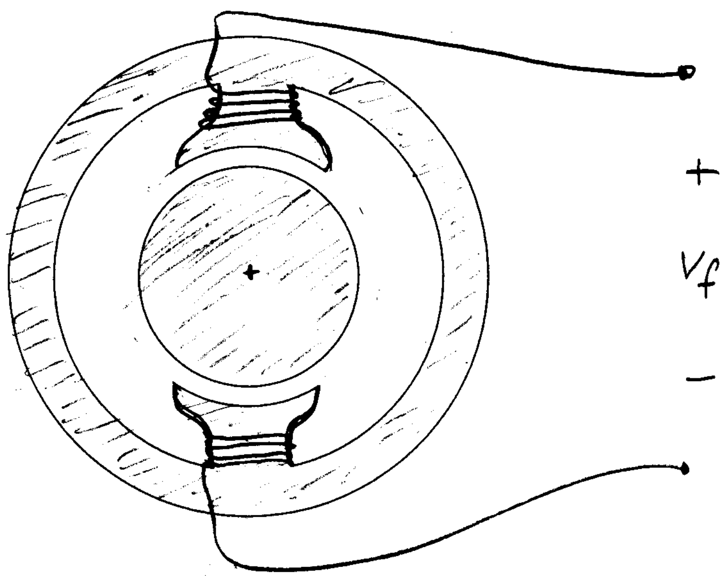
- Model as an ideal power converter

$$\text{mech'l pwr } P_m = T\,\omega = \left(K_a \Phi I_a\right)\left(\frac{E_a}{K_a \Phi}\right)$$

$$= I_a E_a = P_e \quad \text{electrical power}$$

losses in friction, R_a

- the field (for a dc machine this is on the stator)
  - small machines often have a permanent magnet to create the field
  - larger ones use field windings



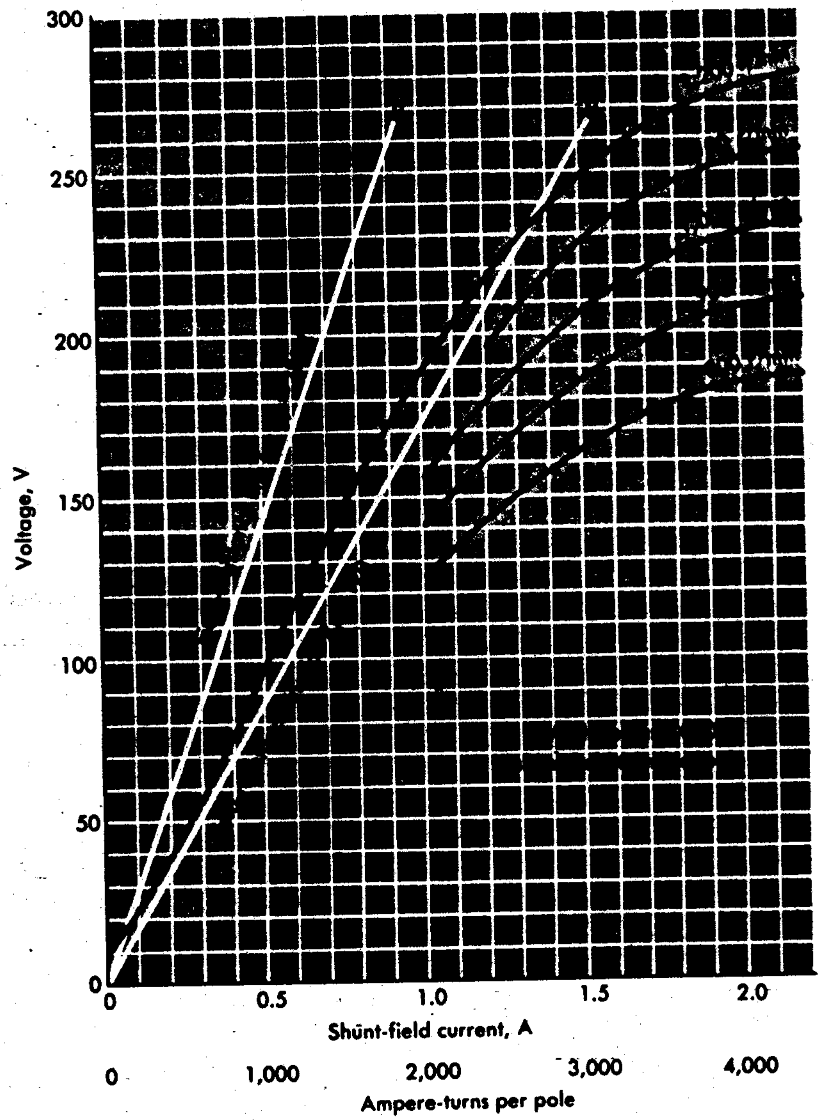
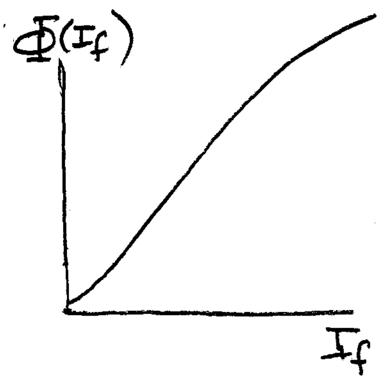


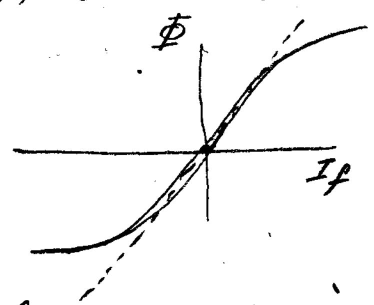
FIGURE 15-2 Magnetization curve for a 230-V 1,200 r/min machine, typical for a 15-kW generator or 15-hp motor.



$I_f = \frac{V_f}{R_f}$ in st. st.

inductance L_f creates time constant L_f/R_f

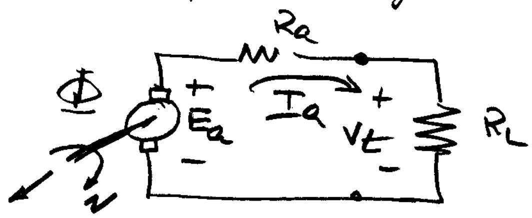
Often linearized as $\Phi \cong K_p I_f$, so $E_a \simeq K_a K_p I_f n$



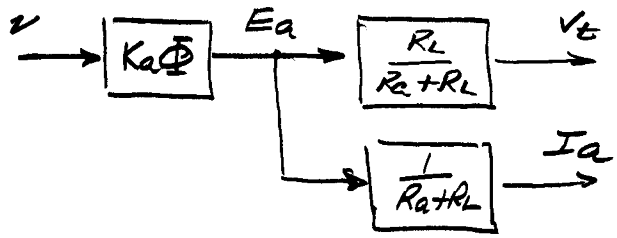
field model, including inductance, is

$$V_f \longrightarrow \boxed{\frac{1}{R_f + sL_f}} \xrightarrow{I_f} \boxed{K_p} \xrightarrow{\Phi} \quad \text{or} \quad V_f \longrightarrow \boxed{\frac{K_p}{R_f + sL_f}} \xrightarrow{\Phi}$$

- example – constant field dc generator
  - output voltage depends on shaft speed



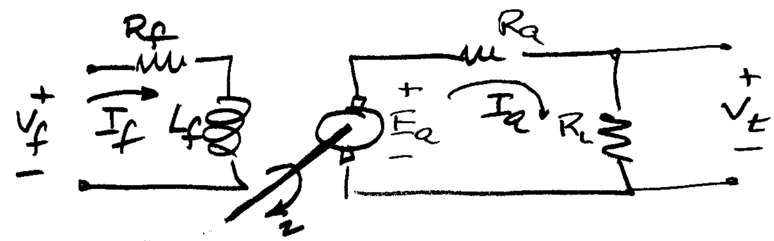
- take ν as input

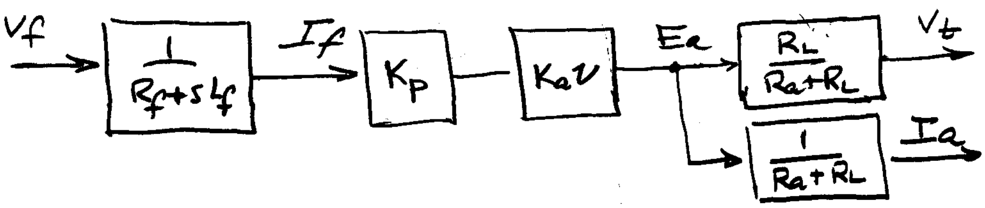


we can calculate
- power to load
- power dissipated
- efficiency
- electromagnetic torque
- P_m after mech'l losses.

$V_t(s) =$

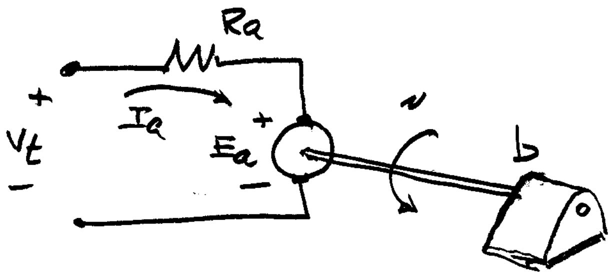
- example – dc generator, constant speed
  - output voltage depends on field voltage
  - linearize the field





$V_t(s) =$

- example – dc motor, fixed field
  - viscous friction load, no inertia

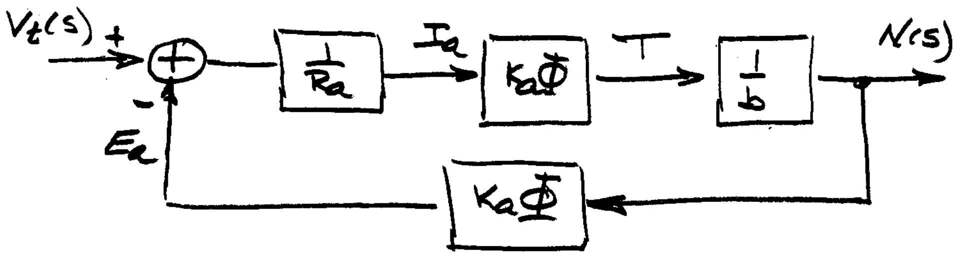


- current I_a depends on shaft speed through E_a
- shaft speed depends on I_a through T
- simultaneous equations

$$V_t = I_a R_a + K_a \Phi \nu$$

$$b\nu = K_a \Phi I_a$$

- or by diagram

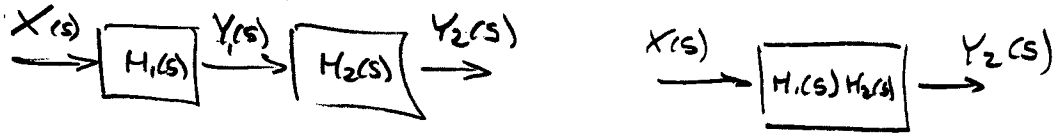


- a loop?
- we saw a loop before in the PLL. we'd better study these diagrams before continuing

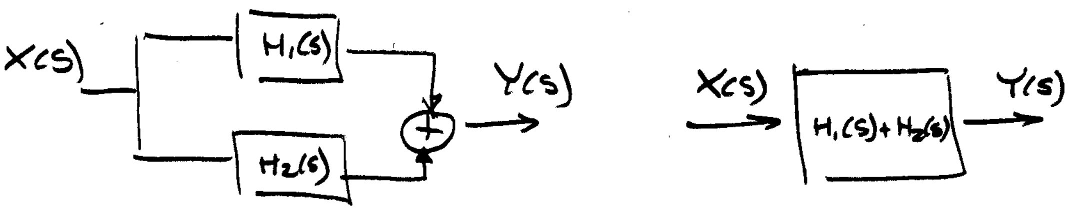
6.2 Block Diagrams

- Represent subsystems by blocks containing their transfer function. Connect using the interface variables.

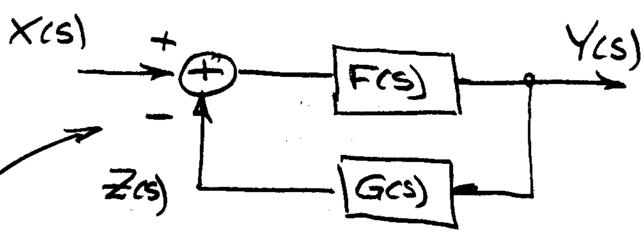
- Simplify by combining cascades



and adding parallel branches



- Feedback



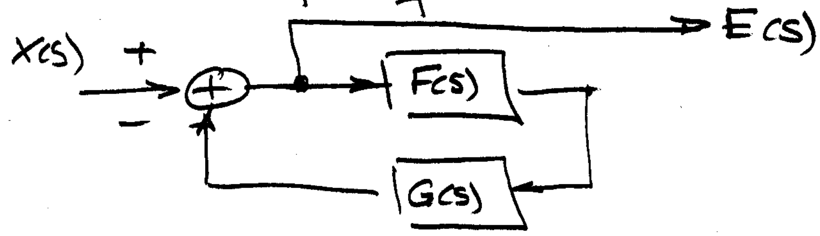
note signs

$$Y(s) = F(s)\big(X(s) - Z(s)\big) = F(s)\big(X(s) - G(s)\,Y(s)\big)$$

$$Y(s) = \frac{F(s)}{1 + F(s)\,G(s)}\,X(s)$$

$$= \frac{\text{forward gain}}{1 + \text{loop gain}}\,X(s)$$

an example of feedback



$$E(s) = \frac{1}{1 + F(s)G(s)} X(s)$$

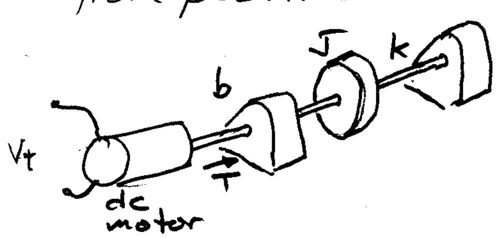
forward gain is 1 here.

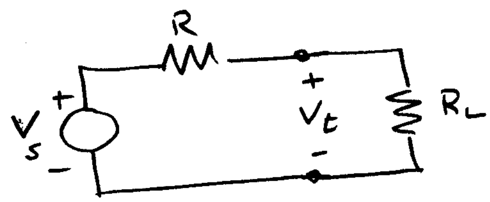
- connections with loading
  - we have become used to concatenating blocks and multiplying transfer functions

$$\rightarrow \boxed{H_1} \rightarrow \boxed{H_2} \rightarrow \boxed{H_3} \rightarrow$$

but this is appropriate only when the following system(s) do not load prior ones — loading affects the interface variable

from p. 5.7.2



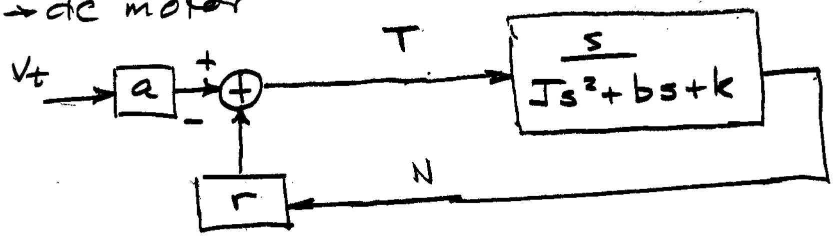


the load causes the motor to slow down, hence not a simple cascade

additional load (increased current, reduced R_L) causes V_t to sag

– To represent loading, use a pair of variables at the interface.

→ dc motor

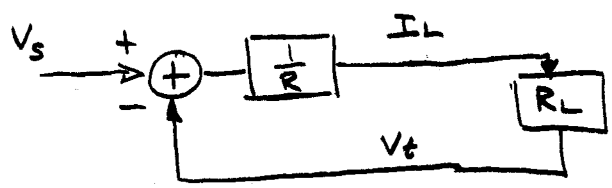


$$\frac{N(s)}{V_t(s)} = a \frac{\frac{s}{Js^2+bs+k}}{1+\frac{rs}{Js^2+bs+k}}$$

$$= \frac{as}{Js^2+(b+r)s+k}$$

agrees w p. 5.7.2 . what about $\frac{T(s)}{V_t(s)}$?

→ similarly the electric circuit

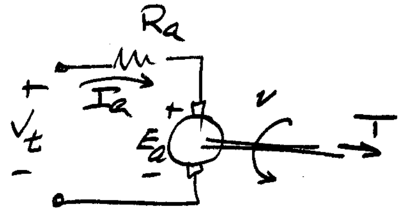


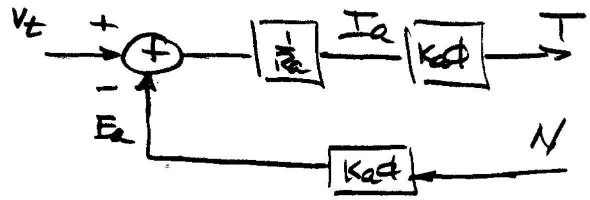
$$\frac{V_t(s)}{V_s(s)} = \frac{R_L/R}{1+R_L/R}$$

$$= \frac{R_L}{R_L+R}$$

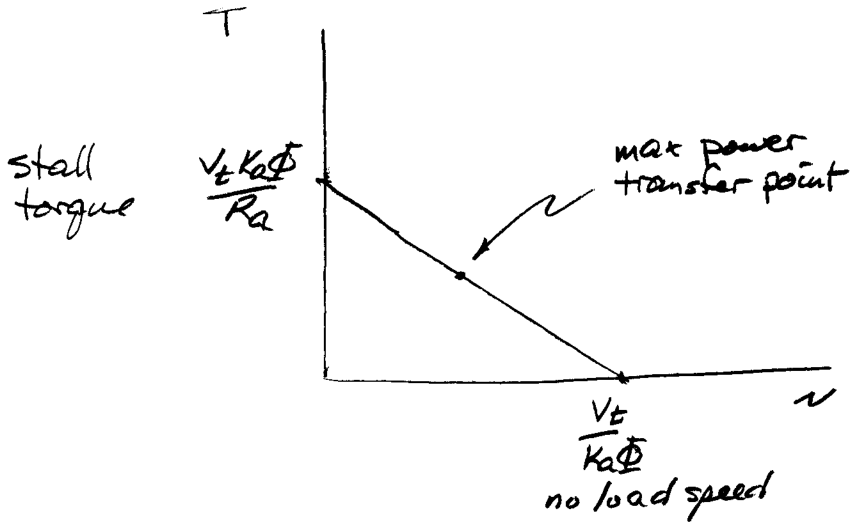
- Revisit the dc motor example, using what we now know of the relationships between Φ, E_a vs ν, I_a vs T.

We want the torque vs speed characteristic:

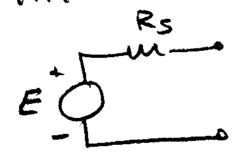




$$T = K_a\Phi\left(\frac{V_t - K_a\Phi\nu}{R_a}\right) = \frac{K_a\Phi}{R_a}V_t - \frac{(K_a\Phi)^2}{R_a}\nu$$



max power is just like Thevenin source



but it's a poor choice of operating point, since equal pwr is dissipated in armature windings

- What is the effect of doubling the field?

- A static (non-dynamic) load curve