

6.4 Feedback Control

M & V 9.2-9.4, 9.8

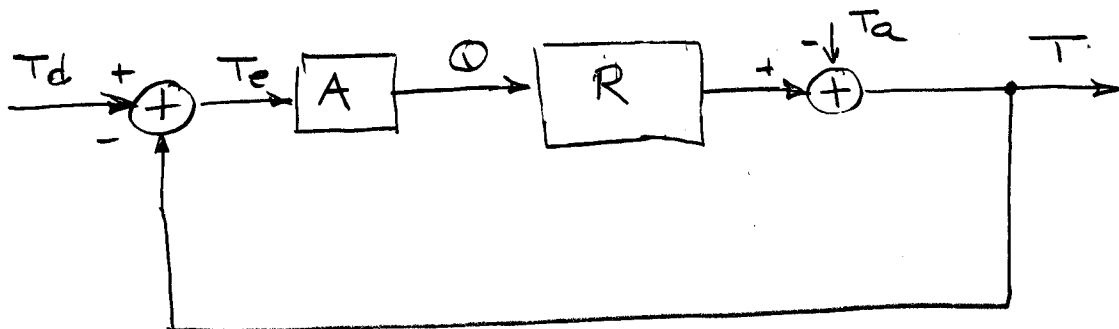
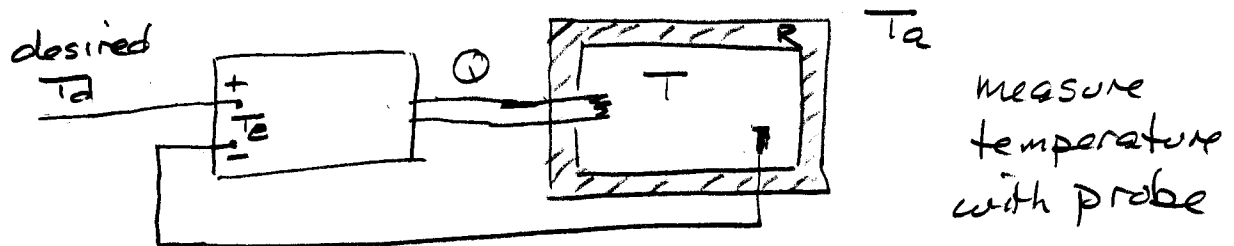
6.4.1

- Feedback can improve the response of systems compared to open loop structures:
 - improved regulation - little variation in the output even if large changes in parameters
 - faster response

On the other hand

- more complex, more expensive
- possible instability in some systems

- First example - the incubator



— superposition and collapsing loops gives

$$T = \frac{AR}{1+AR} T_d - \frac{1}{1+AR} T_a$$

If $AR \gg 1$, then $T \approx T_d$

Regulation in response to ambient changes is now

$\frac{1}{1+AR} \ll 1$ compared to 1 in open loop.

Is T sensitive to the actual value of AR ?

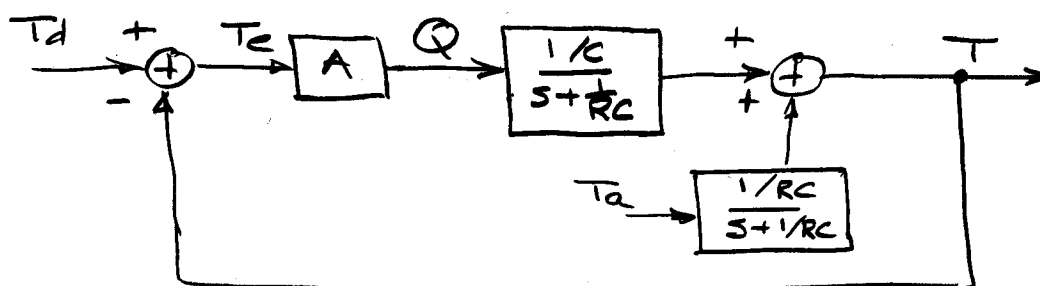
$$\frac{dT}{dAR} \cdot \frac{AR}{1+AR} = \frac{1}{(1+AR)^2} \ll 1$$

little dependence

$$S_{AR} = \frac{1+AR}{AR} \cdot AR \cdot \frac{1}{(1+AR)^2} = \frac{1}{1+AR} \ll 1$$

— Note that T_e must be non zero in this system in order to have non zero Q — hence output doesn't exactly equal input

— Now look at the dynamics



$$T = \frac{\frac{A/C}{s + 1/RC}}{1 + \frac{A/C}{s + 1/RC}} T_d + \frac{1/RC}{s + 1/RC} \cdot \frac{1}{1 + \frac{A/C}{s + 1/RC}} T_a$$

$$= \frac{A/C}{s + \left(\frac{1}{RC} + \frac{A}{C}\right)} T_d + \frac{1/RC}{s + \left(\frac{1}{RC} + \frac{A}{C}\right)} T_a$$

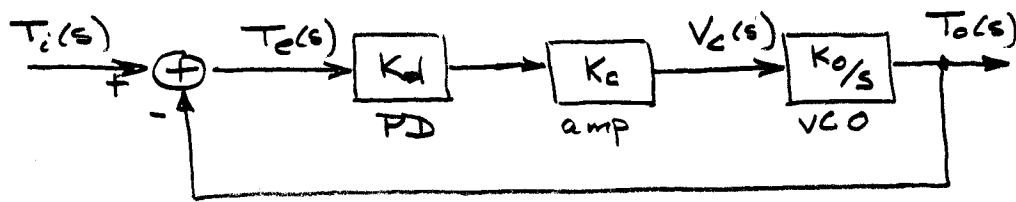
new time constant

$$\tilde{\tau} =$$

Why does it respond more quickly?

- Large A means any change in T_e causes a jump in Q , driving system quickly back to $T \approx T_d$

• Second example - the PLL



- The VCO model $\xrightarrow{V_c(t)} \left[K_o \int^t \right] \xrightarrow{Z_o(t)}$

$$\dot{z}_o(t) = K_o V_c(t)$$

$$s T_o(s) = K_o V_c(s) \quad \text{or} \quad T_o(s) = \frac{K_o}{s} V_c(s)$$

- output timing response

$$T_o(s) = \frac{K_d K_c K_o / s}{1 + K_d K_c K_o / s} T_i(s) = \frac{K_d K_c K_o}{s + K_d K_c K_o} T_i(s)$$

time const $\alpha =$

- timing error response

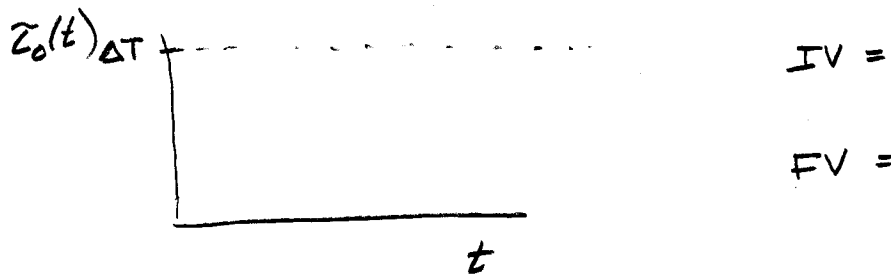
$$T_e(s) = \frac{1}{1 + K_d K_c K_o / s} T_i(s) = \frac{s}{s + K_d K_c K_o} T_i(s)$$

- Consider the response to a step change in input timing:

$$T_i(s) = \frac{\Delta T}{s}$$

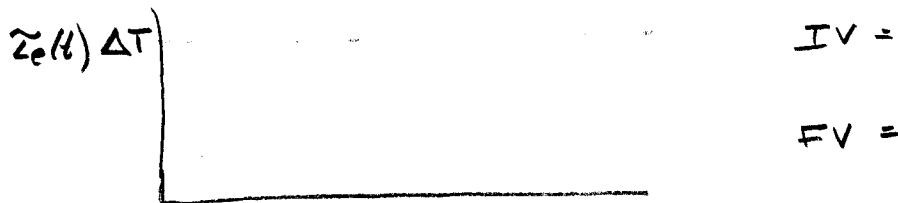
$$\text{so } T_o(s) = \frac{\Delta T}{s} \cdot \frac{K_d K_c K_o}{s + K_d K_c K_o}$$

To get $Z_o(s)$, invert the transform
or simply note that it's the step response
of a first order lowpass.



and the corresponding error response

$$T_e(s) = \frac{\Delta T}{s} \cdot \frac{s}{s + K_d K_c K_o}$$



- No steady state error ($z_e(\infty) = 0$), unlike the previous system. Why?

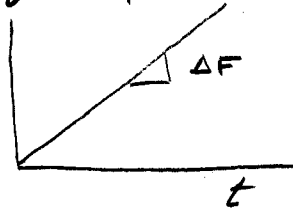
There's an integrator in the loop. The output tuning $z_o(t)$ can maintain its new value even if v_c is zero (in fact, if $v_c \neq 0$, then it can't keep its tuning constant). The control $v_c = 0$ if $z_e = 0$, that is, no error.

Here the gain K_c is not needed to make the final error small — it simply adjusts the speed of response

$$\alpha = \frac{1}{K_d K_c K_o}$$

- what if there is a frequency offset at the transmitter,

so $z_i(t) = \Delta F t$



?

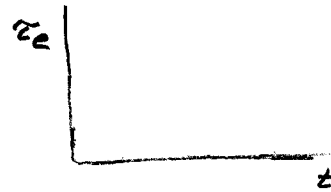
Then $T_i(s) = \frac{\Delta F}{s^2}$

and $T_e(s) = \frac{\Delta F}{s^2} \cdot \frac{s}{s + 1/\alpha} = \frac{\Delta F}{s(s + 1/\alpha)}$

IV =

FV =

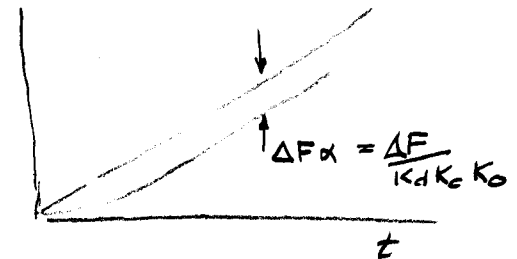
so $z_e(t) = \Delta F \alpha (1 - e^{-t/\alpha})$



Also, the output phase is

$T_o(s) = \frac{\Delta F}{s^2} \cdot \frac{1/\alpha}{s + 1/\alpha}$

$z_o(t) = \Delta F (t - \alpha(1 - e^{-t/\alpha}))$



- what if the input timing has sinusoidal swings?

$z_i(t) = \Delta T \sin(2\pi f_i t) = \Delta T \sin(\omega_i t)$

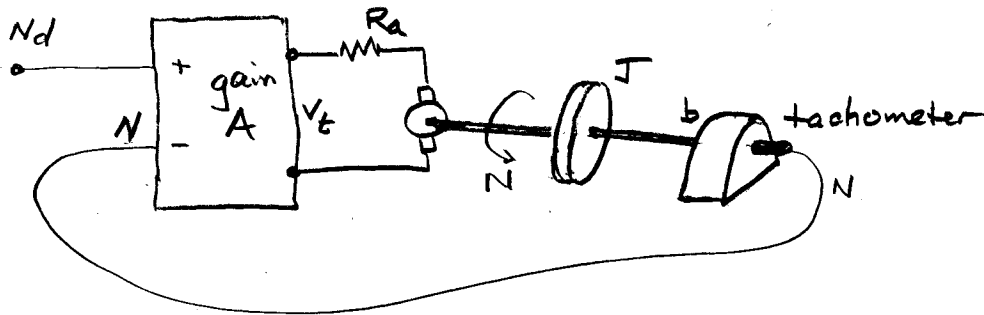
$T_i(s) = \Delta T \frac{\omega_i}{s^2 + \omega_i^2}$

so $T_e(s) = \frac{\Delta T \omega_i}{s^2 + \omega_i^2} \cdot \frac{s}{s + 1/\alpha}$

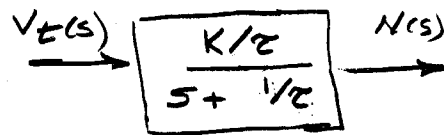
Invert by partial fractions or residues

$z_e(t) = \frac{\Delta T \omega_i \alpha}{1 + \omega_i^2 \alpha^2} \left(\cos(\omega_i t) + \omega_i \alpha \sin(\omega_i t) - e^{-t/\alpha} \right)$

• Third example - speed control of dc motor



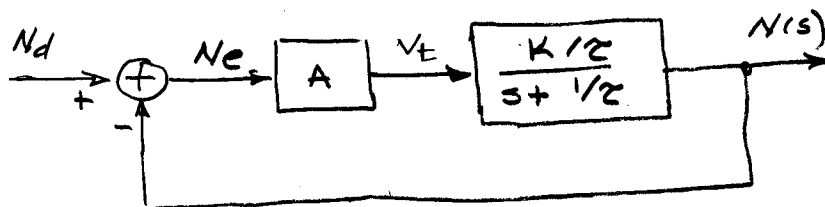
- From section 6.3, the motor and load are



$$\text{where } K = \frac{K_a \Phi}{R_a b + (K_a \Phi)^2}$$

$$\tau = \frac{R_a J}{R_a b + (K_a \Phi)^2}$$

- So the control system is



looks like
the thermal
system

$$N(s) = \frac{\frac{AK/\tau}{s + 1/\tau}}{1 + \frac{AK/\tau}{s + 1/\tau}} \cdot Nd(s) = \frac{AK/\tau}{s + \frac{1}{\tau}(1 + AK)} \cdot Nd(s)$$

- DC gain = $\frac{AK}{1 + AK} \approx 1$ if $AK \gg 1$ good regulation

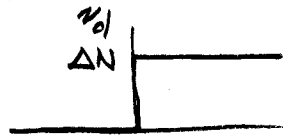
Again we need a steady state error
to produce V_t so it keeps turning.

- time constant is $\frac{\tau}{1+AK}$ faster than the original

- Is it reasonable to improve it further by making A huge? Consider the terminal voltage V_t

$$V_t(s) = \frac{A}{1 + \frac{AK/\tau}{s + 1/\tau}} \cdot N_d(s) = \frac{A(s + 1/\tau)}{s + \frac{1}{\tau}(1+AK)} \cdot N_d(s)$$

Suppose the desired speed setting $n_d(t)$ jumps by ΔN



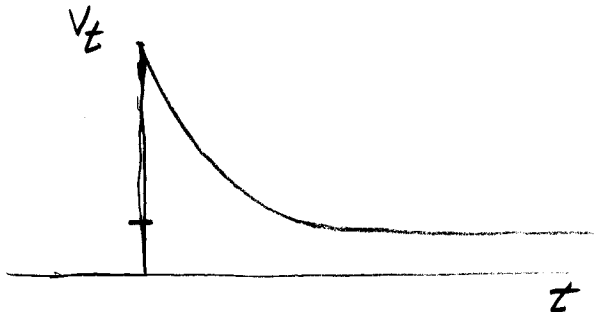
$$\text{so } N_d(s) = \frac{\Delta N}{s}$$

Then

$$V_t(s) = \frac{A(s + 1/\tau)}{s + \frac{1}{\tau}(1+AK)} \cdot \frac{\Delta N}{s}$$

$$IV =$$

$$FV =$$



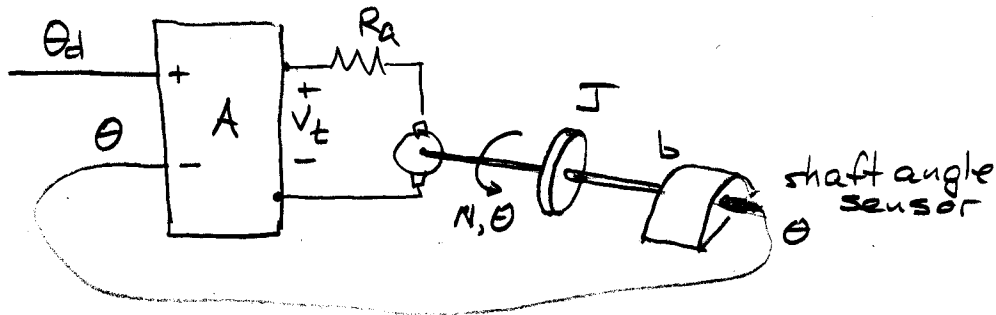
This huge spike in V_t may not be

- possible (how to generate it?)

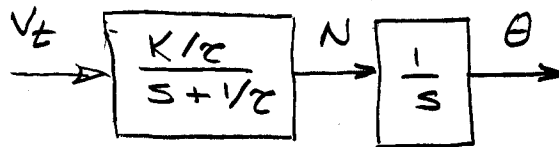
- desirable (armature current, shaft torque)

but what if we need large A to keep the steady state error small?

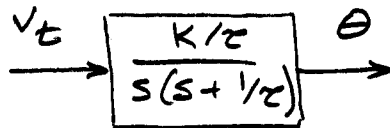
- Fourth and last example — position control



- The motor and load are

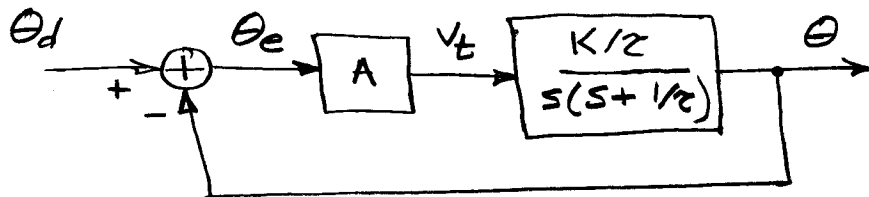


$$K = \frac{k_a \Phi}{R_a b + (k_a \Phi)^2}$$



$$\tau = \frac{R_a J}{R_a b + (k_a \Phi)^2}$$

- So the control system is

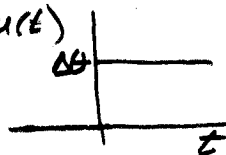


$$\Theta(s) = \frac{\frac{AK/\tau}{s(s + 1/\tau)}}{1 + \frac{AK/\tau}{s(s + 1/\tau)}} \Theta_d(s) = \frac{AK/\tau}{s^2 + \frac{s}{\tau} + \frac{AK}{\tau}} \Theta_d(s)$$

Now it's a second order system!

- Before further analysis, pick off easy results:

Step input $\theta_d(t) = \Delta\theta u(t)$



$$\theta_d(s) = \frac{\Delta\theta}{s}$$

$$\theta(s) = \frac{\Delta\theta}{s} \frac{AK/\tau}{s^2 + \frac{s}{\tau} + \frac{AK}{\tau}}$$

$$Iv =$$

$$Ev =$$

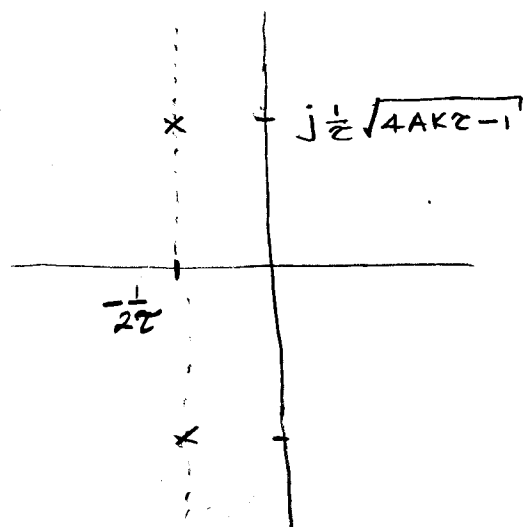
so the integrator in the loop again makes the final error zero.

- Nature of the response? Under or overdamped?

$$\omega_0 = \sqrt{\frac{AK}{\tau}} \quad \zeta = \frac{1}{2\sqrt{AK\tau}} \quad (\text{since } 2\zeta\omega_0 = \frac{1}{\tau})$$

The gain A affects both of them, so we can't choose them independently:

- want ω_0 large for fast response $\Rightarrow AK$ large
- if AK too large, then very underdamped.



increasing A moves poles out along the dotted line

Step response of the position control system:

motor constants $\tau := 1 \cdot \text{sec}$ $K := 20 \cdot \text{Hz}$

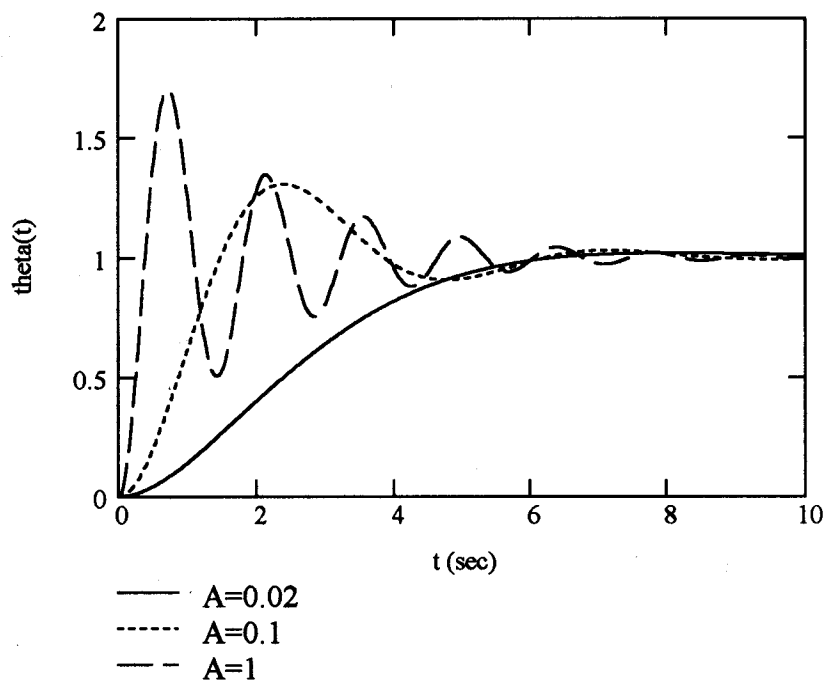
normalized closed loop parameters

$$\omega_o(A) := \sqrt{\frac{A \cdot K}{\tau}} \quad \zeta(A) := \frac{1}{2 \cdot \sqrt{A \cdot K \cdot \tau}} \quad \omega_d(A) := \omega_o(A) \cdot \sqrt{1 - \zeta(A)^2}$$

unit step response

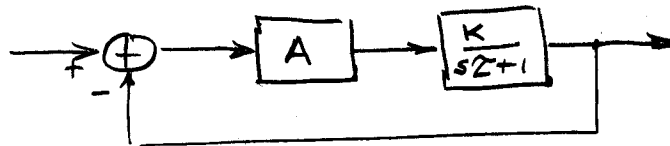
$$\theta(t, A) := 1 - e^{-\zeta(A) \cdot \omega_o(A) \cdot t} \cdot \left(\cos(\omega_d(A) \cdot t) + \frac{\zeta(A)}{\sqrt{1 - \zeta(A)^2}} \cdot \sin(\omega_d(A) \cdot t) \right)$$

$t := 0, 0.01 \dots 10$



- We've seen four examples of systems embedded in a simple feedback loop. Any general observations?

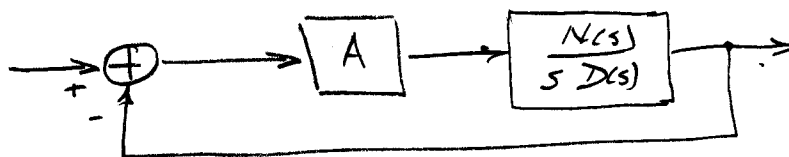
— A first order lowpass in a loop



eg. incubator speed control

- tracks well and responds quickly (still 1st order) for large A
- has a small but non zero steady state error in response to a step
- has a large jump in the drive applied to the system when the input changes

— An integrator in the loop



PLL $\rightarrow \left[\frac{K_0}{s} \right] \rightarrow$

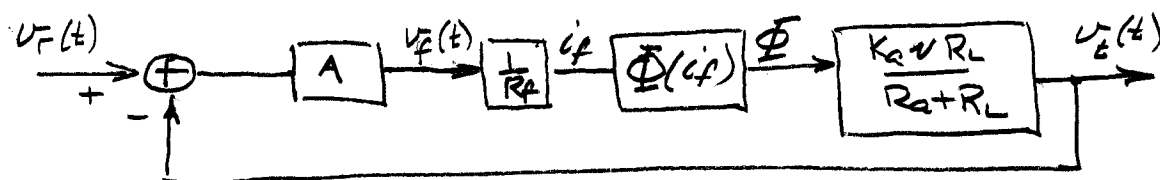
pos'n control

$\rightarrow \left[\frac{K}{s(s^2 + 1)} \right] \rightarrow$

- has zero steady state error in response to step:

transfer function $\frac{1}{1 + \frac{A N(s)}{s D(s)}} \xrightarrow{s \rightarrow 0} 0$

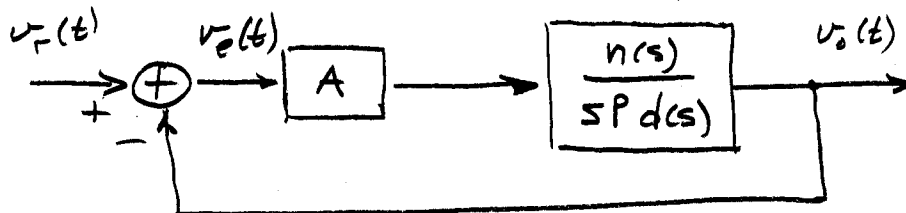
- the controller gain makes the dynamics of the closed loop system differ from those of the open loop system
- closing the loop with the wrong polarity is a bad idea
- since it operates to make the fed-back signal equal the reference signal, feedback reduces the effect of parameter variation (eg. load) and even non linearity.



A dc generator - the loop will drive V_F to the value that makes $V_E \approx V_F$.

- A classification of systems – generalise from our integrator/no integrator observations above

Denote the number of "free integrators" by p



where
 $n(0) \neq 0$
 $d(0) \neq 0$

$$\text{Then } H_e(s) = \frac{V_e(s)}{V_r(s)} = \frac{1}{1 + \frac{A n(s)}{s^p d(s)}} = \frac{s^p d(s)}{s^p d(s) + A n(s)}$$

Examine the final value $v_e(\infty)$ of the error when input is an impulse, step, ramp, parabola etc

$$1 \quad \frac{1}{s} \quad \frac{1}{s^2} \quad \frac{1}{s^3} \quad \frac{1}{s^r}$$

$$v_e(\infty) = \lim_{s \rightarrow 0} s \frac{1}{s^r} \frac{s^p d(s)}{s^p d(s) + A n(s)} = \frac{s^{p-r+1} d(s)}{s^p d(s) + A n(s)}$$

$$= \begin{cases} 0 & \text{if } p > r-1 \\ \text{finite} & \text{if } p = r-1 \\ \infty & \text{if } p < r-1 \end{cases}$$

order of input (r)	Type ($=p$)			
	0	1	2	
0	0	0	0	0
1	finite	0	0	0
2	∞	finite	0	0
3	∞	∞	finite	0

Steady state error