SIMON FRASER UNIVERSITY
School of Engineering Science
ENSC 380 Linear Systems

EXAMPLES OF CONVOLUTION IN CONTINUOUS TIME

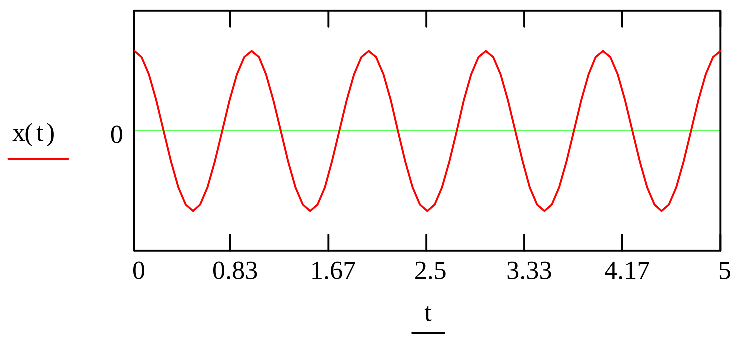
Below are three examples of convolution performed explicitly as an integration. The highlighted regions are values you might want to change and watch the effect on the filter response.

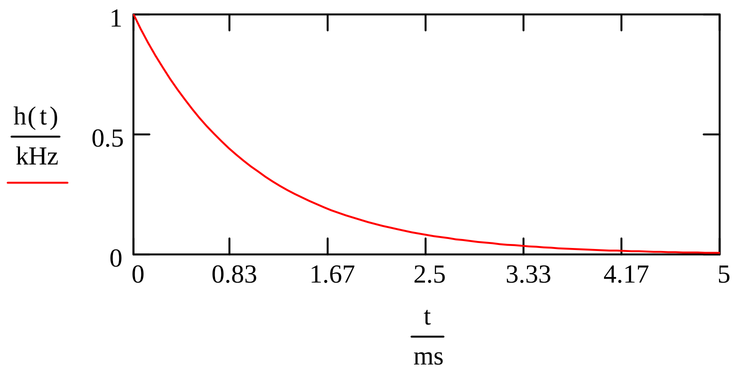
$ms := 10^{-3} \cdot sec$ (have to define this in Mathcad)

Example: First order lowpass system (e.g., an RC circuit), input is a cosine switched on at time zero. Find the response by convolution - "flip, slide and integrate"

$f_c := 1 \cdot kHz$ $\quad x(t) := \cos(2 \cdot \pi \cdot f_c \cdot t)$ $\quad \tau := 1 \cdot ms$ $\quad h(t) := \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}}$

$t := 0 \cdot ms, \frac{1}{16 \cdot f_c} .. 5 \cdot \tau$

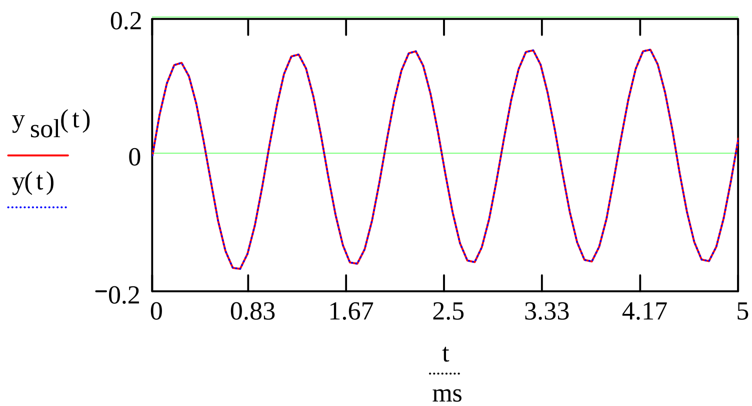




$y(t) := \int_{0 \cdot ms}^{t} x(\alpha) \cdot h(t - \alpha)\, d\alpha$ by convolution

$y_{sol}(t) := \frac{1}{1 + \tau^2 \cdot (2 \cdot \pi \cdot f_c)^2} \cdot \left(\cos(2 \cdot \pi \cdot f_c \cdot t) + 2 \cdot \pi \cdot f_c \cdot \tau \cdot \sin(2 \cdot \pi \cdot f_c \cdot t) - \exp\left(-\frac{t}{\tau}\right) \right)$

solution of differential equation



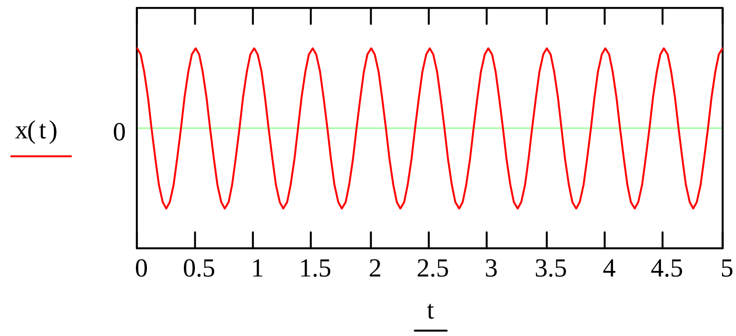
See - the convolution and the DE solution are identical (the traces fall on top of each other).

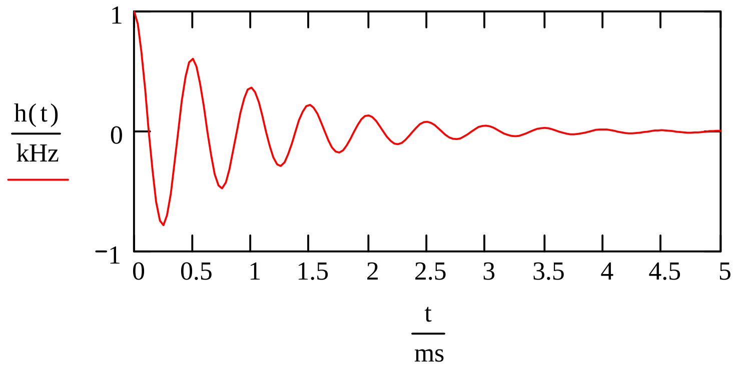
Response climbs to steady state value.

Example: sine wave input to a filter tuned to the same frequency (i.e., resonance)

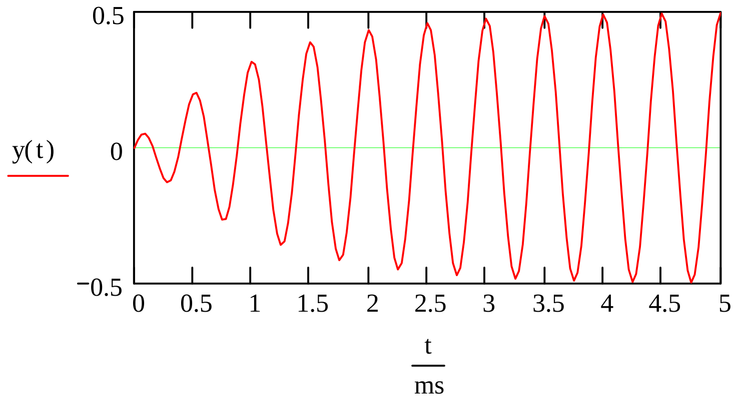
$$f_c := 2 \cdot kHz \qquad x(t) := \cos(2 \cdot \pi \cdot f_c \cdot t) \qquad \tau := 1 \cdot ms \qquad h(t) := \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}} \cdot \cos(2 \cdot \pi \cdot f_c \cdot t)$$

$$t := 0 \cdot ms, \frac{1}{16 \cdot f_c} .. 5 \cdot \tau$$





$$y(t) := \int_{0 \cdot ms}^{t} x(\alpha) \cdot h(t - \alpha) \, d\alpha$$



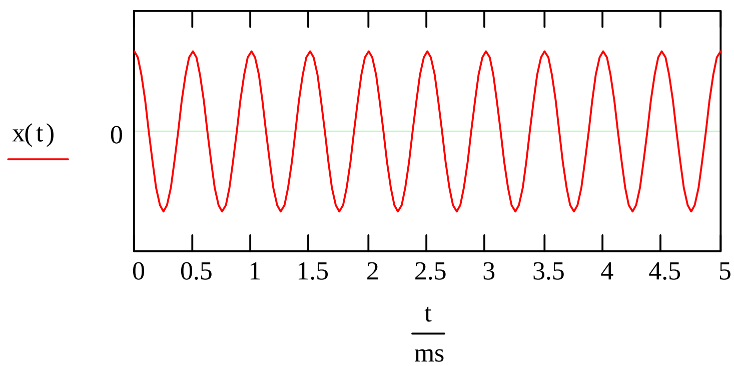
The resonant response builds during the transient portion.

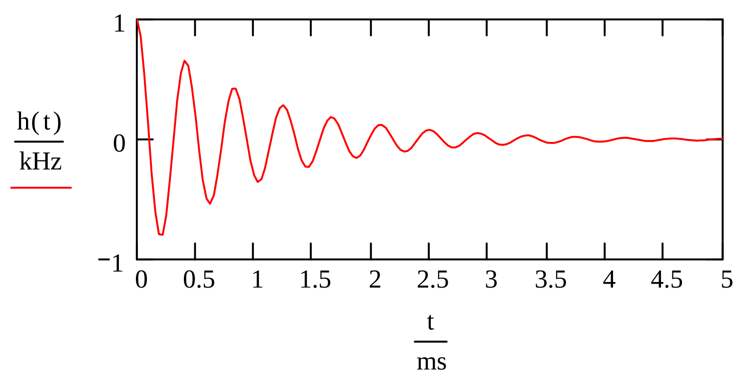
Example: sine wave input to a filter tuned to a slightly different frequency (produces beats)

$f_c := 2 \cdot kHz$ $\quad x(t) := \cos(2 \cdot \pi \cdot f_c \cdot t)$

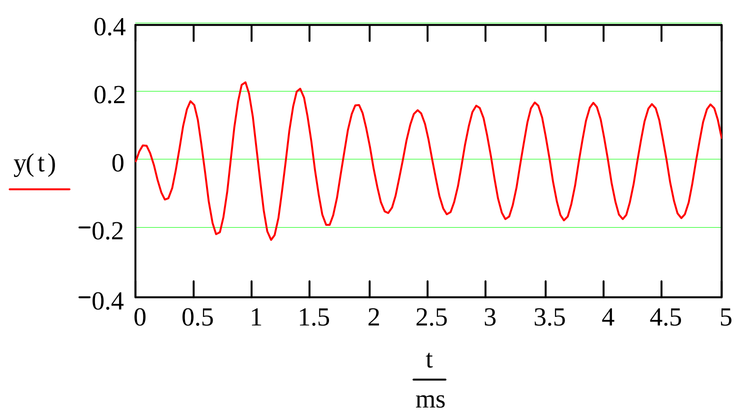
$f_o := 1.2 \cdot f_c$ $\quad \tau := 1 \cdot ms$ $\quad h(t) := \frac{1}{\tau} \cdot e^{-\frac{t}{\tau}} \cdot \cos(2 \cdot \pi \cdot f_o \cdot t)$

$$t := 0 \cdot ms, \frac{1}{16 \cdot f_c} .. 5 \cdot \tau$$





$$y(t) := \int_{0 \cdot ms}^{t} x(\alpha) \cdot h(t - \alpha)\, d\alpha$$



Some beating during the transient portion, but steady state is a sinusoid, as usual.