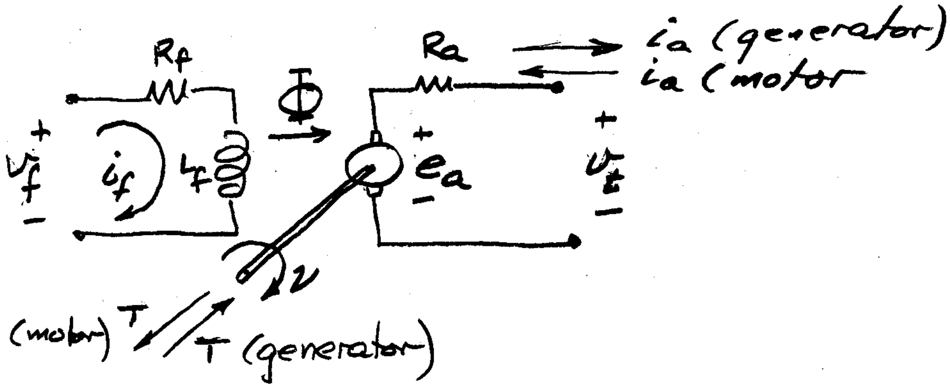
ENSC 380 Final Exam 2000-1 Summary Sheet

Laplace Transforms

$f(t),\ t \geq 0$	$F(s)$
$\delta(t)$	1
1 or $q(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n (n: positive integer)	$\frac{n!}{s^{n+1}}$
e^{-at} (a: real or complex)	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega_0 t$	$\frac{\omega_0}{s^2+\omega_0^2}$
$\cos \omega_0 t$	$\frac{s}{s^2+\omega_0^2}$
$t \sin \omega_0 t$	$\frac{2\omega_0 s}{(s^2+\omega_0^2)^2}$
$t \cos \omega_0 t$	$\frac{s^2-\omega_0^2}{(s^2+\omega_0^2)^2}$
$e^{-at} \sin \omega_0 t$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$
$e^{-at} \cos \omega_0 t$	$\frac{s+a}{(s+a)^2+\omega_0^2}$

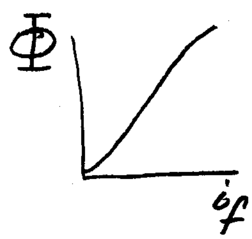
Z Transforms

Sample Values $f(nT)$ for $n \geq 0$	**z-Transform of $f(nT)$**
$f(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \triangleq \delta(n)$	1
1	$\frac{1}{1-z^{-1}}$
$e^{-\alpha nT} = (e^{-\alpha T})^n = K^n$	$\frac{1}{1-e^{-\alpha T}z^{-1}} = \frac{1}{1-Kz^{-1}}$
nT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
nTe^{-anT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
$\sin bnT$	$\frac{(\sin bT)z^{-1}}{1-2(\cos bT)z^{-1}+z^{-2}}$
$\cos bnT$	$\frac{1-(\cos bT)z^{-1}}{1-2(\cos bT)z^{-1}+z^{-2}}$
$e^{-anT} \sin bnT$	$\frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$
$e^{-anT} \cos bnT$	$\frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$



<u>armature</u> $\quad e_a = K_a \Phi \nu \qquad T = K_a \Phi i_a$

<u>field</u>



approximately $\Phi \approx K_p i_f$

<u>combined</u>

$$e_a \approx K_a K_p i_f \nu$$

$$T \approx K_a K_p i_f i_a$$

$$\sum_{k=0}^{\infty} \zeta^k = \frac{1}{1-\zeta}, \quad |\zeta|<1 \qquad \sum_{k=0}^{\infty} (k+1)\zeta^k = \frac{1}{(1-\zeta)^2}, \quad |\zeta|<1$$

N terms: $\displaystyle\sum_{k=0}^{N-1} \zeta^k = \frac{1-\zeta^N}{1-\zeta}$, any ζ

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4n^2-1} = \frac{\pi}{4} - \frac{1}{2} \qquad \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2} = \frac{\pi^2-8}{16}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad \mathrm{Re}[x]\,\mathrm{Re}[y] = \frac{1}{2}\mathrm{Re}[xy^* + xy]$$

$$\cos\theta \cos\varphi = \frac{1}{2}\left(\cos(\theta+\varphi) + \cos(\theta-\varphi)\right)$$

$$\cos\theta \sin\varphi = \frac{1}{2}\left(\sin(\theta+\varphi) - \sin(\theta-\varphi)\right)$$