2.8 Summary of Section

- From $\underline{x}$ and statistics, estimate $\underline{\theta}$. Key properties of $\hat{\underline{\theta}}(\underline{x})$:

  bias, consistency, sufficiency, effeciency

- Two cases:
  - have prior pdf $p_\theta(\underline{\theta})$ ⟶ Bayes est'n
  - don't ⟶ typ ML

- Bayes
  - min risk formulation gives cond'l mean, cond'l median, MAP estimators for three typical cost f'ns. Thus cond'l mean $E_{\theta|x}[\theta]$ always minimizes mse.
  - with symmetry of cost f'n and cond'l prob $p_{\theta|x}(\underline{\theta}|\underline{x})$ (and unimodal) they all have same value – cond'l mean
  - cond'l mean dep on $\underline{x}$, of course, but other cond'l moments, such as cond'l variance (est'n error variance if $\hat{\underline{\theta}}(\underline{x}) = \underline{\mu}(\underline{x})$) may dep on $\underline{x}$. For linear est'n, not.
  - linear est'n with quadratic criterion uses only 1st & 2nd order stats, and it's optimum for Gaussian
  - linear estimation

$$\hat{\underline{\theta}} = E[\underline{\theta}\,\underline{x}^+]\,E[\underline{x}\,\underline{x}^+]^{-1}\underline{x} \qquad \text{a projection.}$$

has a counterpart in all vector spaces

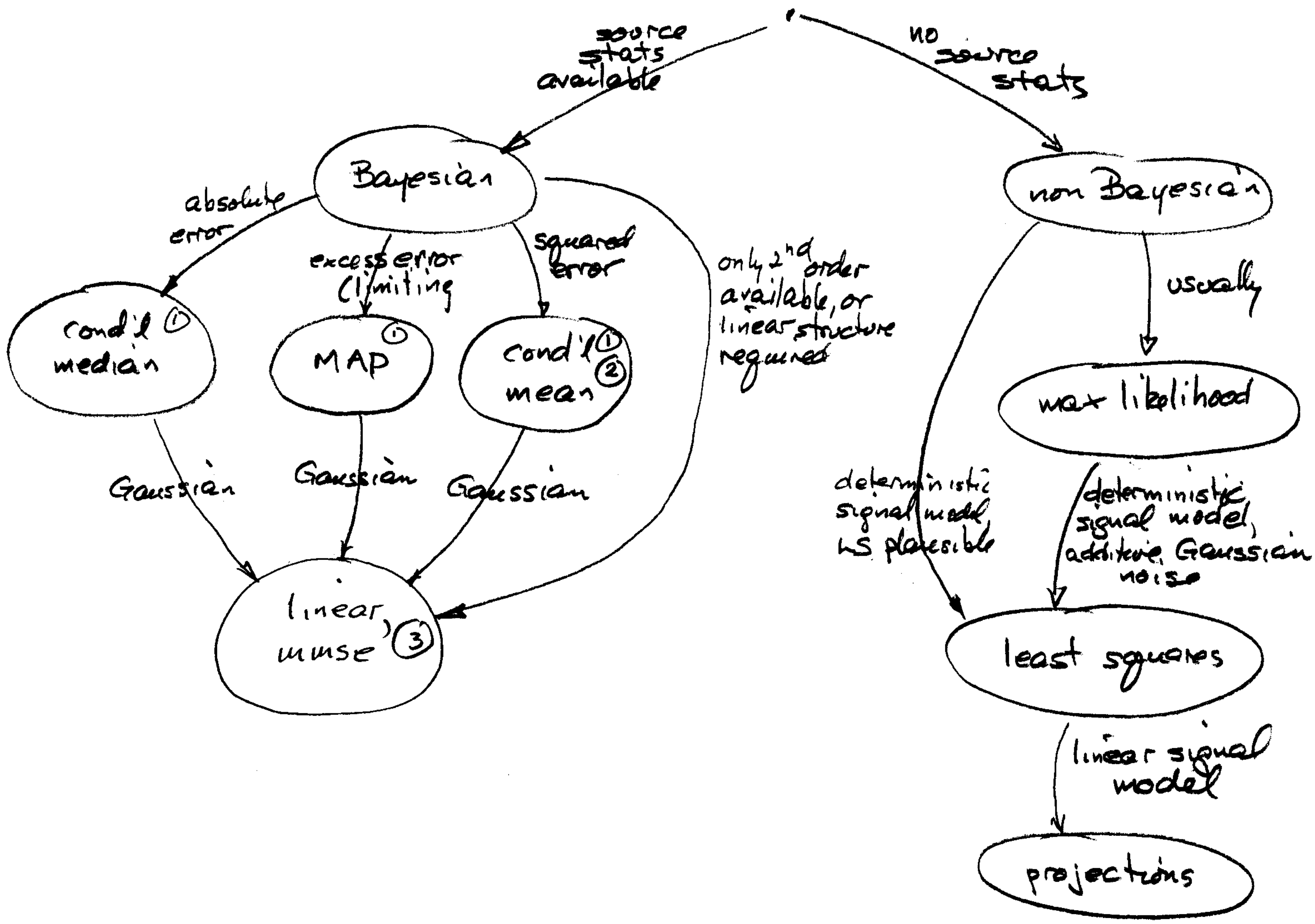
$\underline{e} = \underline{\theta} - \hat{\underline{\theta}}$ is $\perp$ $\underline{x}$ space at opt $\hat{\theta}$

$\|\underline{e}\|^2 = \|\underline{\theta}\|^2 - \|\hat{\underline{\theta}}\|^2$ at opt

error surface is quadratic in case of misadjustment

- maximum likelihood
  - like MAP with uniform $p_\theta(\theta)$
    maximize $p_{x|\theta}(\underline{x} \mid \underline{\theta})$ wrt $\underline{\theta}$
  - ML estimators may or may not be biased
  - with multiple $\underline{x}$ readings, ML is asymptotecally unbiased, consistent, efficient, Gaussian
  - special case is least squares; with linear signal model, get deterministic normal equations, again projections, orthogonality
- Cramér-Rao bound sets minimum est'n error variance.
  Estimators that meet J are "efficient"

• A taxonomy of estimators. We've seen many different, but related, estimation methods. Here's a road map...



① all the same with symmetric, unimodal $p_{\theta|x}(\underline{\theta}|\underline{x})$

② the optimum mmse estimator

③ optimum if Gaussian