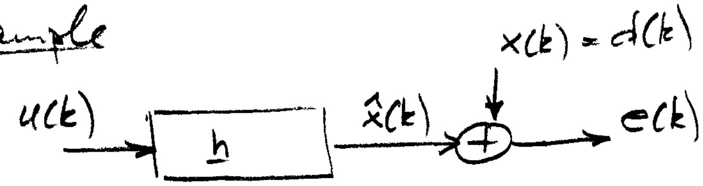
4.8 Block Estimation of Predictor Coefficients

$\boxed{\text{Haykin 6.10}}$

- This is the first point at which we deal with adaptation.
- What if we don't have ensemble statistics, just a data record $u(n)$, $n = 1..N$?
  - no true Wiener filter
  - but can estimate the coeffs of a whitener, hence a process model

- Example



  - we keep records of $u(k)$, $x(k)$
  - How do we process the two arrays to obtain $\underline{h}$?

- A couple of general methods
  - indirect: estimate $r(0), r(1), \ldots$ from data, then run L-D
  - direct: estimate reflection coeffs $\kappa_1, \ldots \kappa_M$ from data.

• Indirect method:

0 | $u(1)$ | $u(2)$ | . . . . . . | $u(N)$ | 0

- the length N of the window is selected to provide quasi stationarity over it
- calculate sample autocorrelations as

$$\hat{r}(i) = \frac{1}{N}\sum_{k=1}^{N-i} u(k)\,u^*(k-i) \qquad i = 0,\ldots,N-1$$

or
$$\hat{r}(i) = \frac{1}{N-i}\sum_{k=1}^{N-i} u(k)\,u^*(k-i)$$

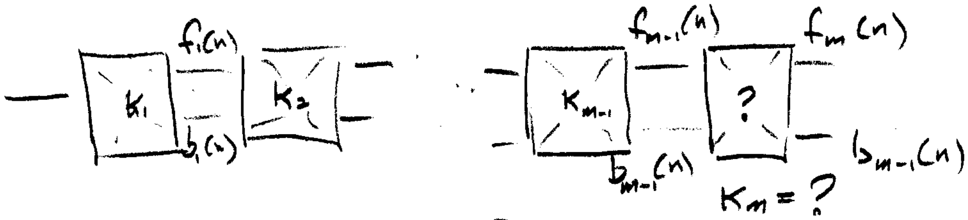
Hmmm... the first one seems to penalize $\hat{r}(i)$ for large i (it's equivalent to Bartlett windowing). But the second doesn't guarantee that $|r(0)| \geq |r(i)|$ $i \neq 0$. Even worse.

- could also allow data to run off the ends

... $u(-2)$ $u(-1)$ $u(0)$ | $u(1)$ | $u(2)$ | ... | $u(N)$ | $u(N+1)$ $u(N+2)$ ...

for an unwindowed estimate. Again, no guarantee that $|r(0)| \geq |r(i)|$, $i \neq 0$

- Direct method:
  - Work with windowed data and estimate κ_m coeffs in the lattice structure
  - Suppose we have stages $1 \ldots m-1$ already estimated.



How to estimate κ_m?

we know

$$f_m(n) = f_{m-1}(n) + \kappa_m^* b_{m-1}(n-1)$$

$$b_m(n) = \kappa_m f_{m-1}(n) + b_{m-1}(n-1)$$

and that, given stationarity, $E[|f_m(n)|^2] = E[|b_m(n)|^2]$

why not choose κ_m to minimise the average of these two variances, or sample variances?

- Denote sample average by $\overline{}$

$$J_m = \overline{|f_m(n)|^2} + \overline{|b_m(n)|^2}$$

$$= \overline{|f_{m-1}(n)|^2} + \kappa_m^* \overline{f_{m-1}^*(n) b_{m-1}(n-1)}$$

$$+ \kappa_m \overline{f_{m-1}(n) b_{m-1}^*(n-1)} + |\kappa_m|^2 \overline{|b_{m-1}(n-1)|^2}$$

$$+ |\kappa_m|^2 \overline{|f_{m-1}(n)|^2} + \kappa_m \overline{f_{m-1}(n) b_{m-1}^*(n-1)}$$

$$+ \kappa_m^* \overline{f_{m-1}^*(n) b_{m-1}(n-1)} + \overline{|b_{m-1}(n-1)|^2}$$

- so take derivative

$$\frac{dJ_m}{dK_m} = f_{m-1}(n)\, b_{m-1}^*(n-1) + K_m^* \, |b_{m-1}(n-1)|^2 + K_m^* |f_{m-1}(n)|^2 + f_{m-1}(n)\, b_{m-1}^*(n-1)$$

or

$$K_m = -\frac{2 \quad f_{m-1}^*(n)\, b_{m-1}(n-1)}{|f_{m-1}(n)|^2 + |b_{m-1}(n)|^2}$$

$$= -\frac{2 \displaystyle\sum_{n=m+1}^{N} b_{m-1}(n-1)\, f_{m-1}^*(n)}{\displaystyle\sum_{n=m+1}^{N} \left(|f_{m-1}(n)|^2 + |b_{m-1}(n-1)|^2 \right)}$$

Lower limit on summation is due to time it takes $u(n)$ to propagate to stage m

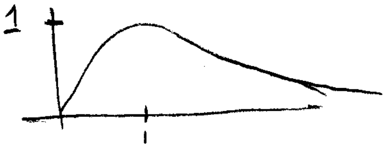
- Estimates ripple down lattice:
  - estimate K_1
  - this permits calculation of $f_1(n)$ $b_1(n-1)$ and therefore K_2
  - this permits calculation of $f_2(n)$, $b_2(n-1)$ and therefore K_3

- Note that this estimate guarantees minimum phase

$$\hat{K}_m = \frac{2\sum_{n=m+1}^{N} b_{m-1}(n-1) f^*_{m-1}(n)}{\sum_{n=m+1}^{N}\left(|b_{m-1}(n-1)|^2 + |f_{m-1}(n)|^2\right)} \le \frac{2\sqrt{B}\sqrt{F}}{B+F} \quad \text{(Schwartz)}$$

where $B = \sum_{n=m+1}^{N} |b_{m-1}(n-1)|^2 \qquad F = \sum_{n=m+1}^{N} |f_{m-1}(n)|^2$

or $K_m = \dfrac{2}{x + 1/x}$ where $x = \sqrt{\dfrac{B}{F}}$



and the max value of $\frac{2}{x+\frac{1}{x}}$ is 1, at $x=1$

Therefore $K_m \le 1$, and minimum phase.

- Operationally ...
  1. run Burg over $\underline{u}$ array to get $K_1, K_2 \cdots K_M$
  2. that generates arrays $\underline{b}_0 \; \underline{b}_1 \cdots \underline{b}_M$
  3. coefficients for joint process estimation determined individually by

$$\eta_i = \frac{\sum_{n=i+1}^{N} x^*(n)\, b_i(n)}{\sum_{n=i+1}^{N} |b_i(n)|^2} = \frac{\underline{x}^+ \underline{b}_i}{\underline{b}_i^+ \underline{b}_i}$$