

TABLE 7.1 SUMMARY OF THE KALMAN VARIABLES

Variable	Definition	Dimension
$\mathbf{x}(n)$	State vector at time n	M -by-1
$\mathbf{y}(n)$	Observation vector at time n	N -by-1
$\mathbf{F}(n+1, n)$	State transition matrix from time n to $n+1$	M -by- M
$\mathbf{C}(n)$	Measurement matrix at time n	N -by- M
$\mathbf{Q}_1(n)$	Correlation matrix of process noise vector $\mathbf{v}_1(n)$	M -by- M
$\mathbf{Q}_2(n)$	Correlation matrix of measurement noise vector $\mathbf{v}_2(n)$	N -by- N
$\hat{\mathbf{x}}(n+1 \mathcal{Y}_n)$	Predicted estimate of the state vector at time $n+1$, given the observation vectors $\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(n)$	M -by-1
$\hat{\mathbf{x}}(n \mathcal{Y}_n)$	Filtered estimate of the state vector at time n , given the observation vectors $\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(n)$	M -by-1
$\mathbf{G}(n)$	Kalman gain at time n	M -by- N
$\boldsymbol{\alpha}(n)$	Innovations vector at time n	N -by-1
$\mathbf{R}(n)$	Correlation matrix of the innovations vector $\boldsymbol{\alpha}(n)$	N -by- N
$\mathbf{K}(n+1, n)$	Correlation matrix of the error in $\hat{\mathbf{x}}(n+1 \mathcal{Y}_n)$	M -by- M
$\mathbf{K}(n)$	Correlation matrix of the error in $\hat{\mathbf{x}}(n \mathcal{Y}_n)$	M -by- M

TABLE 7.2 SUMMARY OF THE KALMAN FILTER BASED ON ONE-STEP PREDICTION

Input vector process

Observations = $\{\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(n)\}$

Known parameters

State transition matrix = $\mathbf{F}(n+1, n)$

Measurement matrix = $\mathbf{C}(n)$

Correlation matrix of process noise vector = $\mathbf{Q}_1(n)$

Correlation matrix of measurement noise vector = $\mathbf{Q}_2(n)$

Computation: $n = 1, 2, 3, \dots$

$$\mathbf{G}(n) = \mathbf{F}(n+1, n)\mathbf{K}(n, n-1)\mathbf{C}^H(n)[\mathbf{C}(n)\mathbf{K}(n, n-1)\mathbf{C}^H(n) + \mathbf{Q}_2(n)]^{-1}$$

$$\boldsymbol{\alpha}(n) = \mathbf{y}(n) - \mathbf{C}(n)\hat{\mathbf{x}}(n|\mathcal{Y}_{n-1})$$

$$\hat{\mathbf{x}}(n+1|\mathcal{Y}_n) = \mathbf{F}(n+1, n)\hat{\mathbf{x}}(n|\mathcal{Y}_{n-1}) + \mathbf{G}(n)\boldsymbol{\alpha}(n)$$

$$\mathbf{K}(n) = \mathbf{K}(n, n-1) - \mathbf{F}(n, n+1)\mathbf{G}(n)\mathbf{C}(n)\mathbf{K}(n, n-1)$$

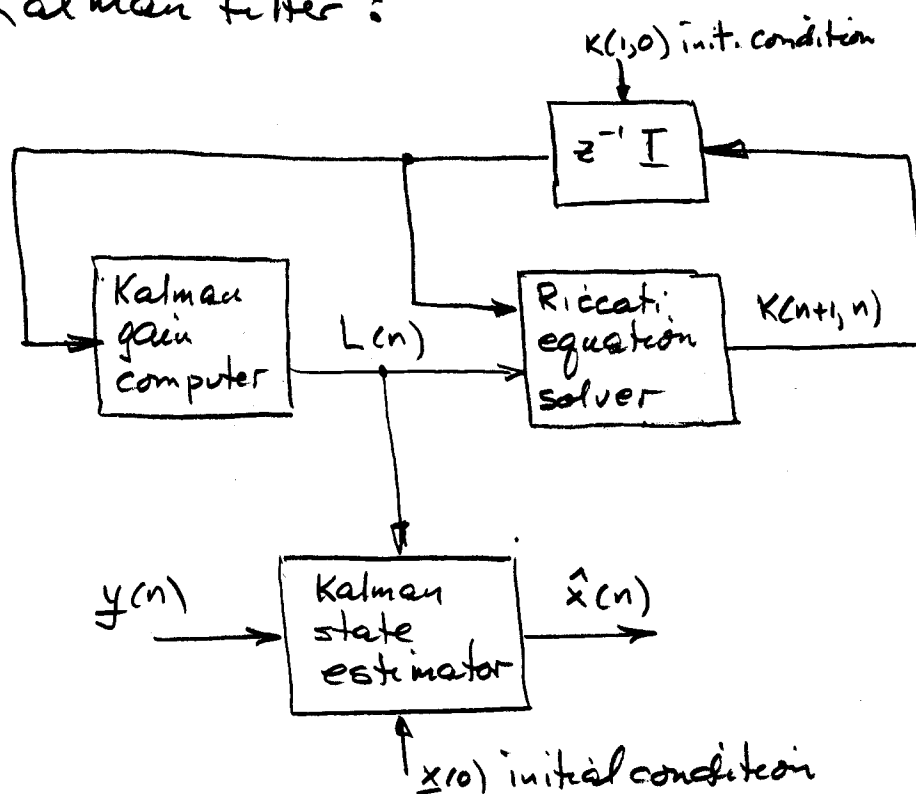
$$\mathbf{K}(n+1, n) = \mathbf{F}(n+1, n)\mathbf{K}(n)\mathbf{F}^H(n+1, n) + \mathbf{Q}_1(n)$$

Initial conditions:

$$\hat{\mathbf{x}}(1|\mathcal{Y}_0) = E[\mathbf{x}(1)]$$

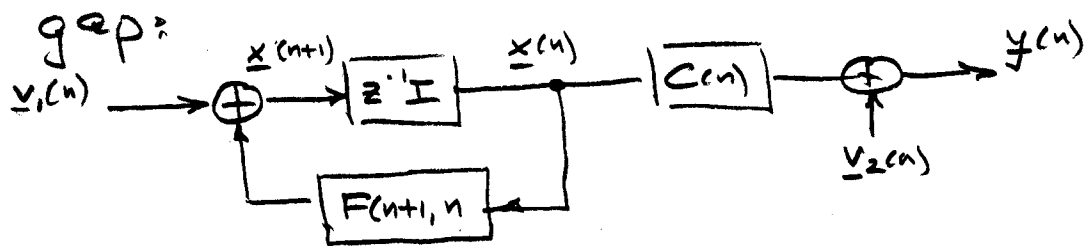
$$\mathbf{K}(1, 0) = E[(\mathbf{x}(1) - E[\mathbf{x}(1)])(\mathbf{x}(1) - E[\mathbf{x}(1)])^H] = \Pi_0$$

- A signal flow diagram for the entire Kalman filter:



- Note that the gain and covariance calculation does not depend on $y(k)$, $k = 1 \dots$. This means they can all be precalculated.
- The quantity $K(n) = E[\epsilon(n, n) \epsilon^T(n, n)]$ is available as a byproduct of the calculation — Kalman filter provides its own accuracy estimation.

- The Kalman filter does not require stationarity
— or even stability!
- The Kalman filter can handle gaps in the measurements, since n is an index, not strict time
 - Suppose $y(n)$ is missing, but we have up to $y(n-1)$ and from $y(n+1)$ on.
 - Just redefine the dynamics to bridge the



$$\begin{aligned}
 \underline{x}(n+1) &= F(n+1, n) \underline{x}(n) + \underline{v}_1(n) \\
 &= F(n+1, n) F(n, n-1) \underline{x}(n-1) + F(n+1, n) \underline{v}_1(n-1) + \underline{v}_1(n) \\
 &= F(n+1, n-1) \underline{x}(n-1) + \underline{v}_1'(n-1)
 \end{aligned}$$

$$\text{where } \underline{Q}_1'(n-1) = F(n+1, n) \underline{Q}_1(n-1) F^T(n+1, n) + \underline{Q}_1(n)$$

- Filtering and prediction come easily.

- Filtering: estimate $\underline{x}(n)$ on the basis of $y^{(1)}, \dots, y^{(n)}$.
We have $\hat{x}(n|y_n)$ as a variable in the Kalman state estimator.

- Prediction: estimate $\underline{x}(n+k)$ on the basis of y_n .
Iterate the state equation, and obtain

$$\underline{x}(n+k) = F(n+k, n)\underline{x}(n) + F(n+k, n+1)\underline{v}_1(n) + \dots + \underline{v}_1(n+k-1)$$

Take conditional expectation (conditioned on y_n), and

$$\hat{x}(n+k|y_n) = F(n+k, n)\hat{x}(n|y_n) \quad \text{easy!}$$

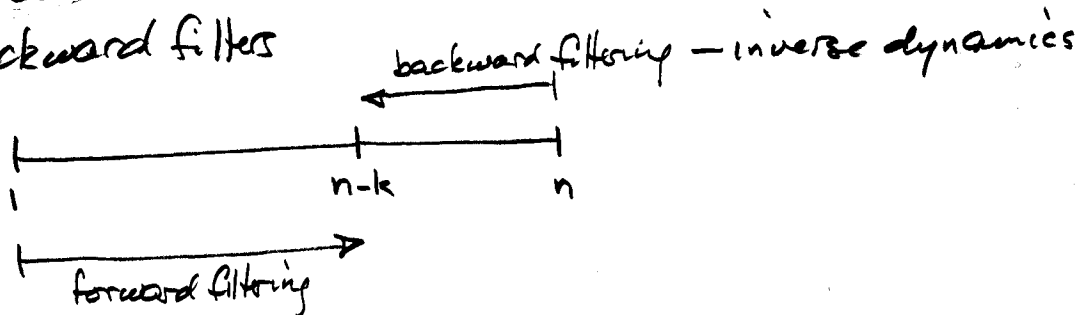
In fact, one of them ($\hat{x}(n+1|y_n)$) is generated inside the Kalman state estimator

- Smoothing is harder.

- Estimate $\underline{x}(n-k)$ on the basis of y_n

Can't mimic the predictor, because the process noises $\underline{v}_1(i)$ $n-k \leq i < n$ are not indep of y_n .

- A common derivation is to use forward and backward filters



see AGolb for more information