

**SIMON FRASER UNIVERSITY**  
School of Engineering Science  
ENSC 810-3 Digital Signal Processing

Final Examination

April 18, 2000

Instructions:

- This is a 3-hour open book examination.
- All 4 questions are worth equal marks.
- Explain your answers. *A bare “yes” or “no” or “3” is worth nothing.*
- If you don't have time to complete your answer, at least jot down your reasoning about how to solve it, in order to gain part marks.

**1. A Space of Matrices**

The sum of squares of all the components of an  $M \times N$  matrix  $\mathbf{X}$  is easily shown to be

$$\text{trace}[\mathbf{X}^H \mathbf{X}] = \sum_{m=1}^M \sum_{n=1}^N |x_{m,n}|^2$$

- (a) Using this observation, define an inner product space of  $M \times N$  matrices. Include definitions of norm, inner product and normalized inner product (the equivalent of correlation coefficient for random variables, or cosine of the included angle for spatial vectors).
- (b) You are given three  $M \times N$  matrices  $\mathbf{X}$ ,  $\mathbf{Y}$  and  $\mathbf{Z}$ . How would you find the best linear approximation of  $\mathbf{X}$
- $$\hat{\mathbf{X}} = a\mathbf{Y} + b\mathbf{Z}$$
- where “best” means minimum sum of squares of the error? Prove your answer.

**2. LMS in Trouble**

Suppose an LMS algorithm is working on data for which the correlation matrix  $\mathbf{R}$  is singular.

- (a) What is the effect on adaptation of  $\mathbf{w}(n)$ ? To what extent does it converge or fail to converge?
- (b) What is the effect on convergence of  $J(n) = E[|e(n)|^2]$ ?

### 3. RLS With Moving Window Average

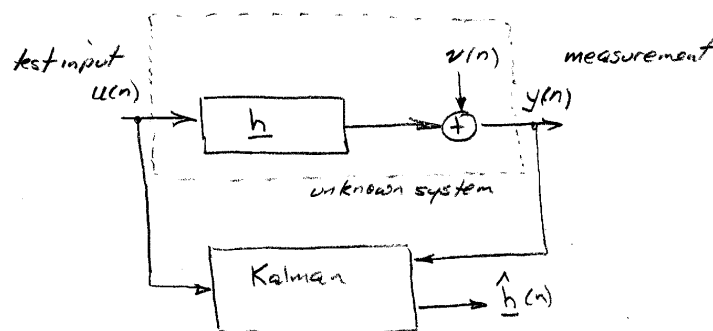
To give RLS some tracking ability, we usually make the forget factor  $\lambda$  just less than unity. An alternative to exponential weighting is use of a length- $K$  moving window average; that is, we choose  $\hat{\mathbf{w}}(n)$  to minimize

$$J(n) = \sum_{i=n-K+1}^n |e(i)|^2 = \sum_{i=n-K+1}^n |d(i) - \hat{\mathbf{w}}^H(i) \mathbf{u}(i)|^2$$

- (a) Write the deterministic normal equations at time  $n$  in both forms: with  $\mathbf{d}$  and  $\mathbf{A}$ , and with  $\Phi$  and  $\mathbf{z}$ . Show how each of the arrays is defined.
- (b) Consider a basic recursive algorithm like that in Section 8.2. Show how  $\Phi$  and  $\mathbf{z}$  can be updated recursively.
- (b) Show how to adapt RLS to this cost function. Note that Woodbury's identity in Section 8.3 is also valid for  $g^2 < 0$ .

### 4. System Identification by Kalman

We would like to determine the impulse response of system. The system is fixed, but unknown, - but at least we know it is an FIR filter  $\mathbf{h}$  of order  $M$ . To see how the system behaves, we drive it with a random signal  $u(n)$  and observe the response  $y(n)$ , which, unfortunately, contains white noise. Because the data record is not very long, we decide that the optimality of the Kalman approach outweighs its complexity.



- (a) Formulate this problem in Kalman terms; that is, give
  - the state equation and output equations;
  - the updates in the Kalman gain computer and the Ricatti equation solver.
- (b) Show how it acts as a filter that whitens  $y(n)$ .

