

SIMON FRASER UNIVERSITY
 School of Engineering Science
 ENSC 810-3 Digital Signal Processing

Final Examination

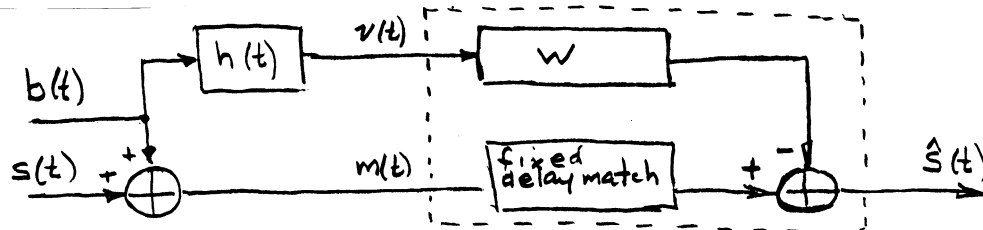
April 9, 1999

Instructions:

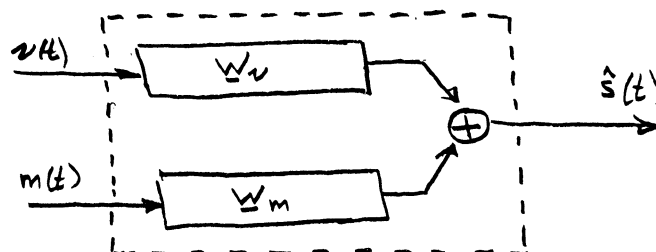
- This is a 3-hour open book examination.
- All 4 questions are worth equal marks.
- Explain your answers. *A bare “yes” or “no” or “3” is worth nothing.*
- If you don’t have time to complete your answer, at least jot down your reasoning about how to solve it, in order to gain part marks.

1. Noise Cancellation

You have a microphone that picks up a noisy version $m(t)$ of a signal $s(t)$ that you want. Fortunately, you have a reference signal – a distorted version $n(t)$ of the true noise $b(t)$ in the microphone input, which is filtered by $h(t)$ to produce the reference. After sampling, of course, all signals are functions of the time index n : $m(n), s(n), n(n), b(n)$. You decide to construct a noise canceller containing a length- M FIR filter with coefficient vector $\mathbf{w}$, as shown below, and adapt it by LMS.



- What properties of $m(n)$ and $n(n)$ affect the convergence speed?
- If M is too small, what is the consequence? If M is too big, what is the consequence?
- Does the filter $h(t)$ affect convergence? If so, how? If not, why not?
- There is an alternative structure for noise cancellation that uses two FIR filters, as shown below. Does it have any advantage over the basic noise canceller if filter $h(t)$ is invertible?



2. Least Squares

You have decided to fit an M^{th} order linear model to measured data: for a length- M vector of measurements $\mathbf{u}$, and a desired output d , you choose the coefficient vector $\mathbf{w}$ so that

$$\mathbf{u}^H \mathbf{w} \approx d$$

You decide to use least squares, and make K measurements for the fitting, where $K > M$; that is, you have more equations than unknowns. In total, you want the set of equations to satisfy

$$\mathbf{A} \mathbf{w} \approx \mathbf{d}$$

where each row of $\mathbf{A}$ is one of the $\mathbf{u}^H$. As it happened, though, your measurements made the columns of $\mathbf{A}$ linearly dependent.

- (a) The usual solution $\mathbf{w} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{d}$ now has a problem. What is that problem and what can you do about it?
- (b) What does the linear dependence do to the error vector $\mathbf{e} = \mathbf{d} - \mathbf{A} \mathbf{w}$?
- (c) What does the linear dependence do to the spectral representation $\mathbf{A} = \sum_i s_i \mathbf{u}_i \mathbf{v}_i^H$?

3. Recursive Least Squares

- (a) Why is RLS insensitive to eigenvalue spread?
- (b) One way to apply least squares to spectrum estimation is use of Burg's method: minimize the sum of squares of the forward and backward estimation errors (Section 7.5 of class notes, Section 11.9 of Haykin). Can RLS be adapted to make this calculation recursive?
- (c) If the RLS forget factor $\lambda = 1$ and the input $u(n)$ is a stationary process, what is the asymptotic form of the Kalman gain vector $\mathbf{k}(n)$ as $n \rightarrow \infty$?

4. Estimation

Is a MMSE estimate unbiased if the signals do not have Gaussian statistics?