SPENDING AND SAVING IN THE RRIF/LIF YEARS
The Retiree's Calculator

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EXPANDED FORMAT

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LIST OF ABBREVIATIONS AND INITIALISMS

Common abbreviations

CRA	Canada Revenue Agency
CPP	Canada Pension Plan
ETF	Exchange-Traded Fund
LIF	Life Income Fund
LIRA	Locked-In Retirement Account
MER	Management Expense Ratio
NPV	Net Present Value
OAS	Old Age Security
RRIF	Registered Retirement Income Fund
RRSP	Registered Retirement Savings Plan
S&P	Standard and Poors
TSFA	Tax-Free Savings Account
TSX	Toronto Stock Exchange

Abbreviations used only in this study

B	Balance in RRIF or LIF
W	Withdrawal amount
GI	gross income
TP	Tax paid
NI	Net income

1. INTRODUCTION

How long will my retirement funds last? The question is an acute one for the millions of Canadians who have no workplace pensions or whose workplace supports only defined contribution plans. For them, RRSPs or workplace LIRAs are at the centre of their retirement finances and they have only guides like "4% withdrawals have a 95% chance of lasting 30 years with inflation." That rule of thumb and others were developed in the well-known Trinity Study [1]-[3].

Unfortunately, in the year of their 71st birthday, retirees must convert RRSPs and LIRAs to RRIFs and LIFs, respectively (or the less-used option of annuities), and that changes everything. CRA imposes minimum withdrawal rates on RRIFs and LIFs, even if the retiree doesn't need the money at that time. LIFs also have a maximum withdrawal rate. There are several pernicious effects:

- In any fund subject to withdrawals, market downturns in the early years sap the subsequent ability of the fund to recover. The forced large withdrawals in early RRIF/LIF years exaggerate this problem and can quickly deflate the portfolio. The Trinity Study guidelines no longer apply exactly.
- The unnecessarily large early withdrawals, even though followed by low withdrawals in later years, cause the retiree to pay more total income tax than if the same amount had been paid in the same number of constant withdrawals.
- The early forced increase in annual income may cause some retirees to suffer OAS "clawbacks."
- The LIF maximum withdrawal rate may prevent retirees from accessing their money, even if they need it. This becomes a problem in later years of the LIF.

If the Trinity Study guidelines to fund longevity do not fit RRIF/LIF withdrawals, then what does? The present document may help. It is a calculator to show how your savings and income evolve (or devolve) in the post-71 world out to age 95, including when the money runs out. If you read it in Mathcad (the package in which I wrote it), then it is interactive - you can change numbers or conditions, and all the calculations and graphs change in response. If you are reading it as a simple PDF, you can mimic all the calculations in your favourite alternative system (e.g., Excel).

Like the Trinity Study, the calculator can incorporate historic market variability. Unlike that study:

- The calculator applies CRA-mandated variable withdrawal rates, instead of fixed withdrawals.
- Because RRIFs and LIFs are intimately tied to income tax, the calculator includes simplified taxation models for retirees, including possible OAS reduction caused by high initial withdrawals.
- The calculator allows use of fixed inflation rates, as well as historical inflation rates.
- The calculator acts to support your target for after-tax spending level for as long as possible, and tracks an external account where you save your forced excess income in the early years.
- The calculator allows you to stress-test your finances with fictitious future market behaviour by appending a selected section of the historical record - good, bad or boring - to follow 2013 of the true record. The Trinity Study stuck to what the market had actually produced over the years.
- **On the drawback side**, it considers only 44 years (1970 to 2013) of market history, instead of the 72 years (1926-1997) of the Trinity Study, although it includes four such markets. It also considers only fixed returns for the fixed income component, instead of bond index returns. Also, it operates in annual time steps, coarser than Trinity's monthly time steps. Later versions may be better.

Most readers will be interested only in this chapter, in Chapter 4, which provides many examples of calculated results for various financial circumstances and in Chapter 5, which gives a short critique of the calculator. Only some will be interested in Chapter 2's component models of the calculator (RRIF/LIF withdrawal schedules, simple taxation models, OAS reduction, markets and inflation) and in the calculator itself, in Chapter 3. To keep the document reasonably small for the majority, I have placed most of Chapters 2 and 3 in "collapsed areas," like the one just below, with the boxed arrow to the left of the line. To expand and read the area, double-click the line. To collapse it again, double-click the line again. Try it here:

This calculator is free, although it works interactively only if you read it in Mathcad. If you have the skills and the interest, I encourage you to improve it or to rewrite it for more accessibility (e.g., in Excel or in Java for server-side operation). If you do, please make the program free to all, just as I have.

Finally, the disclaimers:

- I am not a financial advisor, nor even an MBA. Instead, I am a retired professor of engineering, forced to learn things that didn't interest me because I am at the other end of a defined contribution plan.

- Although I have spent time and thought on it, I make no guarantee whatever about the accuracy of the calculator, and you use it at your own risk. In any case, nothing is hidden - you can read the code yourself to verify what it does.

2. THE COMPONENT MODELS

The overall retirement calculator simply keeps track of various numbers as they change, year by year. However, it contains several nonlinear components, each of which must be modelled reasonably accurately. This section presents models for those key components in the collapsed area below.

Tables for functions used in this section are in the collapsed area at the top of this page. They are also displayed in Appendix A.

2.1 RRIF and LIF Withdrawal Rates

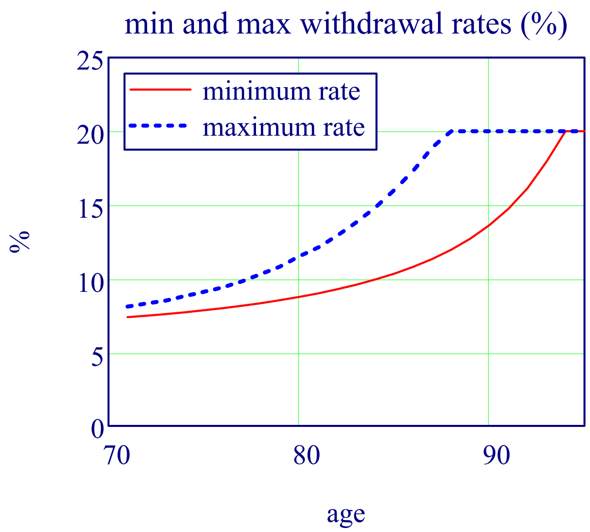
A big factor in retirees' financial lives is how much they withdraw from registered accounts. CRA mandates minimum withdrawal rates for both RRIFs and LIFs [4]. In addition, LIFs have maximum withdrawal rates, which vary by province and with time.

The minimum withdrawal rate applies to both RRIFs and LIFs, and may depend on the carrier. The rates for CIBC [5] are copied as *minratetable* into the collapsed area at the top of this chapter. Column 0 is age, column 1 is min withdrawal rate in percent. It is convenient to convert it to a function:

$$minrate(x) := linterp\left(minratetable^{\langle 0\rangle}, minratetable^{\langle 1\rangle}, x\right)$$

The maximum rate applies only to LIFs. Its table, *maxratetable*, is handled similarly. It is for BC and 2011, from [14]. That reference also shows max rates for other provinces (but only for 2012). Again as a function, Again as a function,

$$maxrate(x) := linterp\left(maxratetable^{\langle 0\rangle}, maxratetable^{\langle 1\rangle}, x\right)$$



If the maximum withdrawal rate never reaches 100%, you can never get *all* of your money out of a LIF.

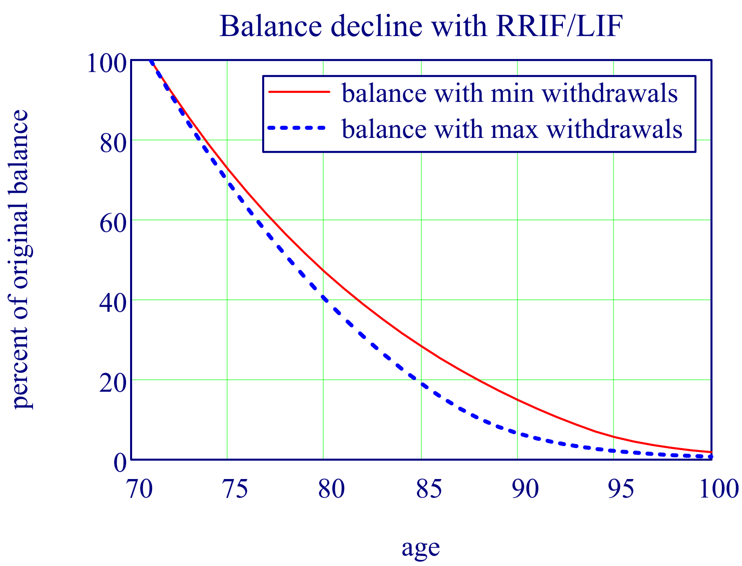
How quickly does balance decline with minimum and with maximum withdrawals? Initialize with balance equal to 1 at age 71, so the sequences below are normalized by the starting balance. They will also be useful in later calculations.

$$age := 72..100$$

$$bmin_{71} := 1 \qquad bmin_{age} := bmin_{age-1} \cdot \left(1 - \frac{minrate(age)}{100}\right)$$

$$bmax_{71} := 1 \qquad bmax_{age} := bmax_{age-1} \cdot \left(1 - \frac{maxrate(age)}{100}\right)$$

They are plotted below. $i := 71..100$

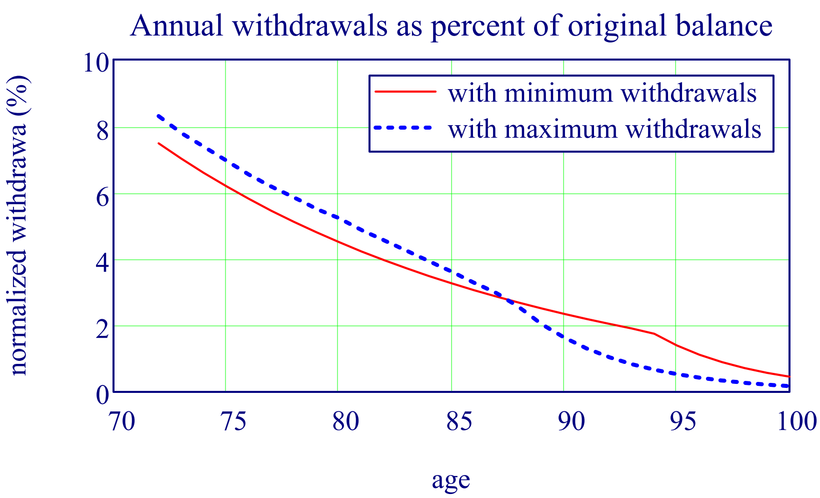


The actual balance lies on or below the upper curve for RRIFs, and on or between the two curves for LIFs.

How big are the annual withdrawals, as a fraction of the starting balance, with min rate and with max rate?

$$wmin_{age} := bmin_{age-1} \cdot \frac{minrate(age)}{100} \qquad wmax_{age} := bmax_{age-1} \cdot \frac{maxrate(age)}{100}$$

These sequences will also be used in the analysis of a later section.



The largest withdrawals are in the early years, even though the withdrawal *rates* increase with age. Many retirees will be forced to take their money out of the RRIF or LIF too quickly, making their income unnecessarily high in the early years. Potential problems:

- They pay more income tax than they would if withdrawals were based only on need.
- If the early large withdrawals happen to occur during periods when the market is down, the fund will not be able recover as easily as it would with smaller, needs-based withdrawals.
- Some retirees will suffer clawback of OAS because their income is artificially inflated.
- They also lose tax-sheltered growth on the amount withdrawn (unless they have enough contribution room in a TFSA).

2.2 Reduction of OAS

OAS (Old Age Security) is an indexed pension, with value dependent on the durations of a person's Canadian citizenship and residence in the country. From the Service Canada website [7], its maximum value in 2012-2013was $546.07 per month ($6.55K per year) for an individual. Unlike CPP (Canada Pension Plan), the payments can decrease if gross income *GI* is too high. Again from the Service Canada website [8], 15% of the amount by which gross income exceeds a threshold (which also appears to be indexed) is recovered in the corresponding year's tax return. The threshold for the 2012 tax year was $69.562 K. Evidently, for an individual gross income of $113.25 K or more, the OAS less the subsequent adjustment nets out to zero.

This is how to calculate the OAS reduction (often called a clawback):

$$gez(x) := if\,(x \geq 0, x, 0)$$ keeps x greater than or equal to zero

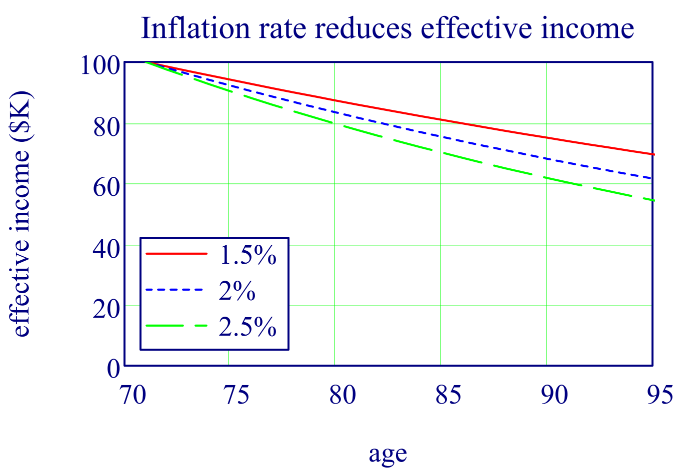
$$OASclaw(GI, OAS) := min\left(OAS, 0.15 \cdot gez\left(GI - T13_6\right)\right)$$

In the above, *T13* is an array of parameters used in CRA tax calculations, and described in Section 2.4. In that array, $T13_6$ is the threshold for OAS reduction.

2.3 Inflation

Buying power of, say, $100, wilts with constant inflation like this:

$$EffIncome(infrate, yrs) := 100 \cdot \left(1 - \frac{infrate}{100}\right)^{yrs} \qquad yrs := 0..25$$



Inflation was savage in the 70s and 80s before the annual rate target became 1% to3%. The Canadian Inflation website [9] shows Consumer Price Index changes over the years. Their table is in the collapsed area at the start of this chapter and plotted below.

$$CPIrate := CPIdata^{\langle 1 \rangle}$$ annual changes in per cent

From the annual inflation rate, calculating the CPI, starting from a value of 1 at the end of 1969, is easy. It's presented as a procedure here, since this type of calculation will be used frequently.

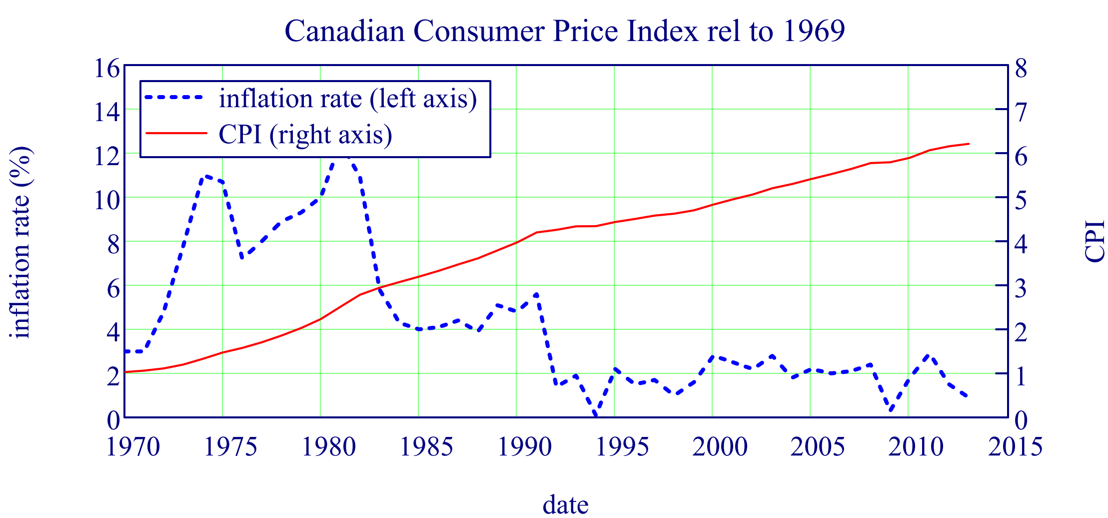
```
getlevel(rate) := | "rate is a vector of % change in each year"
                  | "first rate (for 1969) is assumed to be zero"
                  | level_0 ← 1
                  | " "
                  | " "
                  | for i ∈ 1..rows(rate) − 1
                  |    level_i ← level_(i−1) · (1 + rate_i/100)
                  | return level
```

In the other direction, converting level to annual rate works like this:

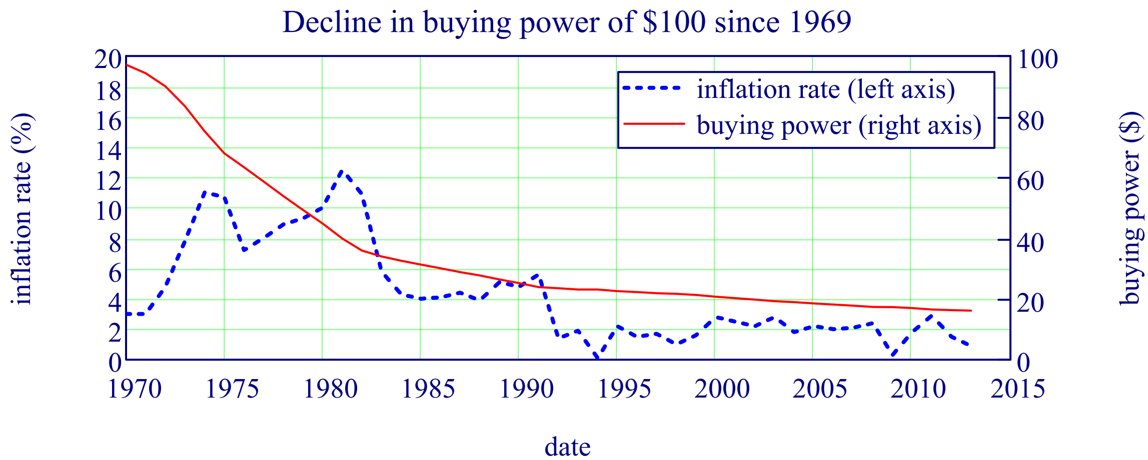
$$getrate(level) := \begin{array}{|l} \text{"level is a vector of index value in each year"} \\ rate_0 \leftarrow 0 \\ \text{"calculate year over year change in per cent"} \\ for\ \ i \in 1 .. rows(level) - 1 \\ \quad rate_i \leftarrow 100 \cdot \left(\frac{level_i}{level_{i-1}} - 1 \right) \\ return\ \ rate \end{array}$$

So... the CPI is $CPI := getlevel(CPIrate)$ *iinfl*

Anyone old enough to be interested in this document will remember how frightening inflation was during the 70s and 80s. It impaired real portfolio growth just as a market downturn would have. The plot below illustrates our inflation history.



Similarly, the buying power of $100 decayed relative to 1969 as the reciprocal of CPI, as shown below:



2.4 The Tax Bite

This section contains two very simplified tax models: for an individual, and for an equal-aged couple who can split their income equally, for example, if it is all from government or private pensions or from RRIFs/LIFs. So no employment income, no non-registered investment income, no income from real estate, no foreign income; similarly, no donations or other complications. This section will be long and tedious, just like the CRA tax forms.

Common abbreviations in this section (note some definitions differ from those on CRA tax forms):

- GI gross income, i.e., gross pre-tax income, CPP, OAS, RRIF, LIF, other pensions;
- NIBA234 CRA's Net Income Before Adjustments, line 234 of the 2013 tax forms, equal to what I have called GI for our simplified retiree models;
- TI260 CRA's taxable income definition, line 260, equal to NIBA234 - OAS clawback;
- TP tax paid;
- NI net income (i.e., after taxes), GI - TP.

All the parameters used in this simplified tax calculation are taken from the 2013 T1 General form and the various Schedules and worksheets and packed into the array *T13* in the collapsed region at the start of this chapter. It is laid out as on the right of the page below:

Parts of *T13* holds the parameters of the piecewise linear (segmental) tax calculations for federal tax and BC tax. For those calculations, it is convenient to convert them to three-column arrays (next page):

$$makesegs(vec) := \left| \begin{array}{l} \text{"vec is submatrix of T13: start and \% for each segment"} \\ \text{"output is 3 cols: start, \%, base; a row for each segment"} \\ Nsegs \leftarrow \dfrac{rows(vec)}{2} \\ Z_{0,0} \leftarrow vec_0 \\ Z_{0,1} \leftarrow vec_1 \\ Z_{0,2} \leftarrow 0 \\ for\ \ n \in 1 .. Nsegs - 1 \\ \quad \left| \begin{array}{l} \text{"segment start, x axis"} \\ Z_{n,0} \leftarrow vec_{2 \cdot n} \\ \text{"segment slope in \%"} \\ Z_{n,1} \leftarrow vec_{2 \cdot n+1} \\ \text{"segment base (start value on y axis)"} \\ Z_{n,2} \leftarrow Z_{n-1,2} + \dfrac{\left(Z_{n-1,1}\right)}{100} \cdot \left(Z_{n,0} - Z_{n-1,0}\right) \\ Z_{n,2} \leftarrow \dfrac{round\left(1000 \cdot Z_{n,2}\right)}{1000} \end{array} \right. \\ return\ \ Z \end{array} \right.$$

0	fed age 0
1	fed age 1
2	fed basic
3	BC age 0
4	BC age 1
5	BC basic
6	OAS threshold
7	fed seg 0 start
8	fed seg 0 %
9	fed seg 1 start
10	fed seg 1 %
11	fed seg 2 start
12	fed seg 2 %
13	fed seg 3 start
14	fed seg 3 %
15	BC seg 0 start
16	BC seg 0 %
17	BC seg 1 start
18	BC seg 1 %
19	BC seg 2 start
20	BC seg 2 %
21	BC seg 3 start
22	BC seg 3 %
23	BC seg 4 start
24	BC seg 4 %

The federal and BC segments are

$$fedsegs := makesegs(submatrix(T13, 7, 14, 0, 0)) = \begin{pmatrix} 0 & 15 & 0 \\ 43.561 & 22 & 6.534 \\ 87.123 & 26 & 16.118 \\ 135.054 & 29 & 28.58 \end{pmatrix}$$

$$BCsegs := makesegs(submatrix(T13, 15, 24, 0, 0)) = \begin{pmatrix} 0 & 5.06 & 0 \\ 37.568 & 7.7 & 1.901 \\ 75.138 & 10.5 & 4.794 \\ 86.268 & 12.29 & 5.963 \\ 104.754 & 14.7 & 8.235 \end{pmatrix}$$

This is how to calculate tax from the three-column arrays that define the segmentally linear functions. Argument *TI260* is CRA's NIBA less the OAS clawback.

$$
SegTax(TI260, S) := \left|
\begin{array}{l}
\text{"S is 3-column array of segments, as above"} \\
return\ \ S_{0,2}\ \ if\ \ TI260 \le S_{0,0} \\
for\ \ n \in 0..rows(S) - 2 \\
\quad return\ \ S_{n,2} + \dfrac{S_{n,1}}{100} \cdot \left(TI260 - S_{n,0}\right)\ \ if\ \ TI260 \le S_{n+1,0} \\
n \leftarrow rows(S) - 1 \\
return\ \ S_{n,2} + \dfrac{S_{n,1}}{100} \cdot \left(TI260 - S_{n,0}\right)
\end{array}
\right.
$$

Next, "age amounts" for those over 65 as function of gross income (income subject to tax), denoted *GI* or *NIBA*

$$fedage(NIBA) := gez\left(T13_1 - 0.15 \cdot gez\left(NIBA - T13_0\right)\right)$$

$$BCage(NIBA) := gez\left(T13_4 - 0.15 \cdot gez\left(NIBA - T13_3\right)\right)$$

Federal and provincial tax credits are the sum of a basic amount, a pension amount and the age amount, all times a multiplier (15% for fed, 5.06% for provincial). Calulation below assumes annual pension is no less than $2,000, so that "pension amount" of $2K and $1K (fed and BC, respectively) can be included. Not making that assumption would have added a lot of code for little return.

$$fedtaxcred(NIBA) := 0.15 \cdot \left(T13_2 + 2 + fedage(NIBA)\right)$$

$$BCtaxcred(NIBA) := 0.0506 \cdot \left(T13_5 + 1 + BCage(NIBA)\right)$$

Net federal tax and BC tax, both functions of *TI260*:

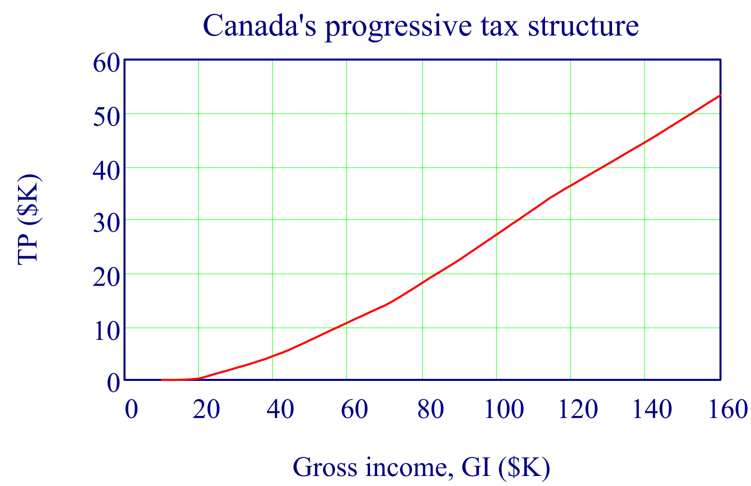
$$NetFedTax(TI260) := gez(SegTax(TI260, fedsegs) - fedtaxcred(TI260))$$

$$BCtax(TI260) := gez(SegTax(TI260, BCsegs) - BCtaxcred(TI260))$$

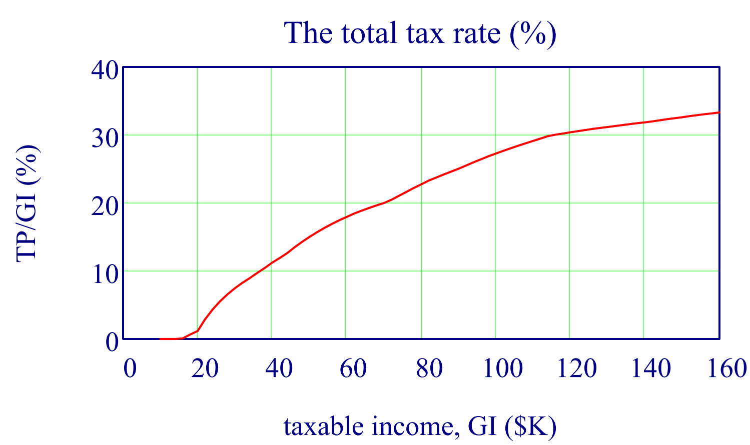
Finally, the tax paid on income GI, which includes CPP, OAS, RRIF, LIF and any other pensions. The OAS component is identified, for any OAS clawback. The function below matched Turbo Tax calculations for the case of a single retiree, 65 years or older, whose only income is those able. Note the dash ' on the function name, for future reference.

$$TaxPaid'(GI, OAS) := \left| \begin{array}{l} \text{"GI is the income subject to tax, CRA's NIBA"} \\ \text{"TI260 is CRA's taxable income, NIBA - OASclaw"} \\ claw \leftarrow OASclaw(GI, OAS) \\ TI260 \leftarrow GI - claw \\ pay \leftarrow NetFedTax(TI260) + claw + BCtax(TI260) \\ return\ \ pay \end{array} \right.$$

View the function for a person with OAS of \$6.5K. This is a progressive taxation structure, as can also be seen in the marginal rates contained in arrays fedsegs and BCsegs. GI



The *total* tax rate TP/GI (not the marginal tax rate) is shown on the next plot.



We need a little more flexibility in the tax function for our purposes. In particular, we need to account for the difference between individuals and couples, and we need to account for the effect of inflation on the tax functions.

To start, distinguish between individuals and couple with the parameter *taxmodel*. It can take these values:

1. for an individual, as above;
2. for the pension-splitting, equal-aged couple, where gross income, tax paid and net income are interpreted as their combined values. Why equal ages? Because withdrawal rates are age-dependent and because the calculator runs out to a fixed age (95), instead of until funds run out. A later version of the calculator might change this.

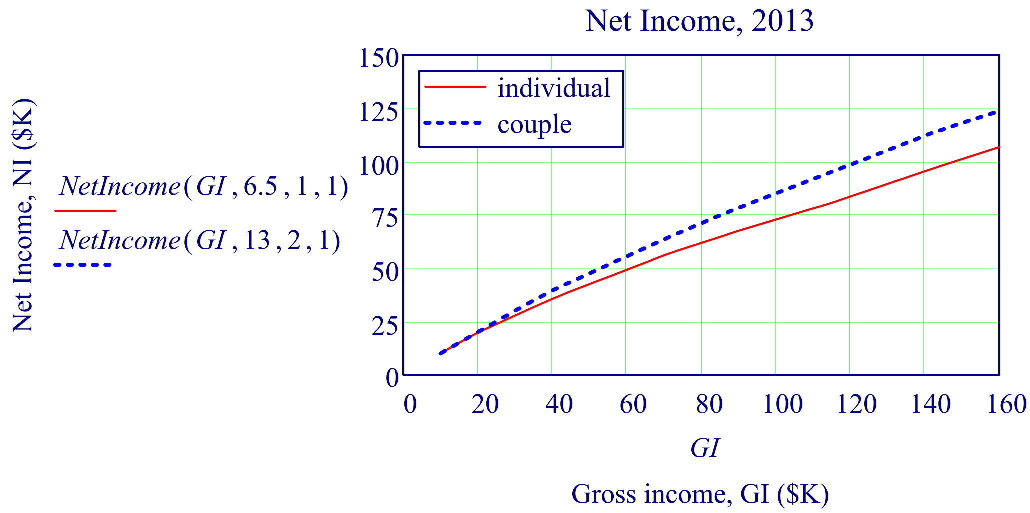
As for the second requirement, CRA evolves its tax function according to inflation, even in the absence of changes in taxation policy. The functions below try to mimic that evolution with the variable *I13*, the inflation level relative to that in 2013. **The assumption is that the *fraction* of GI paid as tax is the same as that in 2013, provided that GI is inflation-corrected back to its 2013 equivalent.** Without this inflation correction, our progressive taxation structure would take too much tax as inflation pushes our gross incomes up. The calculation is in *TaxPaid* (with no dash) below:

```
TaxPaid(GI, OAS, taxmodel, I13) := | if taxmodel = 1
                                   |    | "this is for an individual"
                                   |    | TP ← I13·TaxPaid'(GI/I13, OAS/I13)
                                   | if taxmodel = 2
                                   |    | "this is for a pension-splitting couple,"
                                   |    | "where GI and OAS are combined values"
                                   |    | TP ← 2·I13·TaxPaid'(GI/(2·I13), OAS/(2·I13))
                                   | return TP
```

From the function *TaxPaid*, we get two more useful functions. First, the net income (after taxes)

$$NetIncome(GI, OAS, taxmodel, I13) := gez(GI - TaxPaid(GI, OAS, taxmodel, I13))$$

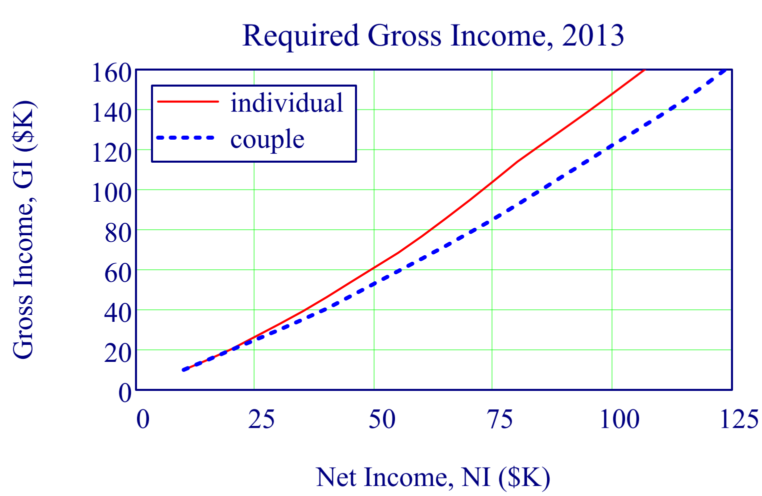
It is plotted below for 2013 (i.e., $I13 = 1$) for an individual and a couple. The GI and OAS of the couple are combined values, twice as large as that of the individual - however, they can split them. Quantities on the axes are shown explicitly as an example of how to use the function.



Finally, the inverse of the *NetIncome* function, i.e., the gross income GI required to generate a given net income NI. It's set up with a root finder. Yes, there are other ways, but this is sort of fun.

$$GrossIncome(NI, OAS, taxmodel, I13) := \left| \begin{array}{l} return\ \ 0\ \ if\ \ NI < 0 \\ \text{"guess 10\% higher than NI"} \\ guess \leftarrow 1.1 \cdot NI \\ GI \leftarrow root(NetIncome(guess, OAS, taxmodel, I13) - NI, guess \\ return\ \ GI \end{array} \right.$$

It is plotted below to confirm that it looks like the net income function above, flipped on the 45 degree line.



2.5 Market Variability

Market volatility is no friend of the retiree. This section collects historic records of market behaviour and presents them graphically with a few comments.

Effect of market variability in withdrawal conditions

The stock market produces the best long-term returns, so its *average* return is good. That's fine for a fund in the years before retirement. However, market *variability* is a big issue when the fund is subject to periodic withdrawals, post-retirement. Withdrawals when the market is down leave the fund proportionally more depleted, and less able to recover when the market goes up. That's bad enough for fixed withdrawals that are just sufficient to meet annual spending needs. It can become serious with the excessive withdrawals mandated for RRIFs and LIFs.

First, consider withdrawals of fixed size (unlike the present study). A well-known study of historical market variations ("The Trinity Study", [1], [2]), showed that a 4% inflation-adjusted withdrawal rate gives a 95% success rate of funds lasting 30 years. This is based on a portfolio of 50/50 equities/ bonds and constant 3% inflation. A 5% withdrawal rate has a 75% chance of lasting 30 years. The inflation adjustment means that we could increase the dollars withdrawn by 3% every year.

The reason portfolio success is expressed as a rate is that the authors tried all 30-year periods in the range 1926 to 1997 and the "rate" is just the fraction of starting years for which the funds lasted for the full period. Note that it's not just the cumulative return over the years that counts - it's when the big and small returns took place. Over the lifetime of the funds, years of high market returns followed by low returns beats by a wide margin a pattern of low returns followed by higher returns.

Another common observation is that the withdrawal rate often varies over time, with a higher rate in the more active early retirement years. The possibility of needing assisted living in later years tends to counters this point, though.

Historical market returns

Obtaining reliable values for historical market returns took a surprisingly long time, for several reasons.

- In some cases, the data weren't available in tabular form, and required reading values from graphs.
- In others, no older data (prior to, say, 1976) seemed be available in either tabular or chart form.
- And in yet others, ostensible data on market returns were available, but were dangerously inaccurate. In particular, the TSX annual returns data at two different websites [10], [11] agreed exactly with each other, but only approximately with the true returns. They produced a reconstructed TSX index that has the same general shape as the real TSX history - but with growth from 1970 to 2013 that is 3 times as great as the true growth. Analysis using those numbers would be far too optimistic.

Below are good historical data for S&P/TSX Composite, S&P/TSX Composite total return, S&P 500 and S&P total return for 1969 to 2013. Bond indexes will be added later. The data are presented as an index level at the *end* of the associated calendar year (typically December 29 to 31), from which the return *during* that year can be calculated. Their provenance is as follows:

- TSX Composite index was obtained primarily by laboriously reading exact end-of-year values from the interactive graph at Globe Investor [12], which spans 1977 to 2013. For 1969 to 1976, the index was obtained from Trading Economics [13] by reading (estimating) end-of-year values from their graphs. The latter source is in reasonably good agreement with Globe Investor from 1977 to 2013, except for wide differences (about 25%) in 1980 and 1981. For January 1, 1975 (which we take as December 31, 1974) it also misses the TSX Composite base value of 1000, but this appears to be the result of a 1-month displacement in the graph. For lack of anything better, this is how the TSX data was synthesized.
- TSX Composite index total return (i.e., accounting for dividends and assuming their reinvestment), courtesy of RBC Dominion Securities. It agrees with the interactive graph at Globe Investor over the latter's span of 1999 to 2013.
- S&P 500 index back to 1978 was also obtained exactly from an interactive graph at Globe Investor. The calculated annual returns agreed exactly with those at the Wikipedia entry for S&P 500, which justified use of the Wikipedia data for the full 1969 to 2013 range. It is presented as the growth in value of $1 invested at the end of 1969 (equivalently, on January 1, 1970.
- S&P 500 total return (i.e., accounting for dividends and assuming their reinvestment) was taken directly from the same Wikipedia site, now assumed to be trustworthy. It is presented as the growth in value of $1 invested at the end of 1969 (equivalently, on January 1, 1970.

The table of these market data is in the collapsed area at the start of this chapter, and a portion of it is shown below, with column headings. A full presentation of the data is in Appendix A.

column:	0	1	2	3	4
data:	year	TSX Comp, end of yr	TSX Comp totl retn end of yr	S&P 500, end of yr	S&P 500 total retn, end of yr

markets =

	0	1	2	3	4
0	1969	1055	2465.997	1	1
1	1970	990	2384.619	1	1.04
2	1971	1020	2421.609	1.108	1.19
3	1972	1255	2998.652	1.281	1.41
4	1973	1205	2865.488	1.059	1.21
5	1974	885	1982.661	0.755	0.89
6	1975	975	1928.41	0.993	1.22
7	1976	1005	1876.624	1.183	1.51
8	1977	1017	1965.399	1.047	1.4
9	1978	1270	2431.473	1.058	...

Columns 5 and 6 are reserved for Canadian bonds and international equity, 7 for synthetic market.

For convenience, give names to these data columns:

$year := markets^{\langle 0 \rangle}$ $TSX := markets^{\langle 1 \rangle}$ $TSXtotal := markets^{\langle 2 \rangle}$

$SP := markets^{\langle 3 \rangle}$ $SPtotal := markets^{\langle 4 \rangle}$

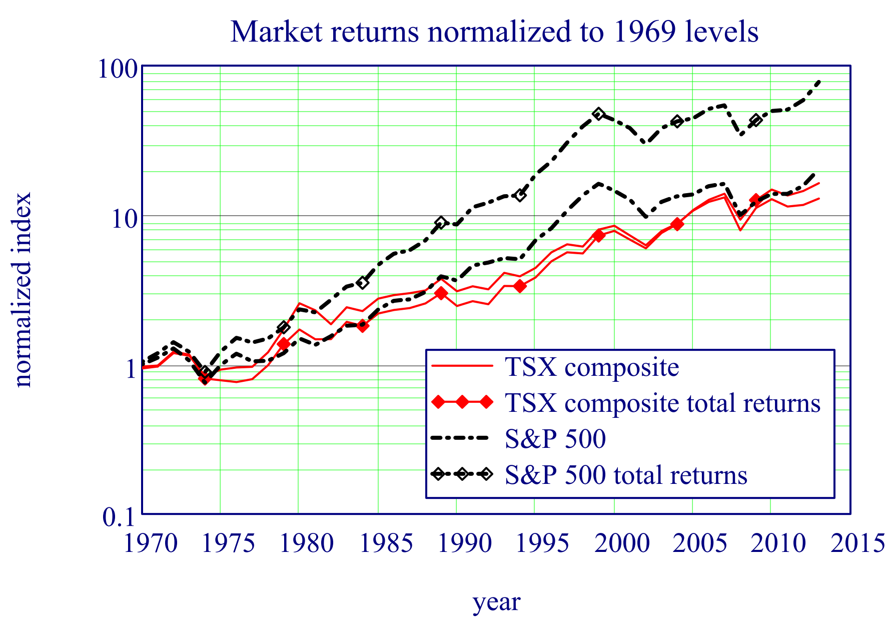
Also for convenience, convert it to a corresponding array of rates (in %):

$$marketretns^{\langle 0 \rangle} := markets^{\langle 0 \rangle}$$

$c := 1 .. cols(markets) - 1$ $marketretns^{\langle c \rangle} := getrate\left(markets^{\langle c \rangle}\right)$

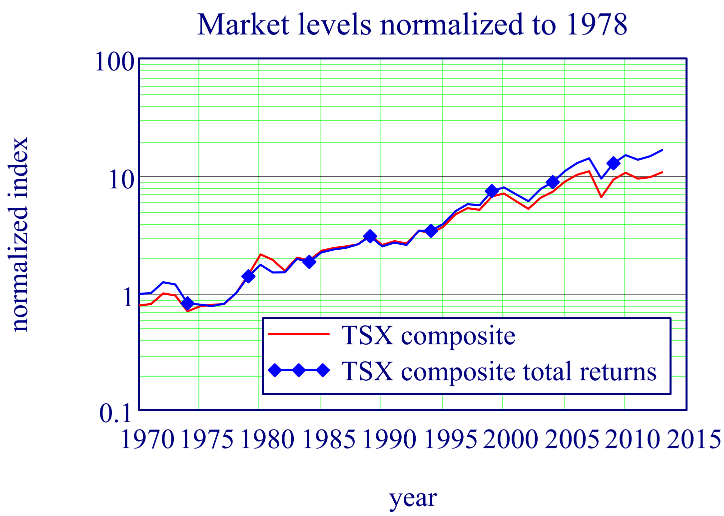
To compare the growth of the indexes, normalize them by the level in their first year (1969), so they all start at the value 1.

$TSX' := \dfrac{TSX}{TSX_0}$ $TSXtotal' := \dfrac{TSXtotal}{TSXtotal_0}$ $SP' := \dfrac{SP}{SP_0}$ $SPtotal' := \dfrac{SPtotal}{SPtotal_0}$



The above plot shows:

- Canadian and American indexes had about the same long-term performance, although they alternated long periods of which grew faster (e.g., S&P 500 grew quickly in the 90s).
- In the last 15 years or so, market swings have been large and the spacing of market peaks (and lows) is somewhat greater than in the past. The greater spacing is even more pronounced when comparing against data earlier in the 20th century.
- Total returns from S&P 500 are considerably better than that of the bare index.
- Total returns in TSX Composite are shown as *lower* than the index, which points to data error, probably in the substituted 1969 to 1976 data in the index. To test this possibility, the plot below normalizes both of them to their values in 1978, after the substituted data.



After 1978, the level of TSX total returns is (almost) consistently higher than the index, as it should be. The only post-78 discrepancy is in 1980 and 1981, the same place that the globeinvestor index record is higher than the tradingeconomics record. Hmm. Overall, though, it appears that the TSX index versus total returns problem is in the 69-76 data, which shows TSX Composite index growth faster than it probably was.

Next, turn to multiyear returns, which can be calculated from an index history like the ones above, by

$$multiyear(mkt, n) := \left| \begin{array}{l} \text{"trailing n-year return from index history in array mkt"} \\ R \leftarrow rows(mkt) \\ for\ \ i \in n..R-1 \\ \quad growth_i \leftarrow 100 \cdot \left(\sqrt[n]{\dfrac{mkt_i}{mkt_{i-n}}} - 1 \right) \\ return\ \ growth \end{array} \right.$$

One-year (i.e., annual) returns are then

$TSXretn := multiyear(TSX, 1)$ $\qquad$ $TSXtotalretn := multiyear(TSXtotal, 1)$

$SPretn := multiyear(SP, 1)$ $\qquad$ $SPtotalretn := multiyear(SPtotal, 1)$

They could equally have been calculated by the function *getrate* in Section 2.3.

Longer multi-year returns (5, 10, 15, 20 years) are calculated similarly below.

$TSXretn5 := multiyear(TSX, 5)$ $TSXretn10 := multiyear(TSX, 10)$

$TSXretn15 := multiyear(TSX, 15)$ $TSXretn20 := multiyear(TSX, 20)$

$SPretn5 := multiyear(SP, 5)$ $SPretn10 := multiyear(SP, 10)$

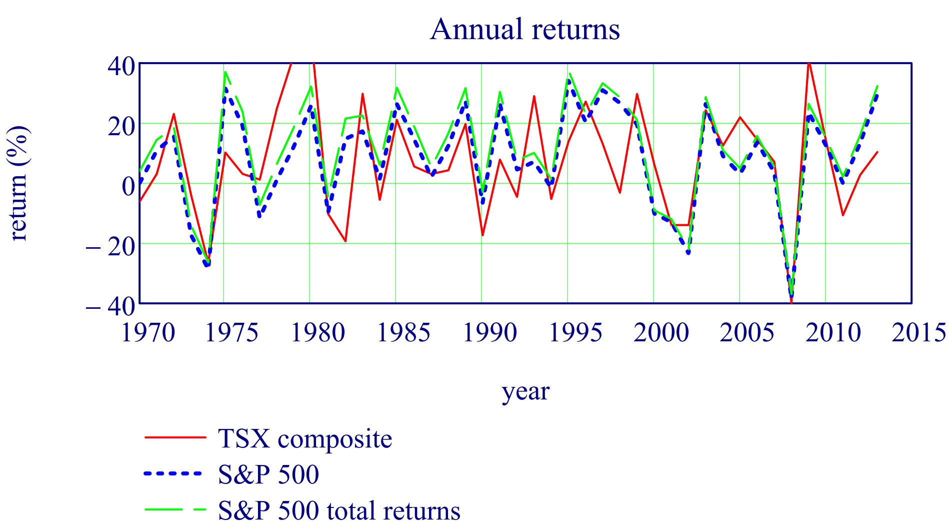
$SPretn15 := multiyear(SP, 15)$ $SPretn20 := multiyear(SP, 20)$

$SPtotalretn5 := multiyear(SPtotal, 5)$ $SPtotalretn10 := multiyear(SPtotal, 10)$

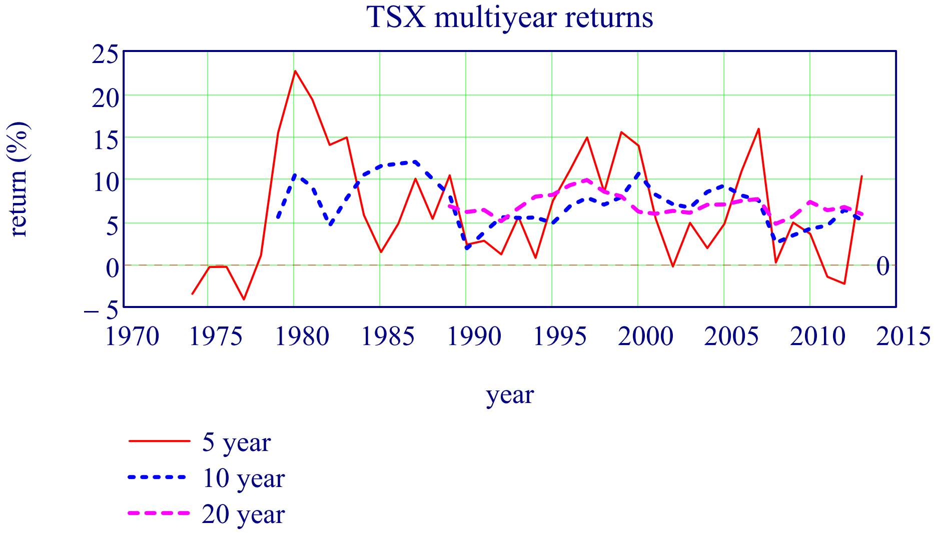
$SPtotalretn15 := multiyear(SPtotal, 15)$ $SPtotalretn20 := multiyear(SPtotal, 20)$

$m5 := 5 .. Rm - 1$ $m10 := 10 .. Rm - 1$ $m15 := 15 .. Rm - 1$ $m20 := 20 .. Rm - 1$

Look at annual returns for all indexes:

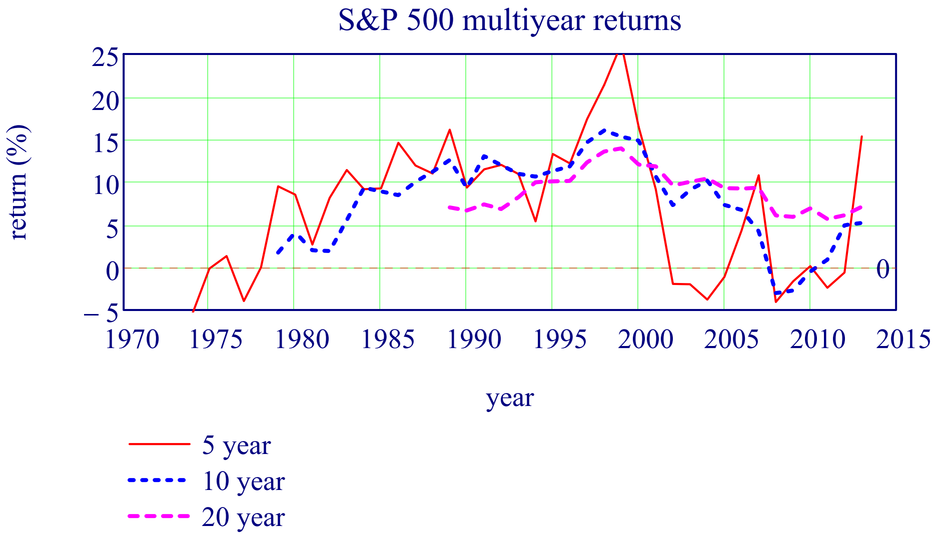


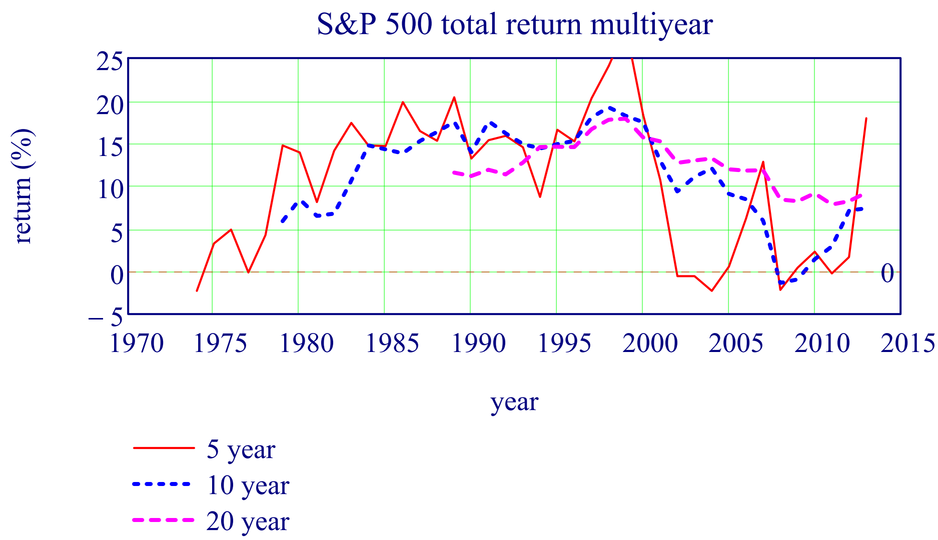
Then multiyear returns for each index separately:



10-year returns are a guide to selecting market extension years (Section 2.6). For TSX Composite:

- best decade, 12.5%: 1978-1987;
- a median decade, 7%: 1993-2002;
- a sub-median decade, 4.8%: 1984-95;
- worst decade, 1.9%: 1981-1990.





These plots show:

- Returns over 5 years can at times be negative. Occasionally, even 10-year returns can go negative. Not good in the withdrawal structure of a RRIF or LIF.
- Multiyear returns have been trending down. On the TSX, 10 and 20 year average returns have been generally between 5% and 10% during the last 5 years. Many analysts predict low returns to continue, perhaps because of inherited turmoil from the early years of this century, or perhaps because of more fundamental issues. However, a calculator program is not the place for this discussion.

Flat Returns

For reference, a market in which the return is the same every year:

$$i := 0 .. rows(year) - 1 \qquad flatrate_i := 1$$

2.6 Sythetic Markets for Stress-Testing the Calculations

Section 2.5 provided historical records for four markets (TSX Composite, TSX Composite Total Return, S&P 500 and S&P 500 Total Return), and two more are planned (Canadian bonds and internation equity). That's a good base on which to test how well combinations of retirement funds and spending aspirations might have played out in the past. However, the historical records are finite; here, they run from 1970 to 2013. Although one can question the relevance of pushing much farther back into historical market behaviour, starting in 1970 does limit the range of market experiences in our testbed. Also, the records are limited to what actually happened - which is of central importance, of course - but one might wonder what the consequences of greater or lesser volatility, or different annualized returns might have been.

This section develops two methods of creating parameterized synthetic markets, ones that have lifelike behaviour and address the two issues noted just above. Specifically:

- The first procedure extends the record beyond 2013 with copies of a selectable segments taken earlier in the record - good, typical or bad.
- The second procedure alters one of the historical records by making it more or less volatile, or with greater or smaller annualized returns.

Both approaches allow stress-testing of the calculator predictions.

Market Extension

The Trinity Study used market returns from 1926 to 1997, a total of 72 years of data. All this report works with is 1970 to 2013, or 44 years of data, which is not really enough for confidence in the signficance of calculated success rates. Worse, there is some evidence (Section 2.5) that market behaviour is changing, with wider swings between market peaks and lows, and a trend toward lower average returns than in the past century.

The calculator in the next chapter runs out to age 95, giving a span of 24 years from the first year of operation of a RRFor LIF at age 72. Even though the calculator lets you select the starting year of historical market returns applied to your account, that year would be limited to 1970 to 1990 - only 21 different years to start it. Again, it doesn't give confidence that you have tested the best or worst market behaviours.

Longer records would be good. Extending backward from 1970 might not be worthwhile, though, if market behaviour in the 21st century really has changed. How about extending forward from 2013? In the absence of a crystal ball, the remedy used here is to let you select a range of years in the historical record, then append the market returns from those years onto the 1970-2013 record, repeating them as necessary to bring it out 25 years to 2038.

The value of this approach is that it lets you stress-test the calculator's forecasts. For example:

- You can select any starting date up to 2014, so that you can see the effect of serious market downturns or the reverse in the early years of your RRIF.
- The post-2013 returns are, of course, conjectural, but you can select what you think is plausible from the past record. Or you can see the worst - for example, with endless repetition of the horror years 2000 to 2008.

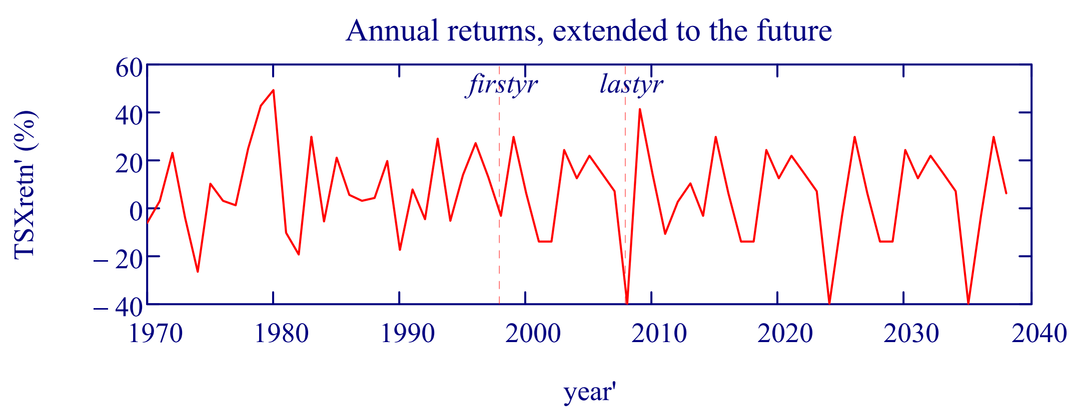
$$
extend(market, firstyear, lastyear) := \left|
\begin{array}{l}
\text{"copy"} \\
mkt \leftarrow market \\
\text{"duration and start index of segment"} \\
length \leftarrow lastyear - firstyear + 1 \\
ifst \leftarrow firstyear - 1969 \\
\text{" "} \\
\text{"base of extension"} \\
R \leftarrow rows(mkt) \\
\text{"extend 25 more years"} \\
for\ \ ixtra \in 0..24 \\
\quad \left| \begin{array}{l} iseg \leftarrow mod(ixtra, length) \\ mkt_{R+ixtra} \leftarrow mkt_{ifst+iseg} \end{array} \right. \\
return\ \ mkt
\end{array}
\right.
$$

As an example, extend the TSX returns data by repetitions of 2000 to 2008:

$firstyr := 1998$ $\qquad$ $lastyr := 2008$

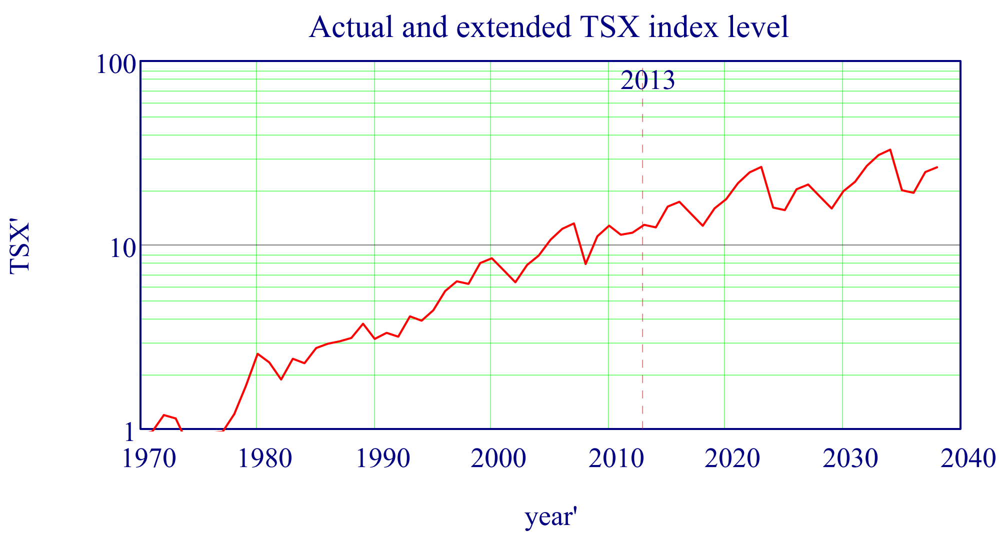
$TSXretn' := extend(TSXretn, firstyr, lastyr)$

And plot it. Note $year'$ is the extended range of years, 1969 to 2038 :



In the plot above, up to 2013 is the actual record, and post-2013 is the extended record.

The effect of extension on the TSX level TSX', where the dash (') denotes an extended record is seen in the next plot.



Adjustable Volatility and Annualized Return

The second stress-testing tool is to alter a true record by modifying its volatility and/or annualized return. If desired, we can also extend it using the method above.

Underlying this approach is the idea that the market can be modelled as the year-by-year product of zero-volatility exponential growth component and a zero-growth market volatility component. We start with a specified market record (e.g., TSX Composite) and separate it into its *post facto* growth and volatility components. After that, we modify the two components, then combine them into a new record with different growth and volatility but still a nodding resemblance to a true market.

More about the model. Suppose we have a record of an index extending over $K+1$ years (for us, K is 44, since it runs 1969 to 2013)

index level I_k $\quad k = 0, 1 .. K \qquad 1969, 1970 .. 2013 \qquad K = 44$

As separate growth and components:

$I_k = G_k \cdot V_k$ $\qquad$ where $\quad G_k = a \cdot g^k$ $\quad$ a is starting value, g is annualized growth factor

Absolute swings in I grow with time, even without changes in the range of V, because G grows. Multiplicative decomposition and exponential growth, so use logs

$$ln(I_k) = ln(a) + k \cdot g + ln(V_k)$$

This suggests fitting values of a and g to the I record. That also reveals V. Least squares should do it.

This is our model:

$$\begin{pmatrix} ln(I_0) \\ ln(I_1) \\ \vdots \\ \vdots \\ ln(I_K) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & K \end{pmatrix} \cdot \begin{pmatrix} ln(a) \\ g \end{pmatrix} + \begin{pmatrix} ln(V_0) \\ ln(V_1) \\ \vdots \\ \vdots \\ ln(V_K) \end{pmatrix}$$

lnI $\qquad H \qquad c \qquad lnV$

The LS estimate gives the decomposition

$$\begin{pmatrix} ln(a) \\ g \end{pmatrix} = \left(H^T \cdot H\right)^{-1} \cdot H^T \cdot lnI$$

$$lnG = H \cdot \left(H^T \cdot H\right)^{-1} \cdot H^T \cdot lnI$$

$$lnV = lnI - lnG$$ multiplicative decomposition in the log domain

The mean of the vector lnV is zero, since the fit includes an additive coefficient $ln(a)$. The standard deviation σ_{lnV} of lnV is one measure of volatility.

Modifying the true record - which is the point of this section - is easily done. To change the underlying annual growth rate, just change the value of g to some new value g'. To change the volatility, multiply the lnV vector by a non-negative factor β which relates the new volatility to that of the true underlying market. If $\beta = 0$, then there is no volatility; if $\beta < 1$, then new volatility is less that true market; if $\beta = 1$ then volatility is that of the true market; if $\beta > 1$ then volatility is greater.

Then new values (indicated by a dash) are:

$$lnV'_k = \beta \cdot lnV_k \qquad V'_k = \left(V_k\right)^{\beta}$$

$$G'_k = a \cdot g'^k \qquad I'_k = G'_k \cdot V'_k = a \cdot g'^k \cdot \left(V_k\right)^{\beta}$$

Nice features of this approach:

- lnV' still has zero mean, like lnV.
- β directly scales the standard deviation σ_{lnV} of the log volatility $\beta\ lnV$.
- Changing g' does not change volatility and changing β does not change growth.
- Decomposition by LS of I' reveals g' and the scaled volatility sequence $lnV' = \beta\ lnV$.

The standard deviation points +/- σ_{lnV} of lnV have counterparts in the percent departure of I' above and below G' with the same probability (or, rather, frequency) of 68%:

$$pct_{\sigma hi} = 100 \cdot \left(e^{\beta \cdot \sigma_{lnV}} - 1\right) \qquad pct_{\sigma lo} = 100 \cdot \left(e^{-\beta \cdot \sigma_{lnV}} - 1\right)$$

The procedure *morphmkt* below does the transformations to change the gain and volatility of a real market record. Its arguments are *mktcol* (selects a column of *markets* (i.e., one of TSX, S&P 500, etc.) for transformation, γ_{pct} the new growth rate in pct, β the volatility scale factor, and *out*, the value of which selects one of several possible outputs according to this:

- $out = 0$: returns calculated values $\left(a \ \ g_{pct} \ \ \sigma_{lnV} \ \ pct_{\sigma hi} \ \ pct_{\sigma lo}\right)$
- $out = 1$: returns derived $augment(lnG, lnV)$ as 2-column array
- $out = 2$: returns derived $augment(G, V)$ as 2-column array
- $out = 3$: returns modified $augment(G', V')$ as 2-column array
- $out = 4$: returns the array *markets*, modified to place I' in column 7
- $out = 5$: returns the array *marketretns*, modified to place the annual gains of I' (%) in column 7

$$morphmarket(mktcol, \gamma_{pct}, \beta, out) :=$$

"get selected market index"

$$I \leftarrow markets^{\langle mktcol \rangle}$$

$$K \leftarrow rows(I) - 1$$

"overhead arrow means componentwise"

$$lnI \leftarrow \overrightarrow{ln(I)}$$

"make H matrix"

$$for\ \ k \in 0..K$$

$$\quad H_{k,0} \leftarrow 1$$

$$\quad H_{k,1} \leftarrow k$$

$$c \leftarrow \left(H^T \cdot H\right)^{-1} \cdot H^T \cdot lnI$$

$$lnG \leftarrow H \cdot c$$

$$lnV \leftarrow lnI - lnG$$

$$G \leftarrow \overrightarrow{exp(lnG)}$$

$$V \leftarrow \overrightarrow{exp(lnV)}$$

""

""

""

""

"calculate a few properties"

$$(a\ \ g) \leftarrow \left(\overrightarrow{exp(c)}\right)^T$$

$$g_{pct} \leftarrow 100 \cdot (g - 1)$$

$$\sigma_{lnV} \leftarrow stdev(lnV)$$

$$\mu_{lnV} \leftarrow mean(lnV)$$

$$pct_{\sigma hi} \leftarrow 100 \cdot \left(e^{\sigma_{lnV}} - 1\right)$$

$$pct_{\sigma lo} \leftarrow 100 \cdot \left(e^{-\sigma_{lnV}} - 1\right)$$

$$return\ \left(a\ \ g_{pct}\ \ \sigma_{lnV}\ \ pct_{\sigma hi}\ \ pct_{\sigma lo}\right)\ \ if\ \ out = 0$$

$$return\ \ augment(lnG, lnV)\ \ if\ \ out = 1$$

$$return\ \ augment(G, V)\ \ if\ \ out = 2$$

"now modify G and V according to parameters"

$\gamma \leftarrow 1 + \frac{\gamma_{pct}}{100}$

for $k \in 0..K$

  $G'_k \leftarrow a \cdot \gamma^k$

  $V'_k \leftarrow (V_k)^\beta$

return $augment(G', V')$ *if* $out = 3$

"modify 'markets' by adding new index as col 7"

$m \leftarrow markets$

$m^{\langle 7 \rangle} \leftarrow \overrightarrow{(G' \cdot V')}$

return m *if* $out = 4$

"modify 'marketretns' by adding new retns as col 7"

$mret \leftarrow marketretns$

$mret^{\langle 7 \rangle} \leftarrow getrate\left(m^{\langle 7 \rangle}\right)$

return $mret$ *if* $out = 5$

return "bad value of 'out'" *otherwise*

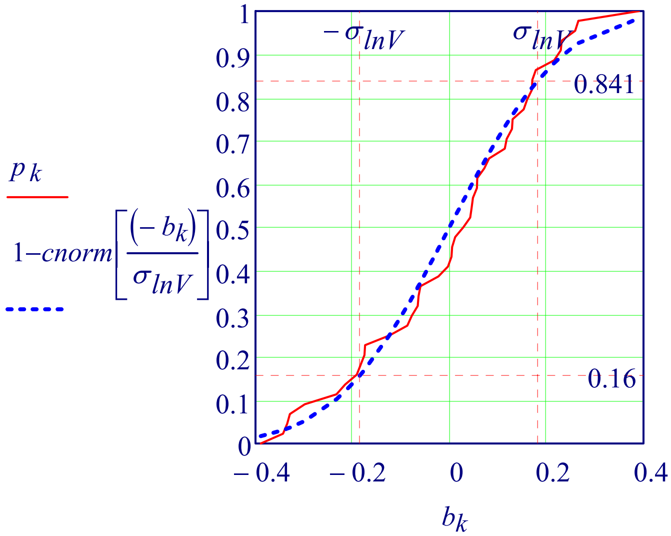
$K := rows(markets) - 1 \qquad k := 0..K$

check derived properties $\left(a \;\; g_{pct} \;\; \sigma_{lnV} \;\; pct_{\sigma hi} \;\; pct_{\sigma lo} \right) := morphmarket(1, 6.9, 1, 0)$

$\left(a \;\; g_{pct} \;\; \sigma_{lnV} \;\; pct_{\sigma hi} \;\; pct_{\sigma lo} \right) = (882.888 \;\; 6.854 \;\; 0.184 \;\; 20.176 \;\; -16.789)$

look at distribution of lnV $\quad R := morphmarket(1, 7, 1, 1)$

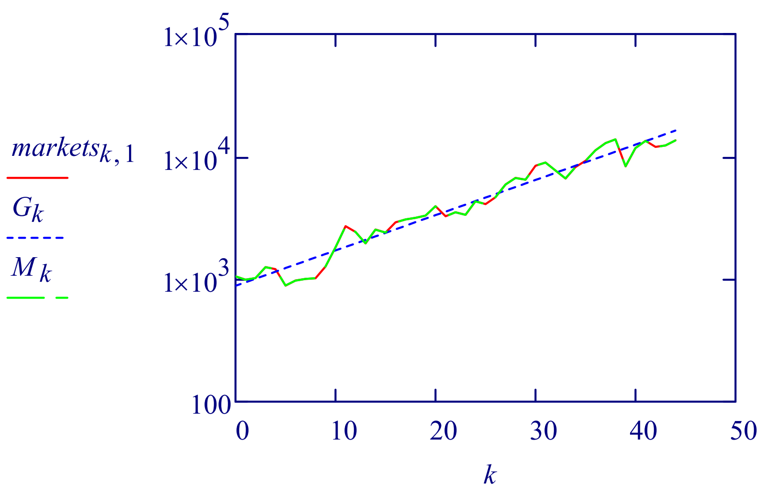
cdf of lnV $\quad b := sort\left(R^{\langle 1 \rangle}\right) \quad p_k := \frac{k}{K}$



Very roughly Gaussian, since +/- $\sigma_{\ln V}$ (horiz axis) are almost equal to Gaussian cdf values (vert axis); however, mean (0) is slightly different from the median (see right). If Gaussian, then V components are log-normal.

check decomposed and resynthesized market

$R := morphmarket(1,0,1,2)$ $\quad G := R^{\langle 0\rangle}$ $\quad V := R^{\langle 1\rangle}$ $\quad M := \overrightarrow{(G\cdot V)}$

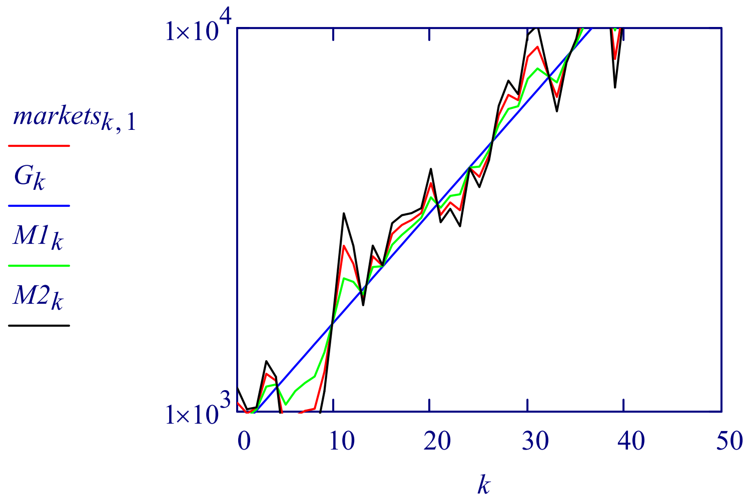


reassembled and the original match up

check effect of β scaling

$R1 := morphmarket\left(1, g_{pct}, 0.5, 3\right)$ $\qquad R2 := morphmarket\left(1, g_{pct}, 1.5, 3\right)$

$M1 := \overrightarrow{\left(R1^{\langle 0\rangle}\cdot R1^{\langle 1\rangle}\right)}$ $\qquad M2 := \overrightarrow{\left(R2^{\langle 0\rangle}\cdot R2^{\langle 1\rangle}\right)}$



Volatility does decrease if β<1 and increase if β>1.

In per cent, the upper and lower pct vals (rel to the mean gain) corresponding to +/- σlnV depend on β like this

$$pcthi = 100\cdot\left(e^{\beta\cdot\sigma_{lnV}} - 1\right)$$

$$pctlo = 100\cdot\left(e^{-\beta\cdot\sigma_{lnV}} - 1\right)$$

3. A MONEY FLOW CALCULATOR FOR THE RRIF/LIF YEARS

3.1 About the Calculator

The big question: How long will our money last? The calculator in this chapter should give you some insight, although it doesn't claim to be exact - and of course the future will do whatever it will do.

The well-known Trinity Study [1],[2], which led to the "4% withdrawal in inflation supports 30 years with 95% success rate" rule of thumb, gave many and various answers to the big question. Contrast it with studies you can conduct yourself with this calculator:

- The Trinity Study considered only fixed-size (optionally inflation-adjusted) withdrawals. The calculator instead accounts for the CRA-mandated variable withdrawal rates (and inflation), which force larger withdrawals from a RRIF/LIF in the early years.
- The Trinity Study considered 72 years (1926-1997) of recorded rates of market return for bonds and equities (stocks). The calculator instead considers only 44 years (1970 to 2013) of stock market returns (though of four such markets) and only flat (constant) return values for fixed income.
- The Trinity Study operates on a monthly time scale. The calculator operates on a coarser annual time scale, corresponding to single annual withdrawals, which is somewhat unrealistic.
- The Trinity Study stuck to what the market had actually produced over the years. The calculator additionally allows you to stress-test your finances with two hypothetical markets. One analyzes a true market record, then lets you tweak the growth and volatitility. The other creates a post-2013 market behaviour by appending a selected section of the historical record - good, bad or boring - to follow 2013 of the true record.
- The Trinity Study considered only historical inflation rates. The calculator also allows use of fixed inflation rates of various values.
- The calculator also includes a simple model of your personal finance, including your other sources of taxable income and what you do with the excess funds from the early large withdrawals.
- The calculator also accounts for: RRIF, LIF or unconstrained withdrawals; Canadian taxation of seniors; selectable post-tax spending targets; and it acts to minimize the tax paid.

This chapter develops the calculator package, which consists of the main program *trajectories*(*A*) and ancillary programs, including:

- *trajectories*(*A*), the calculator itself, which calculates all important quantities over the years;
- *argcheck*(*A*), which checks the validity of the argument values provided to the calculator;
- *goodbadyears*(*A*), which runs *trajectories*(*A*) over many different values of starting year *mktyear1* and records the age (your age) out to which each starting year can support the target spending level;
- *spendhistocdf*(*A*,*extflag*), which is similar to *goodbadyears*(*A*), but presents the results as a histogram of how many starting years support a given age out to which full spending at the target level is supported.

In the collapsed area below is the actual calculator package, since this document is written in Mathcad. Read it, and you can verify its operation for yourself.

3.2 Inherited From Chapter 2

Useful functions:

minrate(*age*) min RRIF or LIF withdrawal rate (%)

maxrate(*age*) max LIF withdrawal rate (%)

TaxPaid(*GI*, *OAS*, *taxmodel*, *I13*) tax paid on gross income *GI* ($K)

NetIncome(*GI*, *OAS*, *taxmodel*, *I13*) net income from gross income *GI* ($K)

GrossIncome(*NI*, *OAS*, *taxmodel*, *I13*) gross income needed to give net income *NI* ($K)

OASclaw(*GI*, *OAS*) the OAS "clawback" calculation

getlevel(*rate*) converts an array of annual rates to a vector of corresponding levels, with element 0 (the first one) set to 1

getrate(*level*) converts an array of index levels to a vector of corresponding annual rates

extend(*market*, *firstyear*, *lastyear*) extends a vector of true market returns (here, 1969 to 2013) out to 2038 by appending the section of returns from *firstyear* to *lastyear*, repeating if necessary

Useful arrays, inherited as global values

markets 44 x 5 array of market indexes, with columns: 0, year; 1, TSX Composite; 2, TSX Composite total return; 3, S&P 500; 4, S&P 500 total return. Index values are taken on the last day of the year.

marketretns 44 x 5 array of market returns obtained by converting columns 1 to 4 of *markets* to annual growth in percent to end of year.

flatrate flat (constant) returns (for reference)

CPIrate Canadian CPI annual changes (%) from 1969 to 2013

3.3 The Calculator Argument List

The longevities of the funds and the "trajectories" of account balances depend on many, many variables. Those variables are termed the *arguments* of the calculator. They are grouped and defined below.

About you

taxmodel	Use 1 for an individual, 2 for a pension-splitting, equal-aged couple.
myagenow	Your age (in range 71 to 94). This lets you use the calculator at some point during those years, when you wonder how long the remaining funds will last. An age younger than 71 is changed to 71 (the opposite of the fountain of youth). Calculation of new values begins at *myagenow* + 1. If you are calculating for a couple with widely different ages, the calculator is of limited value.
thisyear	What year is it right now, when you are using the calculator? The year is used to adjust tax rates relative to 2013. If you don't want that adjustment, make *thisyear* less than 2014 and the calculator will use the 2013 tax table directly.

About your RRIF/LIF account

RLNflag	"RRIF, LIF, Neither" Use 0 if your account is a RRIF, with minimum required withdrawals, 1 if it is a LIF, with both minimum and maximum withdrawals, and 2 if it is neither, with no constraints on withdrawals. Many people have both RRIFs and LIFs, but the calculator considers only one or the other.
equitysplit	This is the percent of your account in equities (stocks, mutual funds, ETFs, etc). The remainder is considered to be in fixed income (individual bonds, GICs, etc), though not bond funds. Account is continually rebalanced to maintain the split.
Bstart	The starting balance of the account, in thousands ($K). "Starting balance" means the value at age *myagenow*.
InvFee	Fees for maintaining your RRIF or LIF can take many forms, such as an annual fixed charge, or an annual percentage (e.g., investment advisor fees), or transaction fees for buying and selling securities, or MERs in mutual funds and (to a much lesser extent) in ETFs. In the calculator, they are approximated as a single annual "investment fee" as a percentage of the balance in the account.

About market returns and inflation

FIretn — The annual growth rate (%) that applies to the fixed income portion of the account. For simplicity, it is fixed.

SaveRetn — The annual rate of return (%) of the savings or non-registered investment account that holds the excess income from the forced withdrawals from your RRIF or LIF. If it is negative, the rate of return is taken to be the same as the market applied to the RRIF or LIF.

whichmkt — Which market affects your equities? Choose from: 1, TSX Composite; 2, TSX Composite total return; 3, S&P 500; 4, S&P 500 total return; 5, 6: reserved for Cdn bonds, int'l equity; 7: synthetic market with selectable gain and volatility (Secn 2.6).

basemkt

g_{pct}

β

For stress-testing your portfolio, you can have the calculator create a synthetic market with the feel of a real one by selecting *whichmkt* = 7 (see above). The calculator modifies *basemkt* (a *whichmkt* value 1,2,...,6, see above) by separating its growth and volatility components. The synthetic market is then given annualized growth g_{pct} (%) and volatility β times that of the base market.

mktseg1

mktseg2

Also for stress-testing, the calculator extends the historic market record out to 2038 by repeating a selectable segment of the historic record. The first and last years of this segment are specified by *mktseg1* - in the range 1970 to 2013 - and *mktseg2* - in the range *mktseg1 + 1* to 2013.

A guide to selecting market extension years might be the annualized 10-year returns of the TSX composite index (Section 2.5). They included:

- best decade, 12.5%: 1978-1987;
- a median decade, 7%: 1993-2002;
- a sub-median decade, 4.8%: 1984-95;
- worst decade, 1.9%: 1981-1990.

Of course, you are free to select whatever years you want.

mktyear1 — The first year of historic market returns to be applied to the equities portion of your RRIF or LIF. Use a value in the range 1970 to 1990 to stick to true historic returns. Values from 1991 to 2013 pick up increasing amounts of the hypothetical market behaviour in the extended market history, and 2014 is market extension alone.

InfRate — Annual inflation rate (%). If it is negative, historical inflation rates are used, starting at *mktyear1*.

About your personal finances

SaveStart — The savings or non-registered investment account that holds the excess income from the forced withdrawals from your RRIF or LIF will be called *Save*. Its initial value (i.e., at *myagenow*) is *SaveStart* , to represent your savings at that age.

IPstart — The initial annual value (i.e., at *myagenow*) in $K of all your pensions or taxable annuities that are indexed to inflation, including CPP and OAS. If you are calculating for a pension-splitting couple (*taxmodel* equals 2), make it your combined indexed pensions. If partially indexed, move the appropriate portion to non-indexed pension just below.

NIP — The annual value in $K of all your taxable pensions that are not indexed to inflation. If you are calculating for a pension-splitting couple (*taxmodel* equals 2), make it your combined non-indexed pensions.

OASstart — The initial annual value (i.e., at *myagenow*) in $K of the OAS component of your indexed pensions *IPstart*. OAS is also indexed to inflation, but is identified separately to allow clawback calculations. If you are calculating for a pension-splitting, equal-aged couple, make it your combined OASes. The calculator treats the two OAS streams as equal.

SpendTarget — Your annual spending *Spend* ***in post-tax $K*** is indexed to inflation by the calculator. Its initial target value is *SpendTarget*. It is a key quantity, since the calculator works to support your specified spending level. Of course, the higher the spending, the sooner the funds run out.

The argument array

Placing all the arguments explicitly in the calculator's argument list would be clumsy and would take up too much space. Instead, we'll put them in a 6x4 argument array, defined as follows:

$$A = \begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix}$$

Packing arguments like this allows procedures to be called with a single argument *A*. The only drawback is that they have to be unpacked within the procedure, but that's easy.

Packing arguments like this allows procedures to be called with a single argument *A*. The only drawback is that they have to be unpacked within the procedure, but that's easy.

3.3 General Description of the Calculator Package

More about the calculator operations

Within *trajectories*, the calculator program, calculations follow a straightforward pattern of year-by-year computation of RRIF/LIF income generation and spending/saving outside the RRIF/LIF. In words, it looks pretty much like this:

```
trajectories_in_words(A) = | "unpack arguments from A"
                           | "extend the market and inflation record to 2038"
                           | "initialize values of Save, IP, OAS, Spending and taxation..."
                           | "...at age myagenow, using their start values"
                           | " "
                           | "initialize historic mktyr from mktyear1"
                           | for  age ∈ myagenow + 1 .. 95
                           |     | "increase IP, OAS, Spending and I13 by inflation"
                           |     | "calculate withdrawal from RRIF/LIF (a bit tricky, see below)"
                           |     | "update balances in RRIF/LIF and Save"
                           |     | "apply market changes to RRIF/LIF and Save..."
                           |     | "... and increment the historic mktyr"
                           | return  "all age-varying quantities as cols of an array 'future' "
```

How to calculate the withdrawal amount

Calculation of the withdrawal value is noted as "a bit tricky" above. Here's why:

- In the early years, it's not hard, since the minimum withdrawal is sufficient to support the target spending level. At some point, it requires drawing on the accumulated excess income in *Save*.
- When *Save* is run down to zero, withdrawals must be greater than the minimum to support target spending...
- ... but if those withdrawals hit the maximum amount (in a LIF) or if the RRIF is depleted below the required level, we have to take out what we can, and cause spending to be lower than desired.

Those calculations depend the *current* values of the age-indexed quantities

B	the balance in the RRIF/LIF
GIother	gross income from all sources other than the RRIF/LIF
Save	the balance in the savings account
Spend	the spending level
OAS	the OAS level

RLNflag	0 for a RRIF, 1 for a LIF, 2 for neither
age	your age at this step of the calculator
taxmodel	1 for individual, 2 for pension-splitting, equal-aged couple
I13	inflation level relative to that of 2013 - used only in tax calculations

Note: $0 \le B$ $\quad 0 \le W_{min} \le W_{max} \le B$ $\quad$ with equalities iff $B = 0$

W_{ideal} not bounded, above or below

To keep the procedure from sprawling across more than one page, pack the arguments as

$$Wargs = (B \;\; GIother \;\; Save \;\; Spend \;\; OAS \;\; RLNflag \;\; age \;\; taxmodel \;\; I13)$$

And here's the procedure to calculate the withdrawal W:

```
calcW(Wargs) := | "unpack arguments - note they have already gone through argcheck"
                | (B  GIother  Save  Spend  OAS  RLNflag  age  taxmodel  I13) ← Wargs
                | "ideally, take out just enough to meet the after-tax spending target"
                | W_ideal ← GrossIncome(gez(Spend − Save), OAS, taxmodel, I13) − GIother
                | W_min ← B·minrate(age)/100
                | W_max ← B·maxrate(age)/100
                | if RLNflag = 0
                |    | "it's a RRIF"
                |    | W ← max(W_min, min(W_ideal, B))
                | if RLNflag = 1
                |    | "it's a LIF"
                |    | W ← max(W_min, min(W_ideal, W_max))
                | if RLNflag = 2
                |    | "it's neither a RRIF nor a LIF"
                |    | W ← max(0, min(W_ideal, B))
                | return W
```

The calculator output

The calculator generates a 12-column array containing the significant quantities as they evolve over the years. There is a row for each year, from *myagenow* to 95, and a column for each such quantity:

0	1	2	3	4	5	6	7	8	9	10	11	12	13
age	B	Save	W	GI	NI	Spend	GIother	OASclaw	IP	OAS	I13	TP	InflSoFar

3.4 The Calculator Itself

$trajectories(A) :=$

"unpack arguments"

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} \leftarrow A$$

$localretns \leftarrow morphmarket\left(basemkt, g_{pct}, \beta, 5\right)$

$mkt \leftarrow extend\left(localretns^{\langle whichmkt \rangle}, mktseg1, mktseg2\right)$

$infl \leftarrow extend(CPIrate, mktseg1, mktseg2)$

"initialize historical market year index"

$hmyi \leftarrow mktyear1 - 1969$

"other initializations for myagenow; calculations begin in the next year"

$age \leftarrow myagenow$

"indexing is written for clarity, not for minimum memory use"

$\left(B_{age} \quad Save_{age} \quad IP_{age}\right) \leftarrow (Bstart \quad SaveStart \quad IPstart)$

$\left(OAS_{age} \quad Spend_{age}\right) \leftarrow (OASstart \quad SpendTarget)$

$\left(W_{age} \quad GI_{age} \quad NI_{age} \quad TP_{age} \quad agevec_{age}\right) \leftarrow (0 \quad 0 \quad 0 \quad 0 \quad age)$

"inflation since myagenow (the starting age in the calculator)"

$InflSoFar_{age} \leftarrow 1$

"assume 2% inflation to get from 2013 tax year to thisyear"

$$I13_{age} \leftarrow \begin{cases} \left(1 + \dfrac{2}{100}\right)^{thisyear-2013} & \text{if } thisyear > 2013 \\ 1 & \text{otherwise} \end{cases}$$

""

"forge into the future, year by year:"

$for\ \ age \in myagenow + 1 .. 95$

" "

"apply inflation:"

$Irate \leftarrow if\left(InfRate \geq 0, InfRate, infl_{hmyi}\right)$

$Ifactor \leftarrow 1 + \frac{Irate}{100}$

$InflSoFar_{age} \leftarrow Ifactor \cdot InflSoFar_{age-1}$

" "

$IP_{age} \leftarrow Ifactor \cdot IP_{age-1}$

$OAS_{age} \leftarrow Ifactor \cdot OAS_{age-1}$

$Spend_{age} \leftarrow Ifactor \cdot Spend_{age-1}$

$I13_{age} \leftarrow \begin{cases} I13_{age-1} \cdot Ifactor & if\ \ thisyear > 2013 \\ 1 & otherwise \end{cases}$

" "

"gross income from sources other than the RRIF or LIF"

$GIother_{age} \leftarrow IP_{age} + NIP$

"calculate the RRIF/LIF withdrawal"

$Wargs \leftarrow \left(B_{age-1}\ \ GIother_{age}\ \ Save_{age-1}\ \ Spend_{age}\ \ OAS_{age}\ \ RLNflag\ \ age\ \ tax\right.$

$W_{age} \leftarrow calcW(Wargs)$

"and update RRIF/LIF balance"

$B_{age} \leftarrow B_{age-1} - W_{age}$

" "

"gross and net income"

$GI_{age} \leftarrow GIother_{age} + W_{age}$

$NI_{age} \leftarrow NetIncome\left(GI_{age}, OAS_{age}, taxmodel, I13_{age}\right)$

$TP_{age} \leftarrow GI_{age} - NI_{age}$

" "

"update savings and (if necessary) spending"

$available \leftarrow NI_{age} + Save_{age-1}$

" "

"is there enough to support the current spending?"

$if\ \ available < Spend_{age}$

"no - this forces spending to be cut back"

$Spend_{age} \leftarrow available$

"either way, update savings like this:"

$Save_{age} \leftarrow available - Spend_{age}$

" "

"action of the markets changes the balances"

"in RRIF/LIF, split of equities/fixed income affects the return,"

"also does the equivalent of rebalancing"

" "

$$splitrate \leftarrow \frac{equitysplit}{100} \cdot mkt_{hmyi} + \left(1 - \frac{equitysplit}{100}\right) \cdot FIretn$$

"update the balance that was reduced by withdrawal"

$$B_{age} \leftarrow \left(1 + \frac{splitrate}{100}\right) \cdot B_{age}$$

"and pay the house"

$$B_{age} \leftarrow \left(1 - \frac{InvFee}{100}\right) \cdot B_{age}$$

"now the savings acct, find and apply its return, whether fixed or market"

$Srate \leftarrow if\left(SaveRetn \geq 0, SaveRetn, mkt_{hmyi}\right)$

$$Save_{age} \leftarrow \left(1 + \frac{Srate}{100}\right) \cdot Save_{age}$$

" "

"update the historical market year index and build the age vector"

$hmyi \leftarrow hmyi + 1$

$agevec_{age} \leftarrow age$

" "

"OAS adjustment also done here explicitly, just to keep track"

$$claw1 \leftarrow I13_{age} \cdot OASclaw\left(\frac{GI_{age}}{I13_{age}}, \frac{OAS_{age}}{I13_{age}}\right)$$

$$claw2 \leftarrow 2 \cdot I13_{age} \cdot OASclaw\left(\frac{GI_{age}}{2 \cdot I13_{age}}, \frac{OAS_{age}}{2 \cdot I13_{age}}\right)$$

" "

$claw_{age} \leftarrow$ | *claw1* if *taxmodel* = 1 / *claw2* otherwise

" "

"save the interesting quantities as cols in an array, with row for each year"

$future \leftarrow augment(agevec, B, Save, W, GI, NI, Spend, GIother, claw)$

$future \leftarrow augment(future, IP, OAS, I13, TP, InflSoFar)$

"trim off the all the zeros in rows 0 to myagenow-1"

$future \leftarrow submatrix(future, myagenow, rows(future) - 1, 0, cols(future) - 1)$

"release it to the world"

return future

3.5 Ancillary Programs

The value of the main calculator program *trajectories*(*A*) is increased by programs that work on its input or its output.

Working on the inputs to *trajectories*(*A*) are:

- *argcheck*(*A*), which checks each variable in the argument array for validity;
- *ExtendedHistory*(*A*), which jointly extends the selected market and inflation records for a graphical presentation.

Working on the output of *trajectories*(*A*) are:

- *lastfullspend*(*future*), which finds your latest age at which spending at the inflation-adjusted target rate can be sustained;
- *goodbadyears*(*A*), which runs the calculator repeatedly over all market starting years, from 1970 to 2014, and records, for each year, the age out to which the full spending level can be sustained and the balance in the RRIF or LIF at the end of age 95;
- *spendhisto*(*A,extflag,cdfflag*) is like *goodbadyears*(*A*) but returns the results as a histogram of how often each last year of full spending occured over the various market starting years; the flags allow selection of (1) true or true-plus-extended history and (2) display of histogram or cumulative distribution function.
- *meannz*(*x*) calculates the mean of the *non-zero* entries of vector *x*;
- *totalvalue*(*R,col,age,NPVflag*) calculates the total of column *col* of *R* (the output of *trajectories*) from *myagenow* to *age*, inclusive, either as a straight sum or as net present value.
- *overall*(*R,A,exitage,NPVflag*). Despite leading a model life, our retiree will die. For arguments *A*, associated *trajectories* output *R* and given exit age, this procedure returns an array with components: 0 tax paid during lifetime; 1 RRIF/LIF balance at end of exit year; 2 tax paid on that lump sum; 3 net (post-tax) value of the lump sum; 4 total tax paid, lifetime and by estate; 5 remaining balance in savings account; 6 net estate, the savings balance plus net lump sum; 7 lifetime OAS clawback. All optionally corrected to present value by NPVflag.

Argument check

Typing in incorrect values for the calculator's arguments is so easy to do, so hard to spot and leads to so much frustration and lost time. Simple validity tests up front can make your day more pleasant.

$argcheck(A) :=$

" unpack arguments"

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} \leftarrow A$$

return "taxmodel must be 1 or 2" *if* $taxmodel \neq 1 \wedge (taxmodel \neq 2)$

return "myagenow must be in 71..94" *if* $\neg(71 \leq myagenow \leq 94)$

return "myagenow must be an integer" *if* $myagenow - round(myagenow) \neq 0$

return "RLN flag must be 0, 1 or 2" *if* $RLNflag \neq 0 \wedge (RLNflag \neq 1) \wedge (RLNflag \neq 2$

return "equitysplit must be in 0 to 100" *if* $\neg(0 \leq equitysplit \leq 100)$

return "Bstart must not be negative" *if* $Bstart < 0$

return "InvFee must not be negative" *if* $InvFee < 0$

return "FIreturn must not be negative" *if* $FIretn < 0$

return "whichmkt must be in 1 to 7" *if* $\neg(1 \leq whichmkt \leq 7)$

return "whichmkt must be an integer" *if* $whichmkt - round(whichmkt) \neq 0$

return "mktyear1 must be integer in 1970...2014" *if* $\neg(1970 \leq mktyear1 \leq 2014)$

return "mktseg1 must be integer in 1970...2012" *if* $\neg(1970 \leq mktseg1 \leq 2012)$

return "mktseg2 must be integer in mktseg1+1...2013" *if* $\neg(mktseg1 + 1 \leq mktseg2 \leq$

return "SaveStart must not be negative" *if* $SaveStart < 0$

return "IPstart must not be less than OASstart" *if* $IPstart < OASstart$

return "NIP must not be negative" *if* $NIP < 0$

return "OASstart must not be negative" *if* $OASstart < 0$

return "SpendTarget must not be negative" *if* $SpendTarget < 0$

return "basemkt must be integer in 1, 2,..., 6" *if* $(basemkt < 1) \vee (basemkt > 6)$

return "OK, good to go"

Joint extension of market and inflation

For insight when playing with the calculator, the results of market extension for stress-testing (Section 2.6) are shown graphically for both the selected market and the inflation record. The procedure below does it in one place.

$$
ExtendedHistory(A) := \left|
\begin{array}{l}
\text{" unpack arguments"} \\
\begin{pmatrix}
taxmodel & myagenow & thisyear & RLNflag \\
equitysplit & Bstart & InvFee & FIretn \\
whichmkt & mktyear1 & mktseg1 & mktseg2 \\
SaveRetn & InfRate & SaveStart & IPstart \\
NIP & OASstart & dummy & SpendTarget \\
basemkt & g_{pct} & \beta & dummy
\end{pmatrix} \leftarrow A \\
localretns \leftarrow morphmarket\left(basemkt, g_{pct}, \beta, 5\right) \\
mkt \leftarrow extend\left(localretns^{\langle whichmkt \rangle}, mktseg1, mktseg2\right) \\
index \leftarrow getlevel(mkt) \\
infl \leftarrow extend(CPIrate, mktseg1, mktseg2) \\
INF \leftarrow getlevel(infl) \\
for \;\; i \in 0 .. rows(mkt) - 1 \\
\quad year_i \leftarrow 1969 + i \\
return \;\; augment(year, index, INF)
\end{array}
\right.
$$

Last year of full spending

The calculator is set up to maintain spending at the inflation-adjusted target *SpendTarget* as long as it can. When the RRIF/LIF funds run out, the actual spending drops quickly to the level that can be sustained by other pension funds, CPP and OAS. The last year of full spending can be found from the spending record (column 6 of the calculator output) as the first year at which spending drops in the following year.

```
lastfullspend(future) := | " 'future' is the output from the 'trajectories' program"
                         | "column 6 is Spend level"
                         | c ← 6
                         | "year by year, look for a drop"
                         | last ← −1
                         | for  i ∈ 1 .. rows(future) − 1
                         |   if  future[i,c] < future[i−1,c]
                         |      | last ← i − 1
                         |      | break
                         | "if no drop is found, last year is '95 or older' "
                         | last ← i  if  last = −1
                         | age ← future[last,0]
                         | return  age
```

Good and bad starting years

Even with equal withdrawals, as in the Trinity Report, poor market returns in the first few years jeopardize the performance over the full time span. CRA's requirement for accelerated withdrawals in the early years of a RRIF or LIF exacerbates the problem. The calculator lets you select an historical market year to start the effect of market records on your RRSP/LIF, but you do it just one year at a time. Very tedious. The program below reduces the tedium. A little.

In more detail, the calculator runs from *myagenow*+1 out to 95; for example, if you set it up at age 71, then it updates quantities, beginning at age 72, for a total of 24 years. It does so for a specific value of *mktyear1* (the market starting year); for example, if your age 72 corresponded with a market as it was in 1978. The program *goodbadyears*(*A*) returns, for the full range of possible market starting years, what the key outcome - your age out to which full spending can be maintained - for each year is.

Note that 1990 is the latest starting year for which the true market records are used over the 24-year calculator span. Later than that, the record is increasingly drawn from the extended section of the market returns, obtained as copies of a designated segment of earlier, true returns. The record from starting year 2014 is exclusively in the extended section.

The output of *goodbadyears*(*A*) is an array in which column 0 is the starting year (1970 to 2014), column 1 is the latest age out to which full spending can be maintained, and column 2 is the balance of the RRIF at the end of age 95. There's a row for each starting year.

```
goodbadyears(A) := | "unpack arguments"
                   | ( taxmodel     myagenow   thisyear    RLNflag     )
                   | ( equitysplit  Bstart     InvFee      FIretn      )
                   | ( whichmkt     mktyear1   mktseg1     mktseg2     ) ← A
                   | ( SaveRetn     InfRate    SaveStart   IPstart     )
                   | ( NIP          OASstart   dummy       SpendTarget )
                   | ( basemkt      g_pct      β           dummy       )
                   | "working copy of arguments for the calculator runs"
                   | A' ← A
                   | for startyr ∈ 1970 .. 2014
                   |   | " replace mktyear1 with startyr in array A' "
                   |   | A'_{2,1} ← startyr
                   |   | future ← trajectories(A')
                   |   | lastgoodage ← lastfullspend(future)
                   |   | alllastgood_{startyr−1969} ← lastgoodage
                   |   | B95_{startyr−1969} ← future_{24,1}
                   |   | year_{startyr−1969} ← startyr
                   | return augment(year, alllastgood, B95)
```

Last spending year histogram and cumulative distribution function

An alternative way to view the results of *goodbadyears*(*A*), the multi-run program above, is as a histogram. For example, how many starting market years allowed full spending out to age 86? The program *spendhistocdf*(*A,extflag,cdfflag*) constructs that histogram. Its arguments:

- *A*, the argument array, from which the first starting year in the set of runs is *mktyear1* in the array;
- *extflag*, the extension flag; if it is set to 0, the market starting years are confined to the true market record, so 1970 to 1990; if it is set to 1, the starting years can run from 1970 up to 2014;
- *cdfflag* , if set to 0 produces histogram, if set to 1 produces cdf.

```
ones(R, C) := | for r ∈ 0 .. R − 1          zeros(R, C) := | for r ∈ 0 .. R − 1
              |   for c ∈ 0 .. C − 1                       |   for c ∈ 0 .. C − 1
              |     X_{r,c} ← 1                            |     X_{r,c} ← 0
              | return X                                   | return X
```

$$
spendhistocdf(A, extflag, cdfflag) := \left|
\begin{array}{l}
\text{"check argument"} \\
return\ \ \text{"bad extflag"}\ \ \ if\ \ (extflag \neq 0) \wedge (extflag \neq 1) \\
goodbad \leftarrow goodbadyears(A) \\
hist \leftarrow zeros(25, 1) \\
\text{"age 71 is lowest bin in the histogram"} \\
for\ \ i \in 0..24 \\
\quad age_i \leftarrow 71 + i \\
laststart \leftarrow if(extflag, 2014, 1990) \\
for\ \ startyr \in 1970..laststart \\
\quad \left| \begin{array}{l}
lastgoodage \leftarrow goodbad_{startyr-1969, 1} \\
lastindex \leftarrow lastgoodage - 71 \\
hist_{lastindex} \leftarrow hist_{lastindex} + 1
\end{array} \right. \\
hist \leftarrow hist \cdot \left(\sum hist \right)^{-1} \\
if\ \ cdfflag \\
\quad \left| \begin{array}{l}
cdf_0 \leftarrow hist_0 \\
for\ \ i \in 1..24 \\
\quad cdf_i \leftarrow cdf_{i-1} + hist_i \\
return\ \ augment(age, cdf)
\end{array} \right. \\
return\ \ augment(age, hist)\ \ \ otherwise
\end{array}
\right.
$$

Mean of non-zero entries

Occasionally, we want to calculate the mean of just the non-zero elements of a vector:

$$
meannz(x) := \left|
\begin{array}{l}
N \leftarrow length(x) \\
count \leftarrow 0 \\
sum \leftarrow 0 \\
for\ \ n \in 0..N-1 \\
\quad if\ \ x_n > 0 \\
\qquad \left| \begin{array}{l} count \leftarrow count + 1 \\ sum \leftarrow sum + x_n \end{array} \right. \\
\text{" "} \\
return\ \ \dfrac{sum}{count}\ \ if\ \ count > 0 \\
return\ \ 0\ \ \ otherwise
\end{array}
\right.
$$

Total of a time series - straight sum and net present value

The function *trajectories* calculates several important quantities (e.g., RRIF balance, gross income, OAS clawback) year by year, from age 71 to 95. Its output is an array with a column for each such quantity and a row for each year (see ***The calculator output*** in Section 3.3).

An obvious type of question is, "How much in total over the years?" Examples are how much OAS clawback was experienced; how much net income in total; how much tax paid. There are a couple, at least, of ways to answer. The obvious is a straight sum of that quantity over the years. However, inflation erodes the value of the dollars, so $100 at age 95 has less significance than the same amount 24 years earlier. A sum obtained by discounting the amount according to inflation is the net present value. There are other definitions of NPV, such as considering the time series against various investment returns, but we'll stick to the inflation discount here.

The function *totalvalue* below operates on the output *R* of *trajectories* to calculate the total of a given column, either as a straight sum or as net present value, from *myagenow* to *age*, inclusive. The year by year inflation relative to that at age *myagenow* (a.k.a. *InfSoFar*) is in column 12 of *R*.

$$
totalvalue(R, col, age, NPVflag) := \left|
\begin{array}{l}
myagenow \leftarrow R_{0,0} \\
InflSoFar \leftarrow R^{\langle 13 \rangle} \\
x \leftarrow R^{\langle col \rangle} \\
total \leftarrow \displaystyle\sum_{i=0}^{age-myagenow} \frac{x_i}{InflSoFar_i} \quad \text{if } NPVflag = 1 \\
total \leftarrow \displaystyle\sum_{i=0}^{age-myagenow} x_i \quad \text{otherwise} \\
\text{return } total
\end{array}
\right.
$$

Remainder when account is terminated

The calculator grinds away out to age 95, which may be extended in a future version (although most of us using the calculator will have dropped out before then). The question then is how much is left in the RRIF or LIF account, although it will likely be asked by our beneficiaries. The procedure below operates on the output R of *trajectories* and lets you specify an age of death (you are not committed to it!). It returns a length-3 array consisting of the fund balance, which CRA considers to be a single posthumous withdrawal, the tax paid on it, and the net income for the estate. A flag allows it to be corrected to present value (at *myagenow*) if desired.

About timing: one's exit is taken to be early in the year, before all the normal inflation and withdrawal processes take place. It's arbitrary, but had to be at some specific point in the year.

And about whether this is for an individual or a couple... It is both, in a way. If $taxmodel = 2$, for a couple, then this procedure effectively assumes that both members die in the same year. Stupid, yes, but it became complex and cumbersome to do it more correctly, so I just let it go. Sorry.

$$wrapup(R, A, age, NPVflag) := \left|\begin{array}{l} myagenow \leftarrow R_{0,0} \\ lastB \leftarrow R_{age-1-myagenow,1} \\ GI \leftarrow lastB \\ OAS \leftarrow 0 \\ taxmodel \leftarrow A_{0,0} \\ \text{"inflation since 2013, for tax calc"} \\ I13 \leftarrow R_{age-1-myagenow,11} \\ TP \leftarrow TaxPaid(GI, OAS, taxmodel, I13) \\ NI \leftarrow GI - TP \\ \text{"inflation since starting age of calculator"} \\ infl \leftarrow R_{age-1-myagenow,13} \\ return\ \dfrac{(GI\ \ TP\ \ NI)^T}{infl}\ \ if\ NPVflag \\ return\ (GI\ \ TP\ \ NI)^T\ \ otherwise \end{array}\right.$$

$$overall(R, A, exitage, NPV) :=$$

"unpack retiree parameters"

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} \leftarrow A$$

"lifetime tax, start to exitage"

$$lifetimetax \leftarrow totalvalue(R, 12, exitage, NPV)$$

"lump sum handling at end of year exitage"

$$\begin{pmatrix} lumpbal \\ lumptax \\ lumpnetval \end{pmatrix} \leftarrow wrapup(R, A, exitage, NPV)$$

"total tax paid, lifetime and estate"

$$totaltaxpaid \leftarrow lifetimetax + lumptax$$

"what's left in savings account"

$$saved \leftarrow R_{exitage-myagenow, 2}$$

"total to heirs, after tax"

$$netestate \leftarrow lumpnetval + saved$$

"lifetime OAS clawback and total OAS clawback"

$$lifeOASclaw \leftarrow totalvalue(R, 8, exitage, NPV)$$

"lifetime spending"

$$spent \leftarrow totalvalue(R, 6, exitage, NPV)$$

$$returntop \leftarrow (lifetimetax \;\; lumpbal \;\; lumptax \;\; lumpnetval)^T$$

$$returnmid \leftarrow (totaltaxpaid \;\; saved \;\; netestate)^T$$

$$returnbot \leftarrow (lifeOASclaw \;\; spent)^T$$

$$return \;\; stack(returntop, returnmid, returnbot)$$

$$endstats(A, exitage, NPV) :=$$

"unpack arguments"

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} \leftarrow A$$

"working copy of arguments for the calculator runs"

$A' \leftarrow A$

$for\ \ startyr \in 1970 .. 2014$

> " replace mktyear1 with startyr in array A' "
>
> $A'_{2,1} \leftarrow startyr$
>
> $future \leftarrow trajectories(A')$
>
> $iyr \leftarrow startyr - 1969$
>
> $year_{iyr} \leftarrow startyr$
>
> $end^{\langle iyr \rangle} \leftarrow overall(future, A', exitage, NPV)$

$return\ \ augment\left(year, end^{T}\right)$

4. USE OF THE CALCULATOR, WITH MANY EXAMPLES

4.1 The Sandbox Area

This is an area where you can try various argument values and see their effect. The graphs can be copied to the clipboard and pasted into other documents (instructions further below).

Enter the arguments

The argument array is below. You can enter your own values on the right hand side by clicking each number and changing it. Don't touch the left side. Definitions of the variables are in Section 3.3.

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} := \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 1 & 1983 & 1985 & 1992 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 1 & 6.8 & 1 & 0 \end{pmatrix}$$

The next line assigns your values to the argument array variable A:

$$A := \begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix}$$

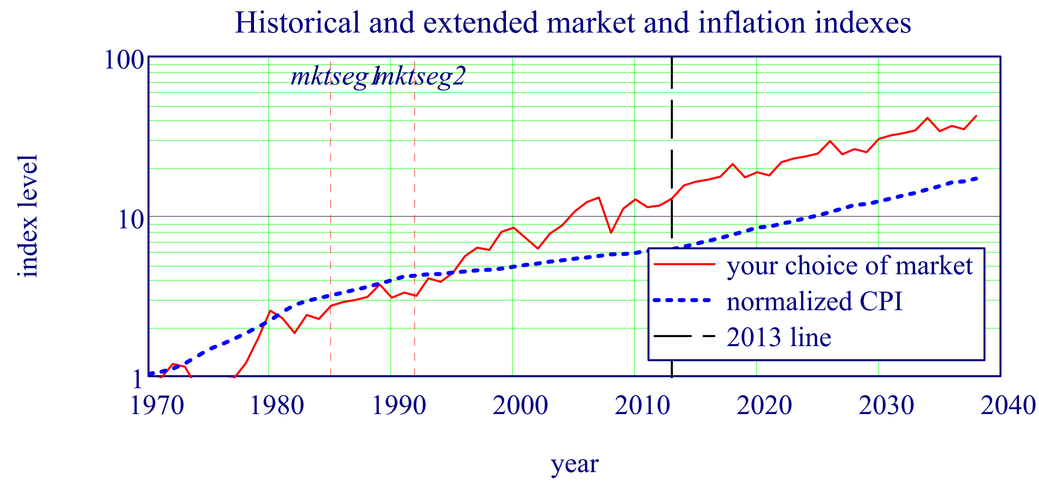
Check them: $argcheck(A) =$ "OK, good to go"

View the results

Now run the calculator $R := trajectories(A)$

Even if you did not select use of historical records of markets and inflation in your arguments above, it is interesting to see the extended market index and extended inflation index. Calculate them, then display.

$H := ExtendedHistory(A)$

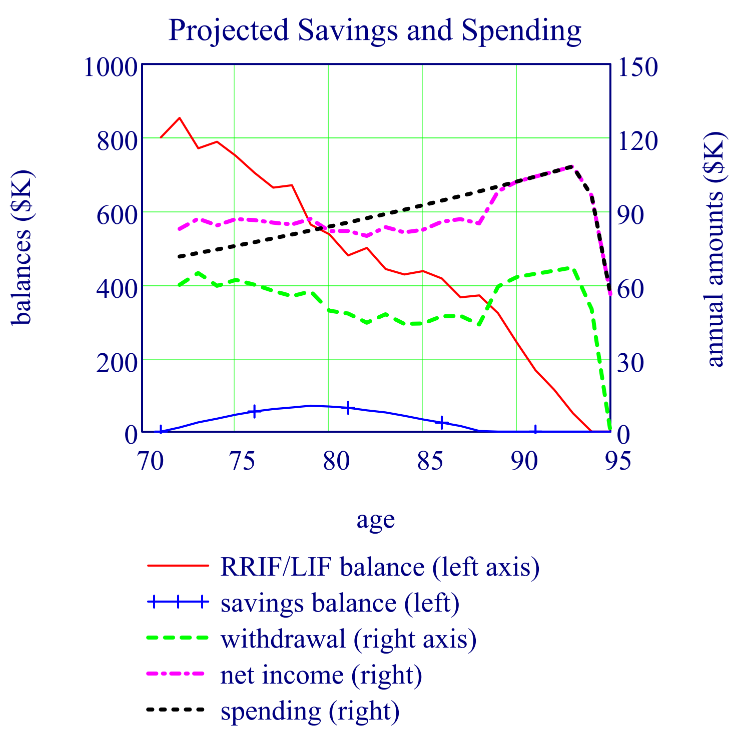


ar

i

They are historical returns up to 2013; after that, an extension by repetition of the selected segment (*mktseg*1 to *mktseg2*) to allow some stress-testing.

This is a detailed plot of key quantities for the selected start year: $mktyear1 = 1983$



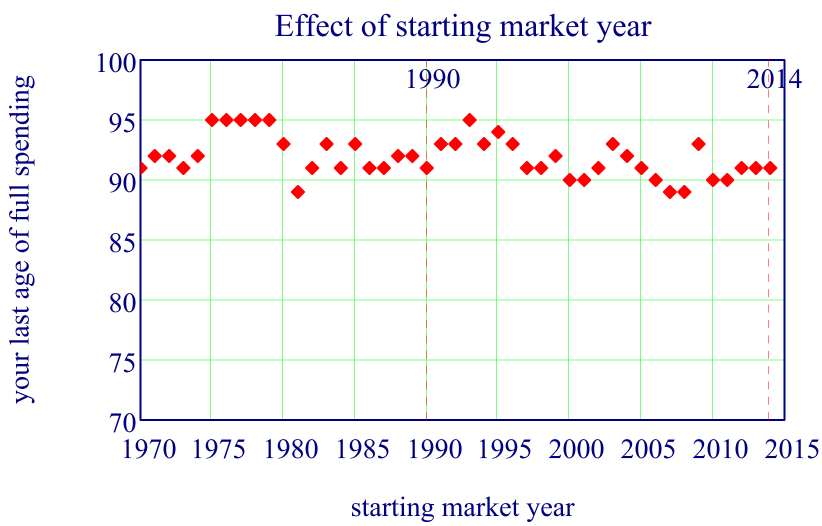
$lastfullspend(R) = 93$

Remember to read the annual amounts (withdrawal, net income, spending) from the right-side axis.

See how the starting market year affects the age to which full spending can be supported. Remember, "95" means "95 or older."

$gb := goodbadyears(A)$

$whichmkt = 1$ $\quad$ $mktseg1 = 1985$ $\quad$ $mktseg2 = 1992$

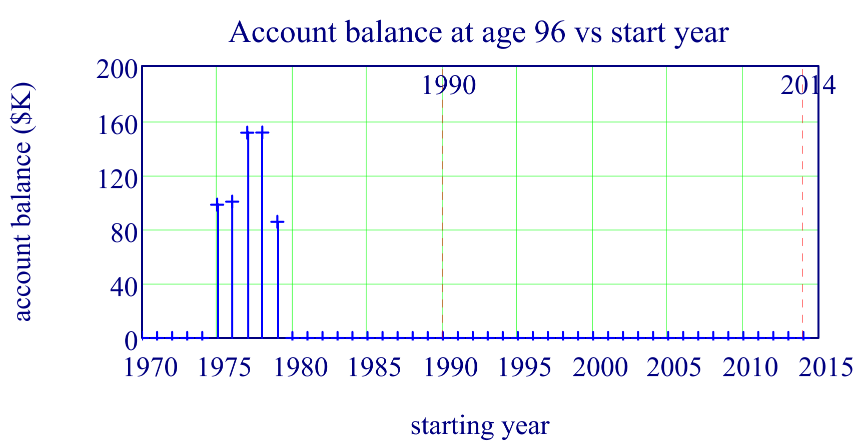


1990 and earlier: only historical market values

1991 to 2014: increasing amounts of the market extension

Age 95 means "95 or older."

A related question: how much is left in the RRIF at the end of age 95? For most of us, the answer is "little or nothing." However, the calculator at present runs only to age 95, so full spending at age 95 must be interpreted as "95 or more." In a future version, the calculator may run out to age 100. In the meantime, we can get some idea of where things stand at that age by the balance in the RRIF or LIF at that age. Here's an example.



1990 and earlier: only historical market values

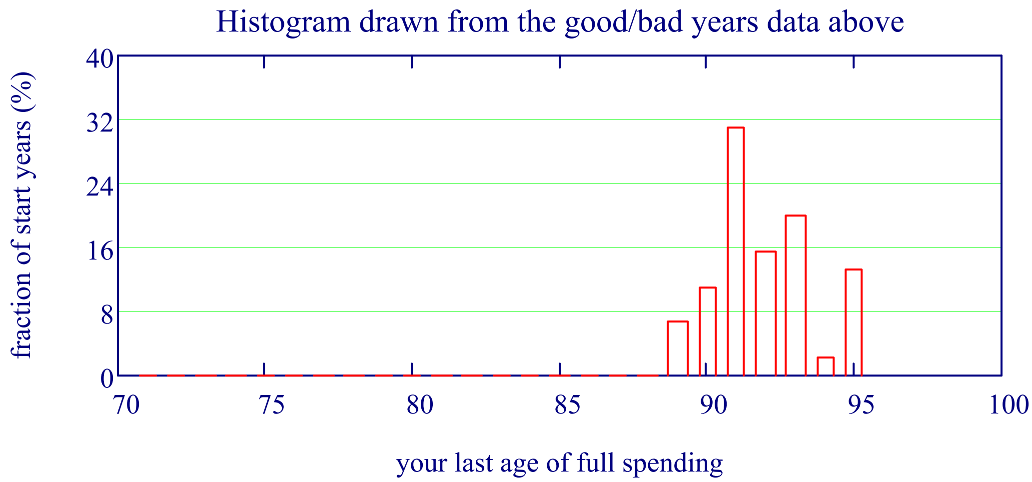
1991 to 2014: increasing amounts of the market extension

The average at start of age 96, over all years and over all years with non-zero balance, respectively:

$mean\left(gb^{\langle 2\rangle}\right) = 12.734$ $K $\quad$ $meannz\left(gb^{\langle 2\rangle}\right) = 117.151$ $K

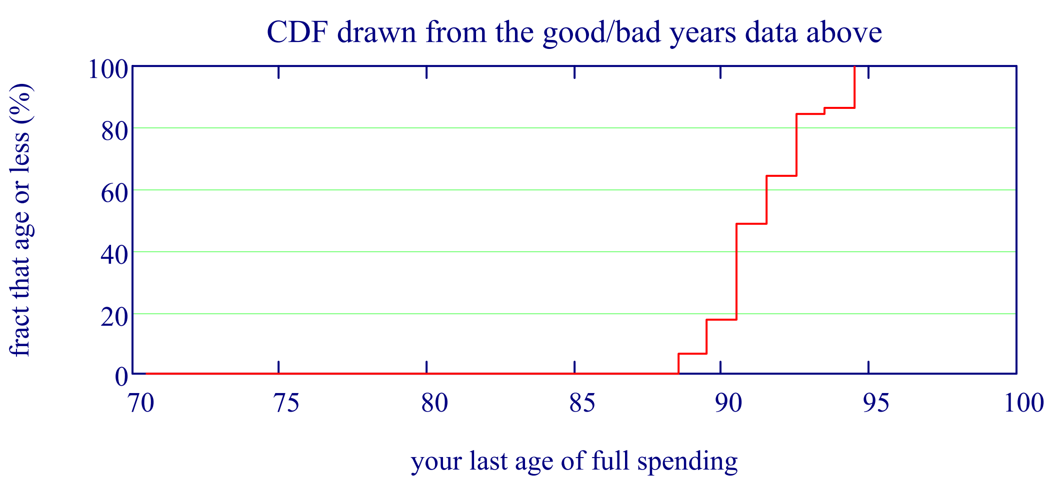
Finally, a histogram of "last age of full spending" data, usi $extflag := 1$ (use 0 to limit to the true historical record)

$luck := spendhistocdf(A, extflag, 0)$



The same histogram information, now presented as a cumulative distribution function (cdf): the fraction of starting years that result in a last age of full spending that does not exceed the age on the horizontal axis.

$luck := spendhistocdf(A, extflag, 1)$



To paste any of these graphs into another document:

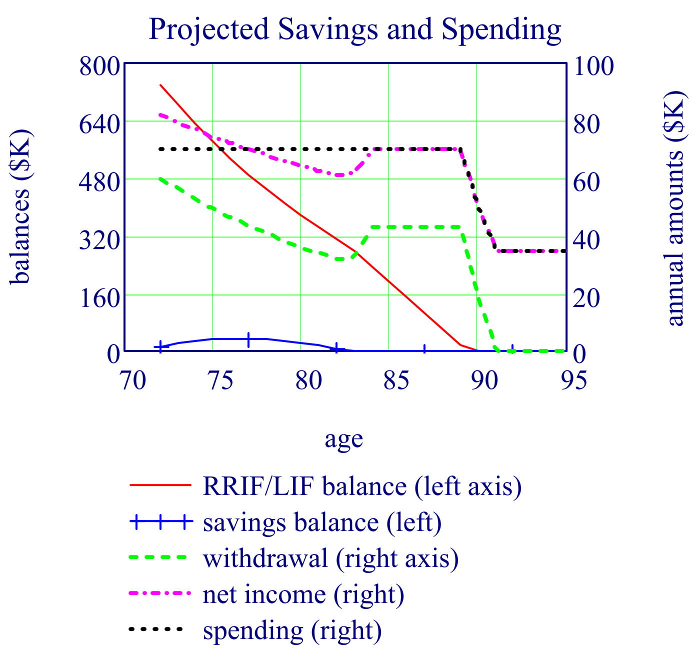
1. Click in an empty spot near the graph, hold the button down, and run mouse over the graph, to enclose it in a dotted rectangle.
2. Copy to the clipboard (ctrl-c will do it).
3. In the other document, click where you want it, and Paste Special > Windows Metafile or bitmap.

4.2 Examples

Super simple RRIF

The Section 2.7 couple again. No inflation, no market returns. Spending target \$70K; RRIF \$800K; combined CPP \$22K and combined OAS \$13K, for a total IP of \$35K.

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 0 & 800 & 0 & 0 \\ 1 & 1985 & 2000 & 2011 \\ 0 & 0 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



What happened:

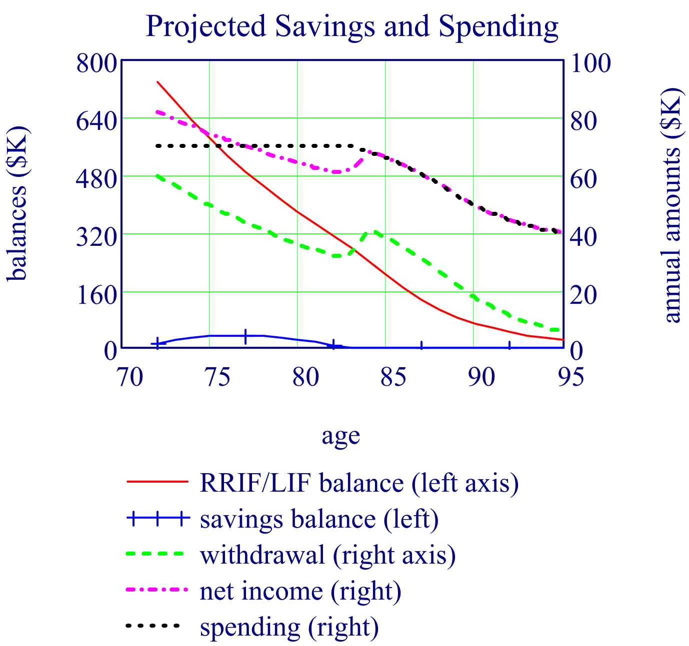
- Initially, min RRIF withdrawals generated excess income, which was put into savings account.
- By age 76, min withdrawals not enough, so drew on savings until exhausted at age 82.
- After age 82, maintained spending by raising RRIF withdrawal above minimum, RRIF then depleted more quickly, up to age 89.
- By age 90, RRIF is exhausted and spending drops to the level supportable by CPP and OAS.
- There was no OAS clawback (not shown).

The size of the saving account indicates by how much the CRA-mandated minimum withdrawals exceeded what the couple actually needed.

Super simple LIF

Same as the super simple RRIF example above, but now the couple has a LIF.

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 1 \\ 0 & 800 & 0 & 0 \\ 1 & 1985 & 2000 & 2011 \\ 0 & 0 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



What happened:

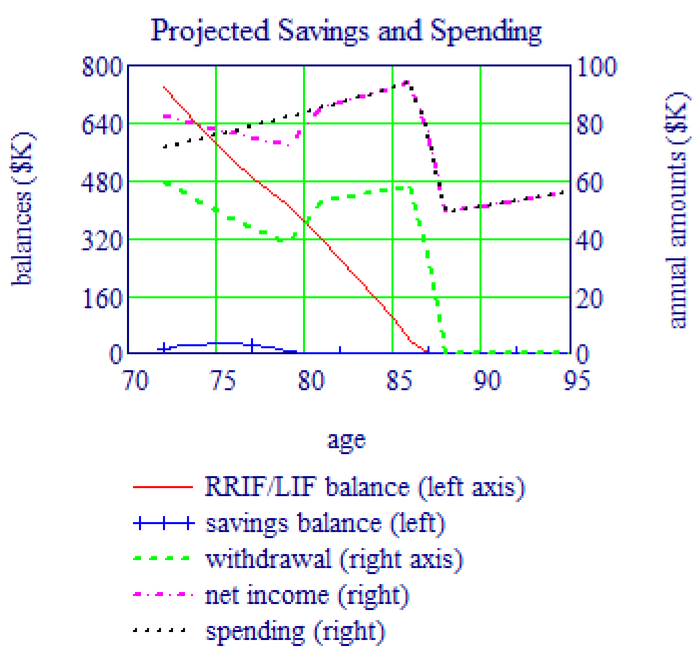
- Initially, min LIF withdrawals generated excess income, which was put in savings account.
- By age 76, min withdrawals not enough, so drew on savings until exhausted at age 82.
- After age 82, it tried to maintain spending by raising withdrawals above minimum, but hit the LIF max withdrawal limit at 83.
- Max withdrawals forced a slow decline of spending from 84 on, asymptotic to the CPP and OAS limit.
- At age 90, the LIF still held almost $67K, but the retiree could get only a little per year.

If the steady decline in spending from age 84 is a worry, perhaps some anticipatory withdrawals in the early (pre-max-withdrawal) years would be helpful, despite the extra tax incurred by the higher gross income.

Super simple RRIF - but with inflation

Back to the RRIF for this couple - but now 2% inflation is working against them, and they have not invested the RRIF funds in either fixed income or equities.

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 0 & 800 & 0 & 0 \\ 1 & 1985 & 2000 & 2011 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



What happened:

- Initially, spending rose on track with inflation, even after the savings buffer ran out and withdrawals increased.
- By age 86 or 87, the RRIF was exhausted, and spending dropped to the CPP and OAS level (which had also increased, because they are indexed to inflation).

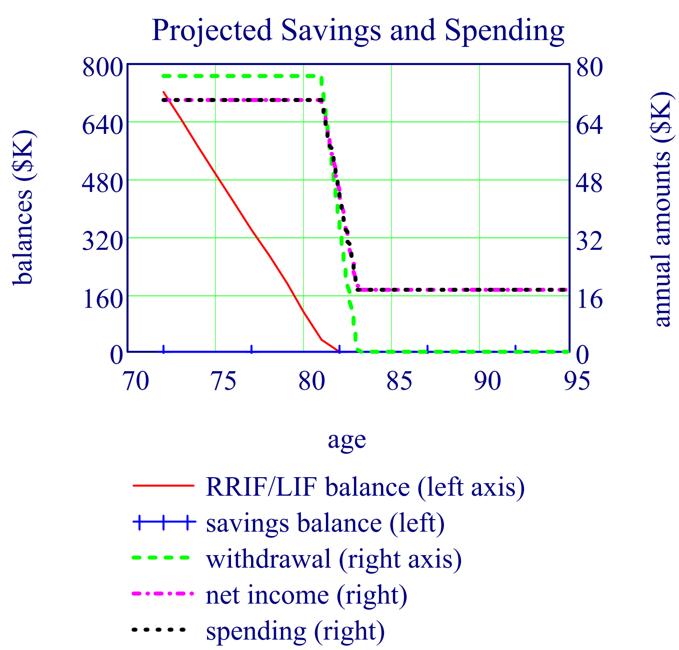
This couple needs to generate some return from their RRIF so that it can sustain their target spending level longer.

Super simple RRIF - an individual

The couple in the examples above benefited from pension splitting. A single person would have a higher gross income, pay more tax and possibly suffer from OAS clawback. Let's check.

Keep RRIF savings and spending target the same, just change the tax model and cut the CPP and OAS in half.

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 1 & 71 & 2014 & 0 \\ 0 & 800 & 0 & 0 \\ 1 & 1985 & 2000 & 2011 \\ 0 & 2 & 0 & 17.5 \\ 0 & 6.5 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



What happened:

- Minimum withdrawals were insufficient in early years, so large withdrawals and no excess income for savings.
- When RRIF is exhausted by age 82, OAS rebounds a bit because no more clawback.

This is a shock - the last year of full spending was 81, compared with 89 for the couple. The reasons:

- The individual had only half of the couple's CPP and OAS.
- All RRIF withdrawals went to one person - so progressive taxation meant much higher taxes.
- The higher gross income caused serious OAS clawback. Over the years, this added up to

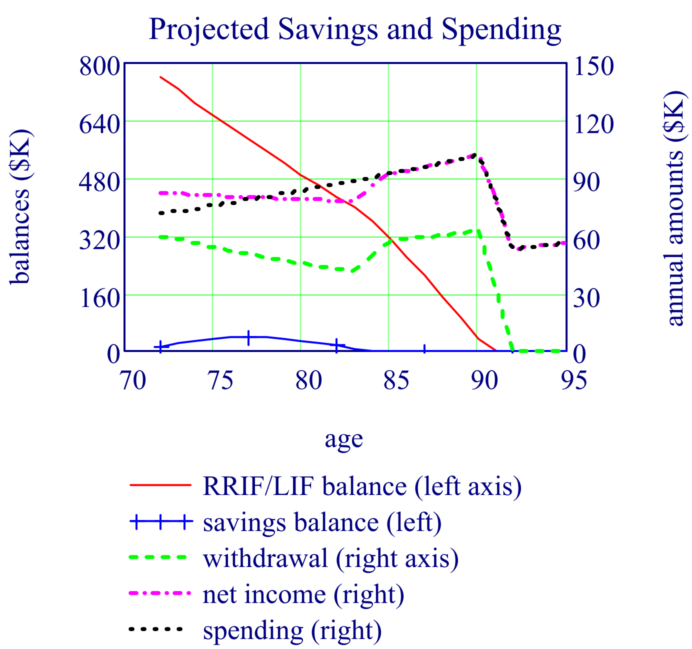
cumulative OAS clawback $\sum R^{\langle 8 \rangle} = 34.629$ $K (straight sum, not NPV)

Worse, it might be harder for one person to accumulate that RRIF than for two people. Reduce spending? Maybe, but not by one half. But see another example later on.

Simple RRIF with fixed income growth and inflation

Back to the couple who found that inflation caused their RRIF to be exhausted at age 86, instead of 89. Now they would like to put their RRIF to work with fixed income investments. Unfortunately, their return is only 3% - certainly better than nothing, but how much does it help?

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 0 & 800 & 0 & 3 \\ 1 & 1985 & 2000 & 2011 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



What happened:

- The FI returns kept the RRIF going longer during the initial min withdrawal period.
- When larger withdrawals were required from age 84, after the savings ran out, the RRIF ran into the ground fairly quickly.
- Nevertheless, it lasted to age 90, instead of age 86.

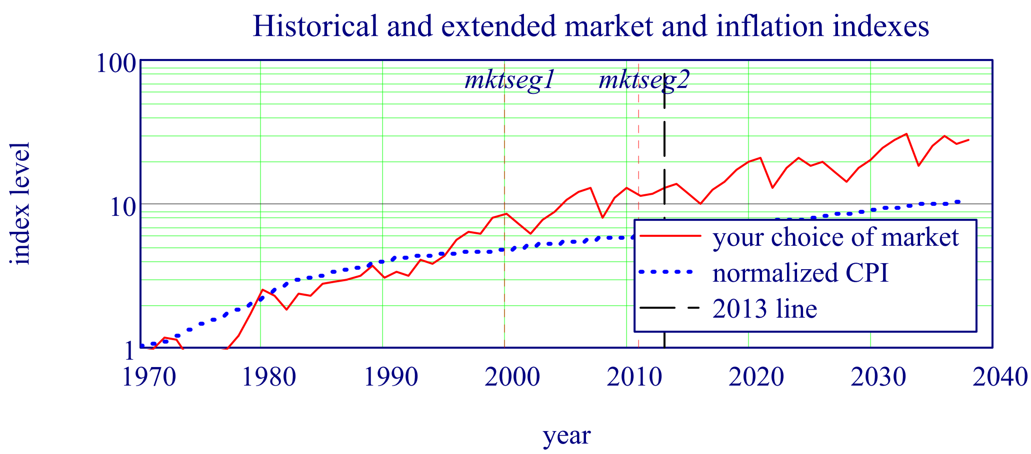
Investment of the RRIF was very important in offsetting the effects of inflation. Two other interesting points (not shown):

- If the fixed income return had been 2%, it would not have been sufficient to cancel the 2% inflation and restore matters to the first super simple RRIF example, where the RRIF ran out at age 89.
- Giving the savings account a 3% return, too, made no noticeable difference. Not surprising, since that account is small, even at its peak value.

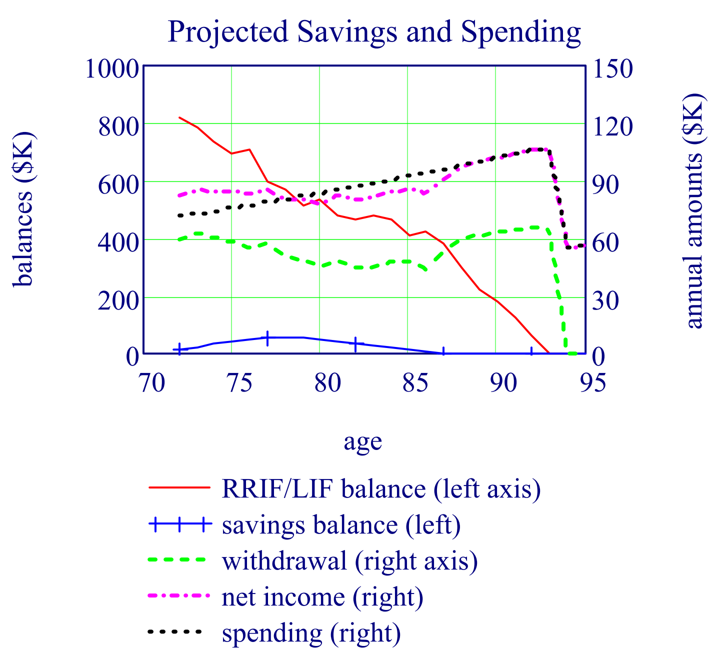
RRIF with market returns - not simple, but rewarding

Now the couple decides to put half of their RRIF into equities; specifically, an ETF for the S&P/TSX Composite. The other half is in fixed income at 3%. Cost of running their investments is 1%.

But what are the market returns? Here, we'll replay the market as if it were starting at 1985 again. For stress-testing, market returns post-2013 will be set to repetitions of the fairly unpleasant 2001 to 2011 years. This is how it looks, together with historical and extended inflation.



Here is the effect on their retirement if the market behaves like a replay from *mktyear1* = 1985



Comments:

- Now, full spending is maintained out to age 93, instead of 90, obtained above with all fixed income at 3%.
- The RRIF balance wobbles in response to the market.
- This plot is for a specific starting market year - so don't draw conclusions until you have seen several historical market records, as shown on next pages.

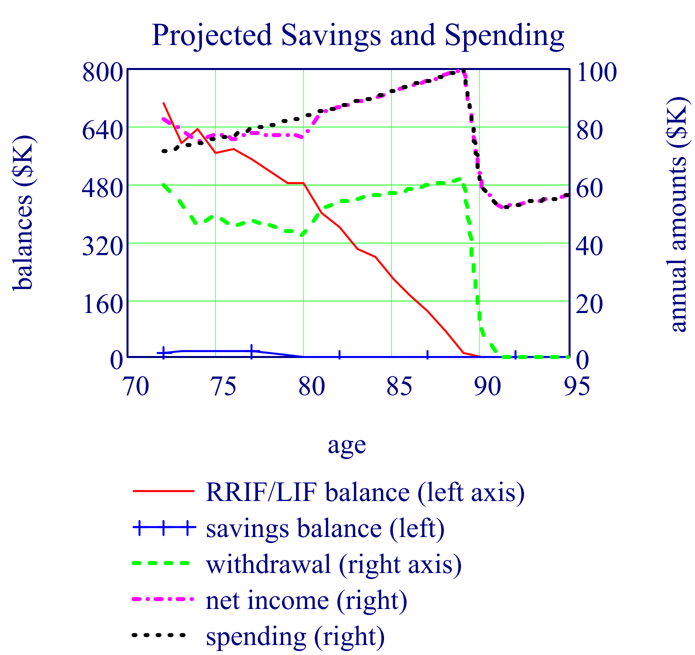
For reference, the argument array for this plot was

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 1 & 1985 & 2000 & 2011 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Much depends on market behaviour in the early years. Now try launching the RRIF in the recession of 1981. Change *mktyear1* in the argument array to 1981. The new argument array:

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 1 & 1981 & 2000 & 2011 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

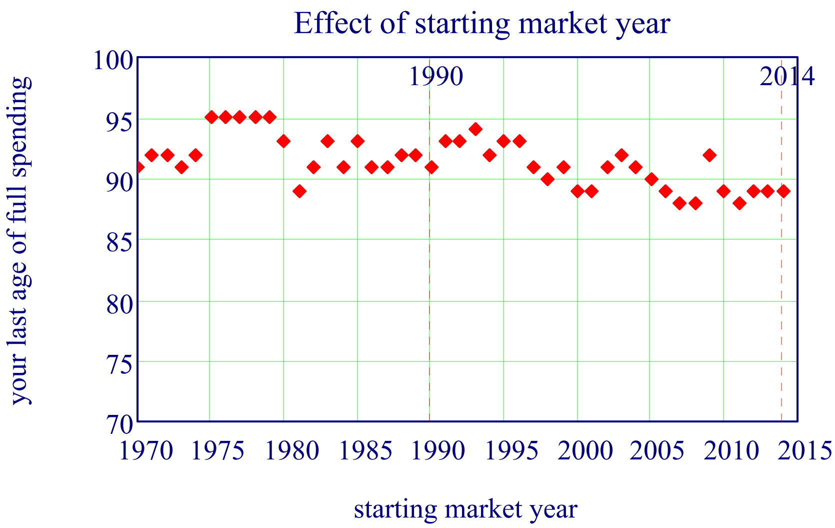
and the results:



$mktyear1 = 1981$

Not great - now the last full-spending age is 88 years, instead of 93, when the market started at 1985.

So, starting year in the market is important. Now try all such years to see how long full spending lasts.



1990 is the last starting market year for which only historical market values affect the outcome.

After 1990, increasing amounts of the extension play a role; by 2014, all returns are from the extension.

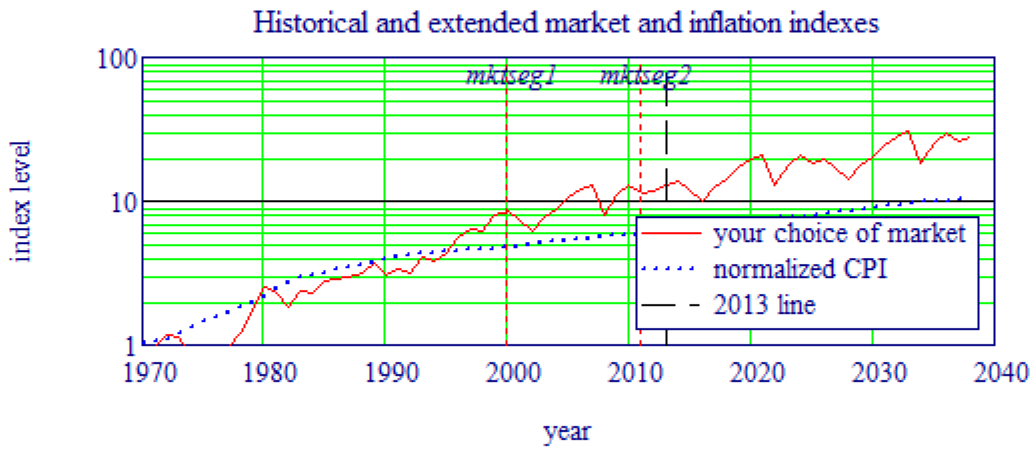
Age 95 means "95 or older."

Just as one would expect from the market, the *average* time the fund lasts has increased, but the *variability* has produced more quite good years (1975 to 1979) and quite bad ones (1981 and several after 2000). About the increasingly worrying performances for starting years 2005 and beyond, remember that they are affected increasingly by the market extension, which was selected as 2000 to 2011, a rather unpleasant time. Many of the years on the right hand side of the plot were therefore affected by them twice.

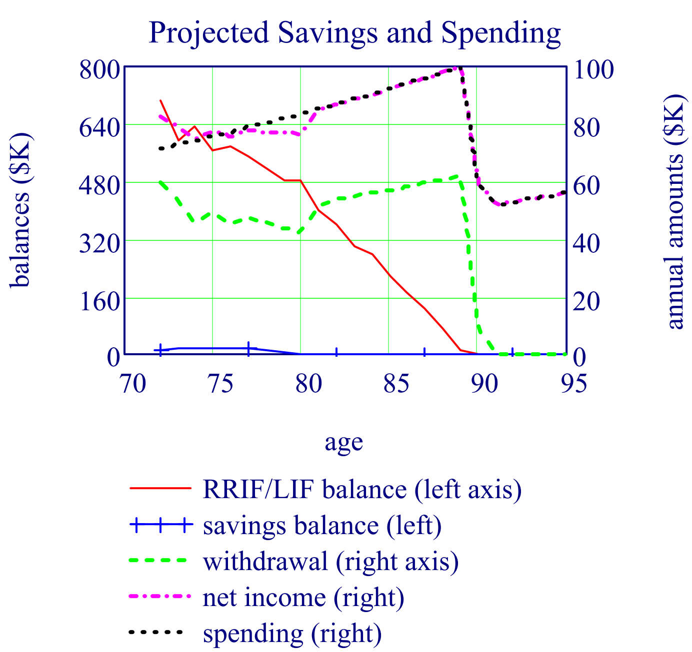
So try extending the market instead with 1985 to 1992, instead. They were not exciting years - not much return, but not much turbulence, either. To compensate, if we want to look at the money graph for a single starting year, we launch the calculator in the awful year 1981. Here are the arguments:

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 1 & 1981 & 1985 & 1992 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Next page shows the market and inflation with their selected extensions:



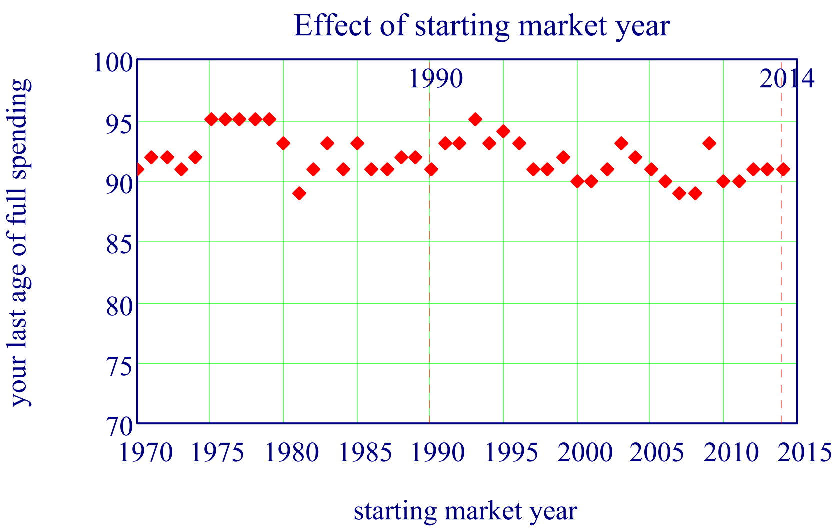
From the plot below, a reminder that launching in 1981 was unfortunate. Note that this year was unaffected by the more benign market extension years, since the 24-year span of the calculator (age 72 to 95) reaches only to 1994, which is before the extensions.



Comments:

- A rapid drop in the RRIF balance in the very early years, because of the recession.
- But from only four years later, balance decline is slower, because the market has improved.
- The party is over after age 89, one year shorter than when the couple had 100% of their RRIF in 3% fixed income.

As usual, though, looking at *all* starting years tells more at a glance. See next page



1990 is the last starting market year for which only historical market values affect the outcome.

After 1990, increasing amounts of the extension play a role; by 2014, all returns are from the extension.

Age 95 means "95 or older."

The fund longevity is much better with the extension by benign years than it was with the awful 2001 to 2011. Only three "failures" (last good age in the 80s), all of them due to launching during market slumps.

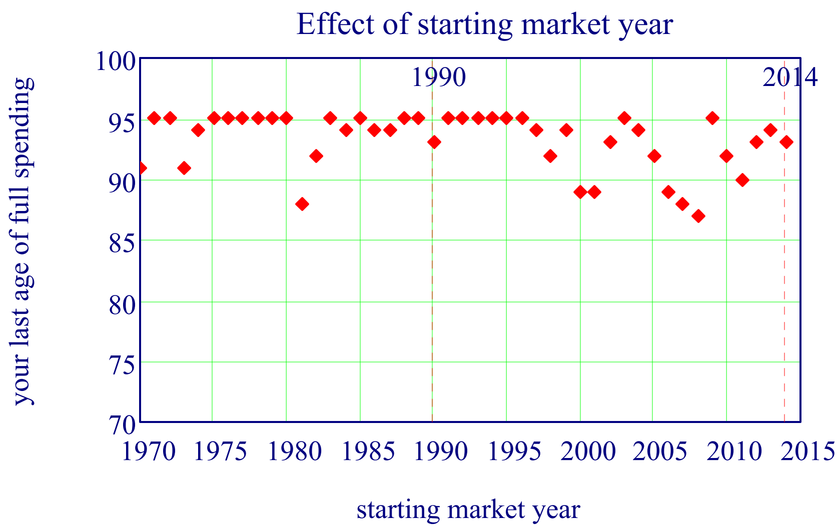
It appear that relying on the market almost always greatly improves the longevity of the funds - but the key word is "almost" - there are occasional unfortunate market outcomes and they must be taken into account.

Juicing it up - an all-equities RRIF

What would happen if the couple had put all their RRIF into equities? We'll keep the same bland extension years, 1985 to 1992, so the arguments are

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 100 & 800 & 1 & 0 \\ 1 & 1981 & 1985 & 1992 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The result can be seen below. The number of "failures" (funds running out in one's 80s) has increased, but but the number of notable "successes" (funds lasting to 95 and beyond) has increased more than that.



1990 is the last starting market year for which only historical market values affect the outcome.

After 1990, increasing amounts of the extension play a role; by 2014, all returns are from the extension.

Age 95 means "95 or older.'

Summary - increasing the fraction of equities significantly improves performance most of the time, but it decreases reliability, since the number of failures increases, too.

Markets and budget - the individual revisited

Recall the example above, "Super simple RRIF - individual," and the shock at finding his or her full spending would last only to age 81, when a couple with the same RRIF could make it last to age 89. Let's see what can be done.

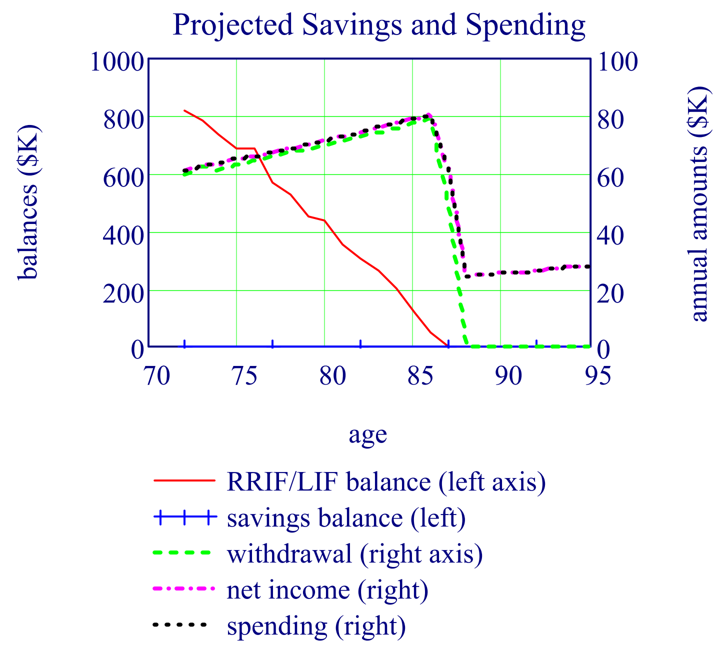
A few changes in succession:

- Set inflation to 2%. That doesn't help, of course. Now the RRIF lasts only to age 80.
- Reduce annual spending from $70K down to $60K. Now it lasts to between 82 and 83.
- Invest the RRIF in 50/50 fixed income and equities. Fixed income annual return 3% (slightly optimistic). Equities in an ETF that follows the TSX Composite. Investment fee 1%.
- For the market returns, extend beyond 2013 with repetition of the bland years 1985 to 1992.

If the market followed the behaviour of the market starting in 1985, the argument array would be

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 1 & 1985 & 1985 & 1992 \\ 0 & 2 & 0 & 17.5 \\ 0 & 6.5 & 0 & 60 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

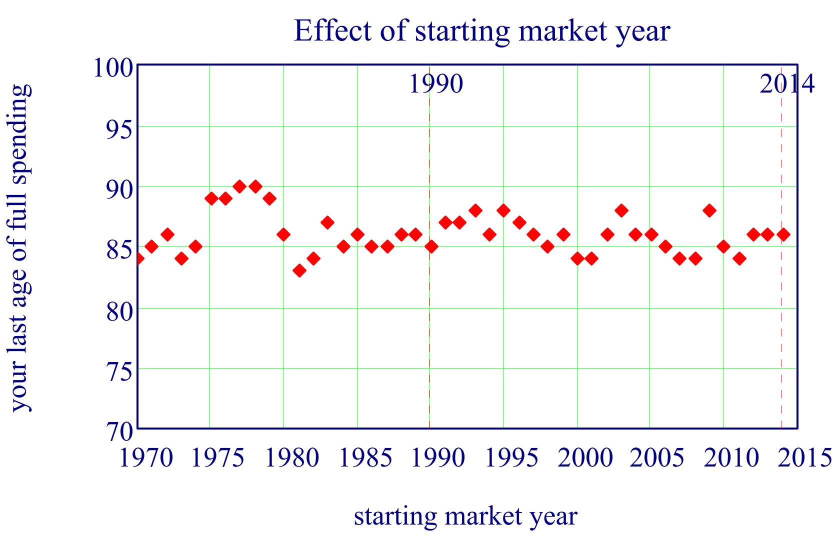
And this would be the experience, given the relatively benign starting year *mktyear1* = 1985



What happened:

- Min withdrawals not enough to meet spending needs, so withdrew more; no excess income, so no savings account.
- RRIF balance decline was slowed by investments.
- Last year of full spending was age 86, a significant improvement.

As always with equities, the market starting year plays a large role, so now we look at the experiences with all starting years, with this result:



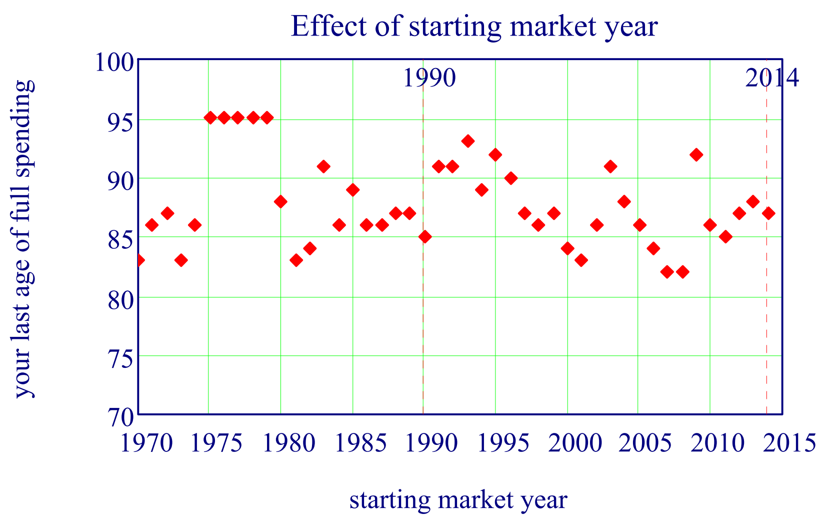
1990 is the last starting market year for which only historical market values affect the outcome.

After 1990, increasing amounts of the extension play a role; by 2014, all returns are from the extension.

Age 95 means "95 or older.'

The RRIF duration is quite variable, ranging from age 83 to 90, which is an improvement over the 82 or 83 achievable with just spending reduction to $60K in 2% inflation. Clearly, investment helped.

And if some investment helped somewhat, would full investment in the market have done even better? We've seen a similar result in the ***Juicing it up*** example. For the RRIF invested 100% in equities, these are the outcomes:



1990 is the last starting market year for which only historical market values affect the outcome.

After 1990, increasing amounts of the extension play a role; by 2014, all returns are from the extension.

Age 95 means "95 or older.'

On average, a big improvement - but there's more scatter, with a few starting years poorer than the 50/50 portfolio case.

Would an American market have been better?

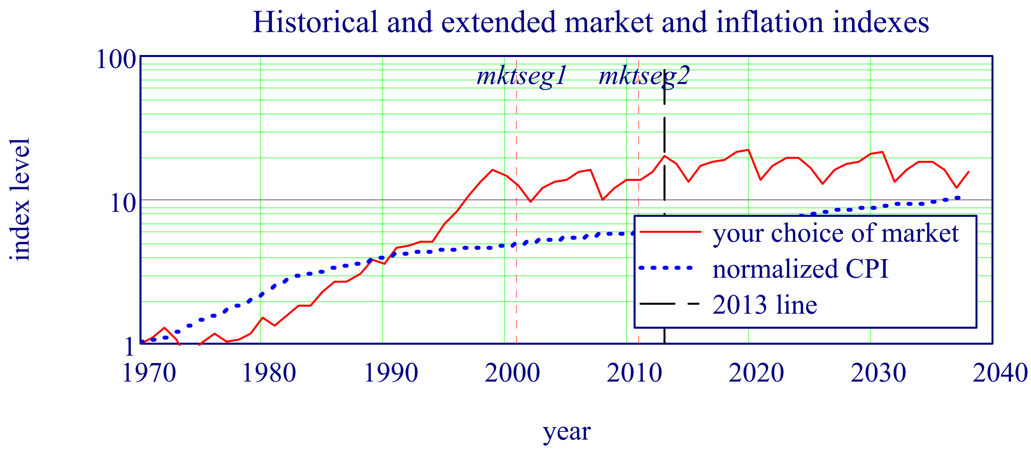
All the market-based examples so far have used the TSX Composite index as the reference market. Let's see if the S&P 500 would have kept the funds going longer.

The pension-splitting, equal-aged couple will be our guinea pigs again, and we'll see if they fare better than they did in the ***RRIF with market returns*** example above. Just change *whichmkt* to 3 (from 1).

Recall that, first, we extended the market with the brutal segment 2001 to 2011. The arguments:

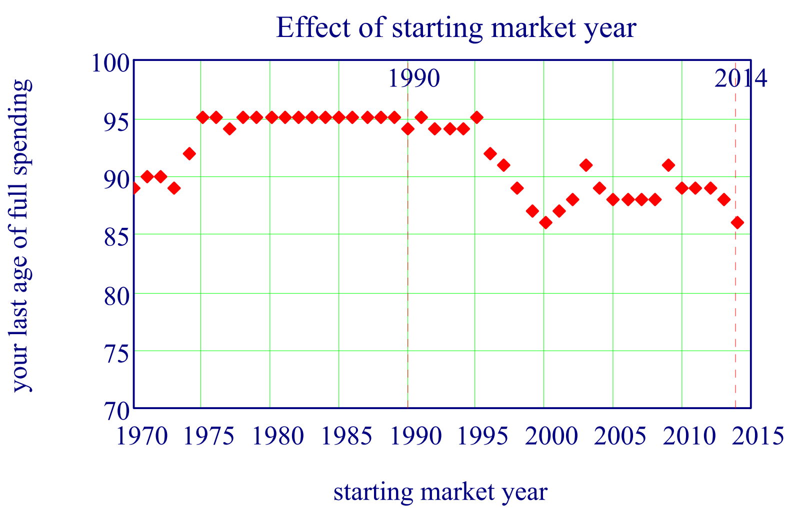
$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 1 & 1983 & 1985 & 1992 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 1 & 6.8 & 1 & 0 \end{pmatrix}$$

With that extension, the S&P 500 looks like this:



and the couple's last age of full spending responded like this:

whichmkt = 3 *mktseg1* = 2001 *mktseg2* = 2011



1990 and earlier: only historical market values

1991 to 2014: increasing amounts of the market extension

Age 95 means "95 or older.'

In the starting years of true historical record (up to 1990), the couple's fortunes were unequivocally improved by use of the S&P 500. However, that index suffered badly during 2000 to 2011, and those are the years we used as extension, so their experience for starting years 1995 and later were much poorer than on the TSX Composite with the same extension years.

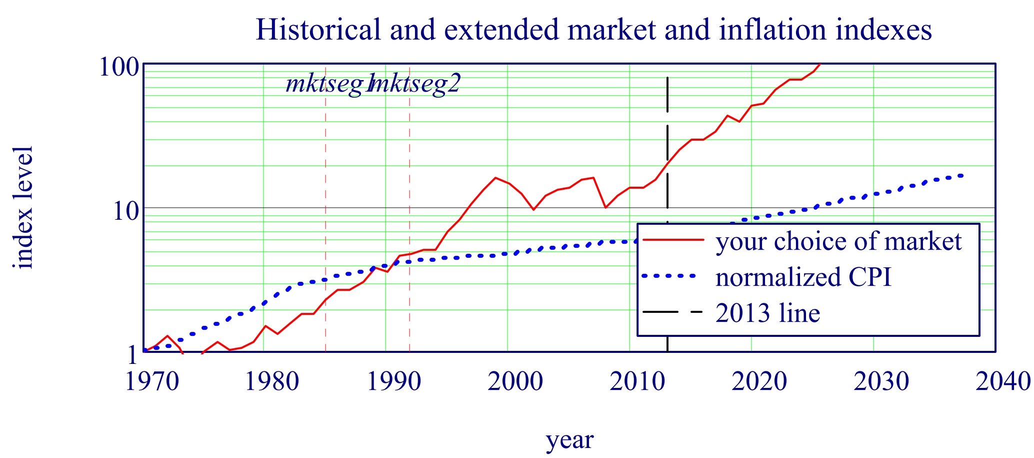
The market extension was meant for stress-testing, and that's just what we have done here.

Caveat: This comparison did not include currency exchange variations. If the couple had bought US dollars and invested in the US market 10 or more years ago, when the Canadian dollar was low, they might have found that the recent rise in the Canadian dollar eroded their extra returns when bringing the money back home.

One more kick at the can. In our earlier example (back to the TSX Composite for the couple), we then extended with the years 1985 to 1992. For comparison, we'll do the same with the S&P 500.

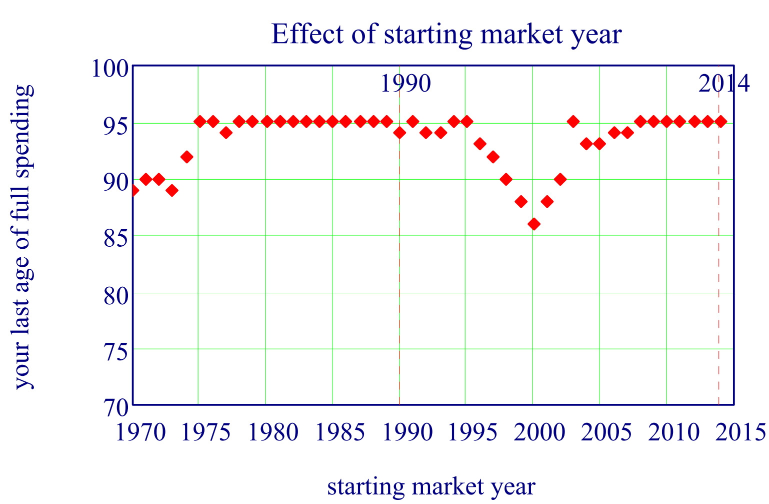
$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 3 & 1985 & 1985 & 1992 \\ 0 & 2 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The S&P 500 with this market extension instead:



Looks like quite a ride coming up...

$whichmkt = 3 \qquad mktseg1 = 1985 \qquad mktseg2 = 1992$



1990 and earlier: only historical market values

1991 to 2014: increasing amounts of the market extension

Age 95 means "95 or older.'

Apart from the tech bubble collapse, this US market would have been much better than TSX.

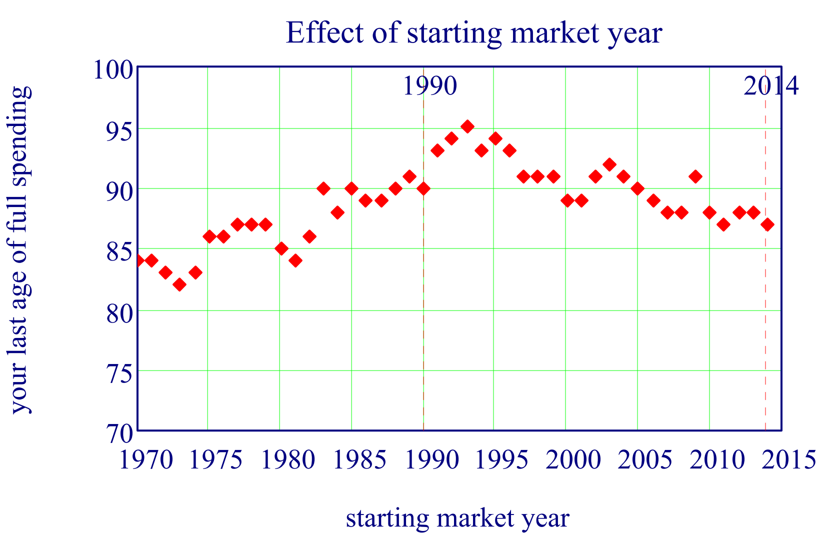
Historic inflation savaged retirees in the 70s and 80s

All the examples so far have used a fixed inflation rate of 2%. That's not a bad model for the last 20 years or so, since a central goal of the Bank of Canada is to keep annual inflation in a band from 1% to 3%. But it was different - and painful - in the 70s and early 80s, when inflation spiked into the low teens. The effect of this loss of buying power was similar that of to a steep market decline, so retirees experience serious hardship, especially those just starting their retirement.

As a reminder of how serious rampant inflation is, we'll repeat one of the previous examples ***RRIF with market returns - not simple, but rewarding*** and replace the 2% inflation setting with the actual inflation over the years. Pick up its second market extension (1985 to 1992) and make its inflation rate negative, to force the calculator to use historic inflation data. The argument array:

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 2 & 71 & 2014 & 0 \\ 50 & 800 & 1 & 3 \\ 1 & 1981 & 1985 & 1992 \\ 0 & -1 & 0 & 35 \\ 0 & 13 & 0 & 70 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The contrast between retirements beginning before and after 1985 is sharp and almost shocking, as the plot below shows. Comparison with the equivalent plot six pages back, which had 2% inflation, shows that inflation in the 60s and early 70s cost the retirees about *8 years* of value from their savings!



1990 and earlier: only historical market values

1991 to 2014: increasing amounts of the market extension

Age 95 means "95 or older.'

5. SHORTCOMINGS AND FURTHER DEVELOPMENT OF THE CALCULATOR

The calculator works well, and it sketches possible futures in ways that other available calculators cannot do. However, it has some shortcomings, and they should be noted as areas for future development:

- It operates to keep the spending level constant in inflation-adjusted dollars. However, spending targets during retirement may vary with time, being larger in the early, active years and, possibly, larger in the later years because of health issues.
- Its tax model is quite simple and it makes an assumption that CRA will continue to adjust it for inflation.
- It assumes that all savings are either in a RRIF or a LIF. However, many retirees will have both. More work needed here.
- It makes use of historic Canadian and US equities returns and historic inflation - but it has no data for indexes of bonds or international equity. That limits its portfolios to a simplistic split between equities, modeled by historic and extended markets, and fixed income, modeled by a fixed annual rate of returns. More work needed, so the user can create diversified portfolios with arbitrary weightings.
- A big shortcoming... although Mathcad is a beautiful package for mathematical and computational explorations, few people have it. The calculator must be ported to another platform, such as Excel or Java.

The scope of the calculator needs attention as well. At present, it is confined to retirees' post-71 experiences, assuming that each year is much like the previous one. The scope really should be expanded in at least two ways:

- The calculator would be much more useful if it began at 65 years. During those early years, many retirees continue to work on a part-time or full-time basis, and even contribute to their RRSPs. It would be difficult for one calculator to capture all variations on how we construct our financial lives, but common patterns ought to be represented.
- Also - hard though it is to think about the prospect - the death of a spouse. As well as the emotional devastation, there are financial consequences for the survivor, and the calculator ought to provide some guidance.

APPENDIX A: TABLES OF DATA USED IN THIS STUDY

RRIF and LIF minimum withdrawals

minratetable =

	0	1
0	65	4
1	66	4.17
2	67	4.35
3	68	4.55
4	69	4.76
5	70	5
6	71	7.38
7	72	7.48
8	73	7.59
9	74	7.71
10	75	7.85
11	76	7.99
12	77	8.15
13	78	8.33
14	79	8.53
15	80	8.75
16	81	8.99
17	82	9.27
18	83	9.58
19	84	9.93
20	85	10.33
21	86	10.79
22	87	11.33
23	88	11.96
24	89	12.71
25	90	13.62
26	91	14.73
27	92	16.12
28	93	17.92
29	94	20
30	95	20

LIF maximum withdrawals

maxratetable =

	0	1
0	55	6.4
1	56	6.5
2	57	6.5
3	58	6.6
4	59	6.7
5	60	6.7
6	61	6.8
7	62	6.9
8	63	7
9	64	7.1
10	65	7.2
11	66	7.3
12	67	7.4
13	68	7.6
14	69	7.7
15	70	7.9
16	71	8.1
17	72	8.3
18	73	8.5
19	74	8.8
20	75	9.1
21	76	9.4
22	77	9.8
23	78	10.3
24	79	10.8
25	80	11.5
26	81	12.1
27	82	12.9
28	83	13.8
29	84	14.8
30	85	16
31	86	17.3
32	87	18.9
33	88	20
34	89	20

Tax parameters for 2013 tax year

$T13 =$

	0
0	34.562
1	6.854
2	11.038
3	32.911
4	4.421
5	10.276
6	70.954
7	0
8	15
9	43.561
10	22
11	87.123
12	26
13	135.054
14	29
15	0
16	5.06
17	37.568
18	7.7
19	75.138
20	10.5
21	86.268
22	12.29
23	104.754
24	14.7

content of each row:

0 fed age 0
1 fed age 1
2 fed basic
3 BC age 0
4 BC age 1
5 BC basic
6 OAS threshold
7 fed seg 0 start
8 fed seg 0 %
9 fed seg 1 start
10 fed seg 1 %
11 fed seg 2 start
12 fed seg 2 %
13 fed seg 3 start
14 fed seg 3 %
15 BC seg 0 start
16 BC seg 0 %
17 BC seg 1 start
18 BC seg 1 %
19 BC seg 2 start
20 BC seg 2 %
21 BC seg 3 start
22 BC seg 3 %
23 BC seg 4 start
24 BC seg 4 %

Market index levels

markets =

year	TSXComp	TSX tot ret	S&P500	S&P500 tot re
1969	1055	2465.997	1	1
1970	990	2384.619	1	1.04
1971	1020	2421.609	1.108	1.19
1972	1255	2998.652	1.281	1.41
1973	1205	2865.488	1.059	1.21
1974	885	1982.661	0.755	0.89
1975	975	1928.41	0.993	1.22
1976	1005	1876.624	1.183	1.51
1977	1017	1965.399	1.047	1.4
1978	1270	2431.473	1.058	1.49
1979	1813	3368.552	1.188	1.77
1980	2706	4216.855	1.494	2.34
1981	2428	3629.947	1.349	2.23
1982	1958	3637.345	1.548	2.71
1983	2541	4744.578	1.815	3.32
1984	2400	4460.988	1.841	3.52
1985	2905	5395.601	2.326	4.64
1986	3066	5703.851	2.666	5.51
1987	3160	5878.936	2.72	5.8
1988	3295	6305.554	3.057	6.76
1989	3943	7388.126	3.89	8.9
1990	3257	6056.488	3.635	8.63
1991	3512	6534.892	4.591	11.26
1992	3350	6231.574	4.796	12.11
1993	4321	8246.293	5.135	13.33
1994	4093	8229.031	5.056	13.51
1995	4661	9412.71	6.78	18.59
1996	5927	12066.122	8.154	22.86
1997	6699	13863.834	10.682	30.48

CPI data

CPIdata =

1969	4.8
1970	3
1971	3
1972	4.8
1973	7.8
1974	11
1975	10.7
1976	7.2
1977	8
1978	8.9
1979	9.3
1980	10
1981	12.5
1982	10.9
1983	5.8
1984	4.3
1985	4
1986	4.1
1987	4.4
1988	3.9
1989	5.1
1990	4.8
1991	5.6
1992	1.4
1993	1.9
1994	0.1
1995	2.2
1996	1.5
1997	1.7

1998	6486	13619.7	13.531	39.19
1999	8414	17959.855	16.174	47.44
2000	8934	19281.629	14.534	43.12
2001	7688	16850.156	12.639	37.99
2002	6615	14749.127	9.685	29.6
2003	8221	18687.324	12.24	38.09
2004	9247	21390.056	13.34	42.23
2005	11272	26546.456	13.74	44.3
2006	12908	31130.744	15.612	51.3
2007	13821	34193.512	16.166	54.12
2008	8311	22901.712	9.947	34.09
2009	11746	30953.192	12.284	43.11
2010	13433	36410.443	13.836	49.61
2011	11995	33226.841	13.836	50.65
2012	12316	35613.926	15.675	58.76
2013	13588	40250	20.315	77.79

1998	1
1999	1.6
2000	2.8
2001	2.5
2002	2.2
2003	2.8
2004	1.8
2005	2.2
2006	2
2007	2.1
2008	2.4
2009	0.3
2010	1.8
2011	2.9
2012	1.5
2013	0.9

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[1] Cooley, Philip L., Carl M. Hubbard, and Daniel T. Walz, "Retirement Savings: Choosing a Withdrawal Rate That Is Sustainable," *American Association of Independent Investors (AAII) Journal*, 10, 3: 16–21, 1998.

[2] Philip L. Cooley, Carl M. Hubbard and Daniel T. Walz, "Sustainable Withdrawal Rates From Your Retirement Portfolio," *Financial Counseling and Planning*, Volume 10 (1), 1999.

[3] Cooley, Philip L., Carl M. Hubbard, and Daniel T. Walz, "Portfolio Success Rates: Where to Draw the Line," *J. Financial Plannin*g , 2011. http://www.onefpa.org/journal/Pages/Portfolio%20Success%20Rates%20Where%20to%20Draw%20the%20Line.aspx

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[6] LIF maximum withdrawal rates for Ontario http://www.fsco.gov.on.ca/en/pensions/policies/active/documents/l200-412.pdf

[7] Service Canada, Old Age Security pension, http://www.servicecanada.gc.ca/eng/services/pensions/oas/pension/index.shtml

[8] Service Canada, The repayment of Old Age Security pension, http://www.servicecanada.gc.ca/eng/services/pensions/oas/payments/repayment.shtml#calculate_repayment_amount

[9] Canadian Consumer Price Idex, http://canadianinflation.com/historical-canadian-inflation-rates/annual

[10] Poor TSX data. A systematic bias of over 3% (huge) made TSX look much better than it was. http://www.ironwealth.ca/usercontent/documents/don%20andrews%20Canada%20Life%20par%20oct%2019th.pdf

[11] More poor TSX data: Their S&P 500 data were also in error, as were those at Boglehead. http://www.ndir.com/cgi-bin/PeriodicTableofAnnualReturns.cgi .

[12] Globe Investor interactive S&P/TSX Composite graph. http://www.theglobeandmail.com/globe-investor/markets/indexes/chart/?q=TSX-I#dashboard/follows/

[13] Trading Economics non-interactive TSX graph.
http://www.tradingeconomics.com/canada/stock-market

[14] Royal Bank of Canada, *Locked-in RRSPs and Your Options* (information booklet 28054), February 2011.