THE EFFECT OF RRIF MINIMUM WITHDRAWALS ON RETIREE TAXATION AND SPENDING

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EXECUTIVE SUMMARY

RRSPs and workplace LIRAs have central roles in the retirement plans of Canadians. However, when a person turns 71, they must be converted to RRIFs and LIFs, respectively, on which CRA has imposed minimum withdrawal rates. That requirement has become a subject of public concern and discussion. There are two withdrawal schemes on the discussion table:

- Flat withdrawals, in which a fixed amount (subject to inflation adjustment) is taken out annually, is the method almost universally considered in retirement planning, to the point of guidelines like "4% annual withdrawals."
- CRA's minimum withdrawal rates, ones that significantly exceed levels usually considered prudent.

The effect of CRA minimum withdrawals, relative to the more natural flat withdrawal schedule, is the topic of this study.

As a basis of comparison, we defined a "testbed," consisting of a model retiree, with specified financial resources and spending aspirations, and a model market, taken from historical market records. The spending aspirations were linked to the corresponding flat withdrawal size as a percentage of the initial RRIF balance, and both withdrawal methods were made to support the same annual spending. On that testbed, we ran a RRIF/LIF calculator program to generate two types of record: the evolution of fund balances over time; and the financial wrap-up when the retiree dies. The results were assessed over all possible historical years at which the model market started.

In comparison to the simpler and more natural flat withdrawal schedule, CRA's minimum withdrawal requirement on RRIFs:

- **imposes higher taxes on the retiree during his or her lifetime;**
- **hastens the collapse of the RRIF when withdrawals are at prudent levels,**
- **which significantly reduces CRA's *total* tax recovery (lifetime and on the lump sum at the retiree's death), commonly by 1/3 to 1/2;**
- **and shrinks the net estate (i.e., post-tax) received by beneficiaries.**

So, everyone loses. Oddly, the biggest loser from the minimum withdrawal regime is CRA itself, in its dramatic under-recovery of total tax. A very strange tax policy!

The study conclusions were robust over various sizes of inital RRIF balance, various spending levels, all possible starting market years, two different markets, two different values of savings account growth, and both straight sum and net present value assessments. Those conclusions are compiled in Chapter 6, perhaps the next stop for the interested reader.

ABOUT THE AUTHOR

Jim Cavers received a bachelor's degree in engineering physics (1966) and a Ph.D. in electrical engineering (1970), both from the University of British Columbia, Canada. From 1970 to 1979 he was an Assistant, then Associate, Professor in the Department of Systems Engineering at Carleton University in Ottawa. After that, he began his second education: industrial positions, first as a Program Manager at MacDonald, Dettwiler and Associates, a Canadian aerospace firm (1989-82), followed by a year as Senior Engineer at Glenayre Electronics. In 1983 he joined the new School of Engineering Science at Simon Fraser University, where he held the rank of Professor until his retirement in 2008. He is now Professor Emeritus. From 1990 to 1994, he was Director of the School.

Jim was the 1998 recipient of the Manning Foundation's Principal Innovation Award, Canada's most prestigious technology prize. He is a Fellow of the Institute of Electrical and Electronics Engineers, and held a Canada Research Chair in Wireless Communications from 2001 until his retirement. He is the author of one book and over 130 publications.

He is nearing age 71 himself, and has a keen interest in the topic of this study.

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LIST OF ABBREVIATIONS AND INITIALISMS

Common abbreviations

CRA	Canada Revenue Agency
CPP	Canada Pension Plan
ETF	Exchange-Traded Fund
LIF	Life Income Fund
LIRA	Locked-In Retirement Account
MER	Management Expense Ratio
NPV	Net Present Value
OAS	Old Age Security
RRIF	Registered Retirement Income Fund
RRSP	Registered Retirement Savings Plan
S&P	Standard and Poors
TSFA	Tax-Free Savings Account
TSX	Toronto Stock Exchange

Abbreviations used only in this study

B	Balance in RRIF or LIF
Save	Balance in non-registered savings account
W	Withdrawal amount
GI	gross income
TP	Tax paid
NI	Net income
IP	Indexed Pension

1. THE PROBLEM OF MINIMUM WITHDRAWAL RATES

How long will my retirement funds last? The question is an acute one for the millions of Canadians who have no workplace pensions or whose workplace supports only defined contribution plans. For them, RRSPs or workplace LIRAs are at the centre of their retirement finances and they have only guides like "4% withdrawals have a 95% chance of lasting 30 years with inflation." That rule of thumb and others were developed in the well-known Trinity Study [1]-[3]. Such flat withdrawals - the same every year but for inflation - are a simple and natural way to manage the funds.

Unfortunately, in the year of their 71st birthday, retirees must convert RRSPs and LIRAs to RRIFs and LIFs, respectively (or the less-used option of annuities), and that changes everything. CRA imposes minimum withdrawal rates on RRIFs and LIFs, initially almost double the "prudent 4%." LIFs also have a maximum withdrawal rate. The minimum withdrawals have several pernicious effects for the retiree:

- In any fund subject to withdrawals, market downturns in the early years sap the subsequent ability of the fund to recover. The forced large withdrawals in early RRIF/LIF years exaggerate this problem and can quickly deflate the portfolio.
- The unnecessarily large early withdrawals, even if followed by lower withdrawals in later years, cause the retiree to pay more income tax in their lifetime than if the same amount had been paid in the same number of constant withdrawals.
- Many retirees have their savings scattered among several RRIFs or LIFs that may contain different asset types and that cannot be combined. The minimum withdrawals apply to each account separately, which may force sales of assets that should be preserved, while leaving untouched more liquid assets in other accounts.

But how serious are those problems? This study provides quantitative answers, based on a head-to-head comparison of the conventional approach of flat withdrawals that are just sufficient to meet spending needs (like the 4% guideline), and CRA's aggressive minimum withdrawal regime. The results show that **everyone loses with CRA's minimum rates**:

- A retiree pays more income tax over his or her lifetime;
- CRA receives signficantly less total tax - lifetime and from the amount left after death of the retiree;
- The estate receives significantly less money for the beneficiaries;
- And the RRIF funds, which form much of the retiree's income, are exhausted sooner.

Rather than consider the funds in isolation, the study takes a more holistic view, accounting for:

- other sources of taxable retireee income, such as CPP and OAS;
- Canadian taxation, with a simplified model for seniors, including OAS clawback;
- historical returns of various market indexes, plus optional synthetic markets for stress-testing;
- specified inflation rates;
- RRIF or LIF constraints on withdrawals, as well as no such constraints (for flat withdrawals);
- all while maintaining a specified annual spending target in after-tax dollars (inflation-adjusted) until it is no longer possible because the funds have been exhausted.

Chapter 2 contains a description of the "engine" that drives this study: the RRIF/LIF calculator [4]. Its code is also available for inspection or use at the same website. In fact, the present document is also written in Mathcad, so it is "live." The graphs and values obtained in Chapters 4 and 5 can recompute and change in response to changes in input values. Nevertheless, you can understand the results of this study without having pored through Chapter 2, so feel free to skim or skip this one on first reading.

Chapter 3 establishes a careful basis for comparison of flat withdrawals and and CRA withdrawals, using a "model retiree" and model market. All subsequent comparisons are within this framework.

Chapter 4 compares, in detail, flat withdrawals and the RRIF/LIF withdrawal regime for historical market returns starting in one specific year. It's a meaty chapter, with plots, tables and analysis. However, confidence in its results cannot be complete, because it is limited to a single market year.

Chapter 5 completes the work of Chapter 4 by comparing flat and CRA withdrawals over all historical years of starting the market, and gathers statistics on fund longevity, the tax paid, the size of the estate, and so on. This chapter is the payoff for all the previous chapters, and it does allow general conclusions.

Chapter 6 summarizes the findings. It expands on the Executive Summary that followed the title page.

This document is written in Mathcad. A great package for mathematical and computational explorations, but only just serviceable as a word processor. That's why it looks the way it does.

Finally, a couple of disclaimers:

- I am not a financial advisor or an accountant. Instead, I am a retired professor of engineering, forced to learn things that didn't interest me because I am at the other end of a defined contribution plan.

- Although I have spent time and thought on it, the embedded calculator has not been verified by another party. However, nothing is hidden - you can read the code yourself at [4] to verify what it does.

2. ABOUT THE CALCULATOR PROGRAM

Reference:C:\Users\Jim\Documents\RRIF minimum withdrawals\RRIF LIF calculator code only 2-0.xmcd(R)

This chapter can be skimmed or skipped on a first reading, although doing so will mean occasionally having to refer back to it for definitions of quantities.

2.1 What It Does

The RRIF/LIF calculator program, incorporated by file reference just above on this page, is fairly straightforward. After it is given the information about a retiree's, or retiree couple's, financial situation, it steps through the years from the starting age out to age 95. Each year, it:

- updates CPP, OAS, the tax function and target post-tax spending level according to inflation;
- calculates the RRIF/LIF withdrawal that, in conjunction with other taxable income and the savings account, *just* supports the target after-tax spending level; then adjusts the withdrawal value if it must meet CRA minimum and maximum rates; see Appendix B for details of withdrawal calculation;
- updates the balances in the RRIF/LIF and non-registered savings accounts;
- updates the RRIF/LIF and non-registered account balances according to investment returns.

When done, it returns an array with a column for each key quantity (e.g., RRIF/LIF balance or tax paid) and a row for each age (i.e., each year).

In concept, it is much like the Trinity Study [1]. Unlike that study, though:

- The calculator applies CRA-mandated withdrawal rates, or optionally it disables them.
- It accommodates other sources of income, like CPP, OAS, other pensions and non-registered savings.
- Because RRIFs and LIFs are intimately tied to income tax, the calculator includes simplified taxation models for retirees, including the progressive taxation structure and possible OAS reduction caused by high initial withdrawals.
- The calculator allows use of fixed inflation rates, as well as historical inflation rates.
- The calculator acts to support your target for after-tax spending level for as long as possible, and tracks a savings account for your forced excess income in the early years.
- The calculator allows the equities component of the account to be driven by choices among different historic market indexes.
- The calculator allows you to stress-test your finances with two types of synthetic market. One has independently selectable growth and volatility drawn from true markets. The other appends a selected section of the historical record - good, bad or boring - to follow 2013 of the true record.
- **On the drawback side**, it considers only 44 years (1970 to 2013) of market history, instead of the 72 years (1926-1997) of the Trinity Study, although it includes four such markets. Also, it currently operates in annual time steps, coarser than Trinity's monthly time steps.

2.2 The Calculator Arguments (Inputs)

The longevities of the funds and the "trajectories" of account balances depend on many, many input variables, termed the *arguments* of the calculator. They are grouped and defined below.

About you

taxmodel	Use 1 for an individual, 2 for a pension-splitting, equal-aged couple.
myagenow	Your age (in range 71 to 94). This lets you use the calculator at some point during those years, when you wonder how long the remaining funds will last. An age younger than 71 is changed to 71 (the opposite of the fountain of youth). Calculation of new values begins at *myagenow* + 1. If you are calculating for a couple with widely different ages, the calculator is of limited value.
thisyear	What year is it right now, when you are using the calculator? The year is used to adjust tax rates relative to 2013. If you don't want that adjustment, make *thisyear* less than 2014 and the calculator will use the 2013 tax table directly.

About your RRIF/LIF account

RLNflag	"RRIF, LIF, Neither" Use 0 if your account is a RRIF, with minimum required withdrawals, 1 if it is a LIF, with both minimum and maximum withdrawals, and 2 if it is neither, with no constraints on withdrawals. Many people have both RRIFs and LIFs, but the calculator presently considers only one or the other.
equitysplit	This is the percent of your account in equities (stocks, mutual funds, ETFs, etc). The remainder is considered to be in fixed income (individual bonds, GICs, etc), though not bond funds. Account is continually rebalanced to maintain the split.
Bstart	The starting balance of the account, in thousands ($K). "Starting balance" means the value at age *myagenow*.
InvFee	Fees for maintaining your RRIF or LIF can take many forms, such as an annual fixed charge, or an annual percentage (e.g., investment advisor fees), or transaction fees for buying and selling securities, or MERs in mutual funds and (to a much lesser extent) in ETFs. In the calculator, they are approximated as a single annual "investment fee" as a percentage of the balance in the account.

About market returns and inflation

FIretn	The annual growth rate (%) that applies to the fixed income portion of the account. For simplicity, it is fixed.

SaveRetn — The annual rate of return (%) of the savings or non-registered investment account that holds the excess income from the forced withdrawals from your RRIF or LIF. If it is negative, the rate of return is taken to be the same as the market applied to the RRIF or LIF.

whichmkt — Which market affects your equities? Choose from: 1, TSX Composite; 2, TSX Composite total return; 3, S&P 500; 4, S&P 500 total return; 5, 6: reserved for Cdn bonds, int'l equity; 7: synthetic market with selectable gain and volatility (Secn 2.6).

basemkt

g_{pct}

β

For stress-testing your portfolio, you can have the calculator create a synthetic market with the feel of a real one by selecting *whichmkt* = 7 (see above). The calculator modifies *basemkt* (a *whichmkt* value 1,2,...,6, see above) by separating its growth and volatility components. The synthetic market is then given annualized growth g_{pct} (%) and volatility β times that of the base market.

mktseg1

mktseg2

Also for stress-testing, the calculator extends the historic market record out to 2038 by repeating a selectable segment of the historic record. The first and last years of this segment are specified by *mktseg1* - in the range 1970 to 2013 - and *mktseg2* - in the range *mktseg1 + 1* to 2013.

A guide to selecting market extension years might be the annualized 10-year returns of the TSX composite index (Section 2.5). They included:

- best decade, 12.5%: 1978-1987;
- a median decade, 7%: 1993-2002;
- a sub-median decade, 4.8%: 1984-95;
- worst decade, 1.9%: 1981-1990.

Of course, you are free to select whatever years you want.

mktyear1 — The first year of historic market returns to be applied to the equities portion of your RRIF or LIF. Use a value in the range 1970 to 1990 to stick to true historic returns. Values from 1991 to 2013 pick up increasing amounts of the hypothetical market behaviour in the extended market history, and 2014 is market extension alone.

InfRate — Annual inflation rate (%). If it is negative, historical inflation rates are used, starting at *mktyear1*.

About your personal finances

SaveStart	The savings or non-registered investment account that holds the excess income from the forced withdrawals from your RRIF or LIF will be called *Save*. Its initial value (i.e., at *myagenow*) is *SaveStart* , to represent your savings at that age.
IPstart	The initial annual value (i.e., at *myagenow*) in $K of all your pensions or taxable annuities that are indexed to inflation, including CPP and OAS. If you are calculating for a pension-splitting couple (*taxmodel* equals 2), make it your combined indexed pensions. If partially indexed, move the appropriate portion to non-indexed pension just below.
NIP	The annual value in $K of all your taxable pensions that are not indexed to inflation. If you are calculating for a pension-splitting couple (*taxmodel* equals 2), make it your combined non-indexed pensions.
OASstart	The initial annual value (i.e., at *myagenow*) in $K of the OAS component of your indexed pensions *IPstart*. OAS is also indexed to inflation, but is identified separately to allow clawback calculations. If you are calculating for a pension-splitting, equal-aged couple, make it your combined OASes. The calculator treats the two OAS streams as equal.
SpendTarget	Your annual spending ***in post-tax $K*** is indexed to inflation by the calculator. Its initial target value is *SpendTarget*. It is a key quantity, since the calculator works to support your specified spending level. Of course, the higher the spending, the sooner the funds run out.

The argument array

Placing all the arguments explicitly in the calculator's argument list would be clumsy and take up too much space. Instead, we'll put them in a 5x4 argument array, defined as follows:

$$A = \begin{pmatrix} \mathit{taxmodel} & \mathit{myagenow} & \mathit{thisyear} & \mathit{RLNflag} \\ \mathit{equitysplit} & \mathit{Bstart} & \mathit{InvFee} & \mathit{FIretn} \\ \mathit{whichmkt} & \mathit{mktyear1} & \mathit{mktseg1} & \mathit{mktseg2} \\ \mathit{SaveRetn} & \mathit{InfRate} & \mathit{SaveStart} & \mathit{IPstart} \\ \mathit{NIP} & \mathit{OASstart} & \mathit{dummy} & \mathit{SpendTarget} \\ \mathit{basemkt} & g_{pct} & \beta & \mathit{dummy} \end{pmatrix}$$

Packing arguments like this allows procedures to be called with a single argument *A*. The only drawback is that they have to be unpacked within the procedure, but that's easy.

Packing arguments like this allows procedures to be called with a single argument *A*. The only drawback is that they have to be unpacked within the procedure, but that's easy.

2.3 The Calculator Program Output

The main calculator program *trajectories*(*A*) follows a straightforward pattern of year-by-year computation of RRIF/LIF income generation and withdrawal needed to support spending outside the RRIF/LIF, all of it subject to inflation correction. It returns an array with a column for each key quantity (e.g., RRIF/LIF balance or tax paid) and a row for each age (i.e., each year).

In more detail, the calculator generates a 14-column array containing the significant quantities as they evolve over the years. There is a row for each year, from *myagenow* to 95, and a column for each such quantity (see the list of abbreviations in this document's front matter):

0	1	2	3	4	5	6	7	8	9	10	11	12	13
age	B	Save	W	GI	NI	Spend	GIother	OASclaw	IP	OAS	I13	TP	InflSoFar

2.4 Ancillary Programs

The value of the main calculator program *trajectories*(*A*) is increased by programs that work on its input or its output.

Working on the inputs to *trajectories*(*A*) are:

- *argcheck*(*A*), which checks each variable in the argument array for validity;
- *ExtendedHistory*(*A*), which jointly extends the selected market and inflation records for a graphical presentation.

Working on the output of *trajectories*(*A*) are:

- *lastfullspend*(*future*), which finds your latest age at which spending at the inflation-adjusted target rate can be sustained;
- *goodbadyears*(*A*), which runs the calculator repeatedly over all market starting years, from 1970 to 2014, and records, for each year, the age out to which the full spending level can be sustained;
- *spendhisto*(*A,extflag,cdfflag*) is like *goodbadyears*(*A*) but returns the results as a histogram of how often each last year of full spending occured over the various market starting years; the flags allow selection of (1) true or true-plus-extended history and (2) display of histogram or cumulative distribution function.
- *meannz*(*x*) calculates the mean of the *non-zero* entries of vector *x*;
- *totalvalue*(*R,col,age,NPVflag*) calculates the total of column *col* of *R* (the output of *trajectories*) from *myagenow* to *age*, inclusive, either as a straight sum or as net present value.
- *overall*(*R*,*A*,*exitage*,*NPVflag*). Despite leading a model life, our retiree will die. For arguments *A*, associated *trajectories* output *R* and given exit age, this procedure returns an array with these components: 0 tax paid during lifetime; 1 RRIF/LIF balance at end of exit year; 2 tax paid on that lump sum; 3 net (post-tax) value of the lump sum; 4 total tax paid, lifetime and by estate; 5 remaining balance in savings account; 6 net estate, the savings balance plus net lump sum; 7 lifetime OAS clawback. All optionally corrected to present value by *NPVflag*.

2.5 Shortcomings of the Calculator Program

The calculator works well, and it sketches possible futures in ways that other available calculators cannot do. However, it has some shortcomings, and they should be noted as areas for future development:

- It operates to keep the spending level constant in inflation-adjusted dollars. However, spending targets during retirement may vary with time, being larger in the early, active years and, possibly, larger in the later years because of health issues.
- Its tax model is quite simple and it makes an assumption that CRA will continue to adjust it for inflation.
- It assumes that all savings are either in a RRIF or a LIF. However, many retirees will have both. More work needed here.
- It makes use of historic Canadian and US equities returns and historic inflation - but it has no data for indexes of Canadian bonds or international equity. That limits its portfolios to a simplistic split between equities, modeled by historic and extended markets, and fixed income, modeled by a fixed annual rate of returns. More work needed, so the user can create diversified portfolios with arbitrary weightings.
- A big shortcoming... although Mathcad is a beautiful package for mathematical and computational explorations, few people have it. The calculator must be ported to another platform, such as Excel, Java or Matlab.

The scope of the calculator needs attention as well. At present, it is confined to retirees' post-71 experiences, assuming that each year is much like the previous one. It would be much more useful if it began at 65 years. During those early years, many retirees continue to work on a part-time or full-time basis, and even contribute to their RRSPs. It would be difficult for one calculator to capture all variations on how we construct our financial lives, but common patterns ought to be represented.

3. A TEST BED FOR ANALYSIS OF MINIMUM WITHDRAWALS

3.1 Objectives of This Chapter

It is not possible to assess the distortions introduced by CRA minimum withdrawals, relative to the more natural flat withdrawals, without a basis of comparison. This chapter provides that basis, in preparation for the tests in Chapters 4 and 5. After a look at how minimum (and maximum, for LIF) withdrawal rates operate, the chapter introduces a test bed for the comparison, in the form of a model retiree and model markets. They will be at the centre of all subsequent calculations.

3.2 Withdrawal Rates

The meaning of withdrawal rates is different in the Trinity Study and in CRA regulations:

- The Trinity Study [1] considered flat withdrawals - constant, or constant when inflation-corrected - and expressed them as a percentage of the *initial* balance in the fund (e.g., the common 4% guideline).
- CRA withdrawal rates are applied against the *current* balance in the fund; in most cases, that balance declines with time as prior withdrawals take their effect, and the amount of the withdrawal usually declines over the years, even though the rate increases with age.

First, consider Trinity's flat withdrawals. Briefly, the authors used historical records of the S&P 500 index and the Consumer Price Index. They structured the portfolio by a few defined mixes of stocks and bonds, continually rebalanced. Launching the portfolio in different years let them see how often it would be successful (i.e., still have some money) at the end of 20-year, 25-year and 30-year spans, which they expressed as success *rates* for each span. Over 25-year spans within a record from 1946 to 1997, a 50/50 stocks/bonds portfolio was always successful with inflation-indexed withdrawals of 4% of the initial balance, but only 65% successful with 5% withdrawals. Appendix A provides a thumbnail summary of more of that study's extensive tabulations.

Turning now to RRIFs and LIFs, their minimum withdrawal rate applies to both RRIFs and LIFs, and may depend on the carrier. The maximum rate, which applies only to LIFs, depends on province and may vary slightly from year to year. In both cases, the rate depends on the retiree's age (Fig. 3.1) and applies to the current fund balance. Since the maximum withdrawal rate never reaches 100%, you can never get *all* of your money out of a LIF.

How quickly does the RRIF or LIF balance decline with minimum and with maximum withdrawals? For simplicity, assume that the fund has no growth or loss. Fig. 3.2 shows the balances as percentages of the initial balance when withdrawals are all at the minimum value or all at the maximum value. The actual balance for any fund lies on or below the upper curve for RRIFs, and on or between the two curves for LIFs. However, we'll see shortly that an all-minimum-withdrawal profile eventually becomes insufficient for most people.

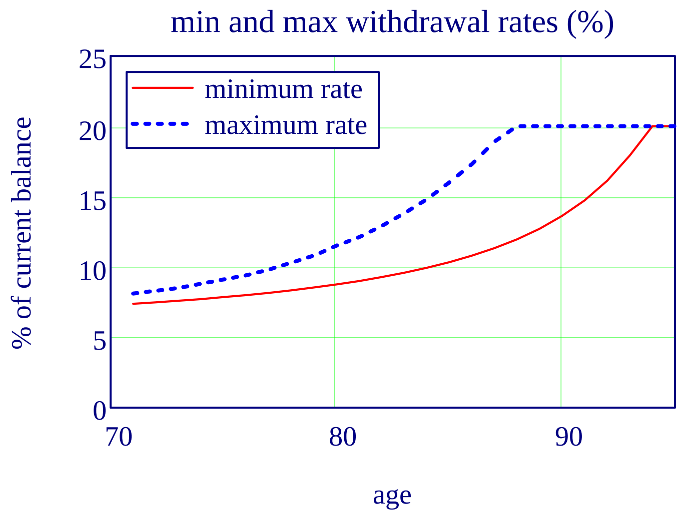
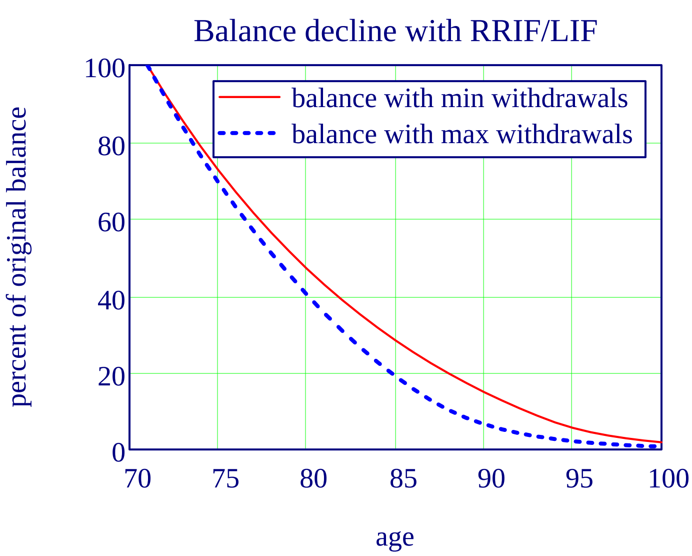


Fig. 3.1. Minimum rate (RRIF and LIF) and maximum rate (LIF) for British Columbia, as percentages of the *current* fund balance.



The balance for *any* allowable withdrawal pattern lies on or below the upper curve for RRIFs, and on or between the two curves for LIFs.

Fig. 3.2. Decline in fund balance with all-minimum (RRIF or LIF) and with all-maximum (LIF) withdrawals. For simplicity, no growth in fund.

Next, how big are the annual withdrawals, as a fraction of the *starting* balance (not the current balance), with all-minimum-rate and all-maximum-rate withdrawals? Fig. 3.3 illustrates the pattern, again for a fund that has no growth or loss. The largest withdrawals are in the early years, even though the withdrawal *rates* increase with age, since rates are applied to the *current* balance. Many retirees will be forced to take their money out of the RRIF or LIF too quickly, making their income unnecessarily high in the early years. The consequences, listed earlier, include increased vulnerability to market downturns, more income tax, possible OAS clawback and portfolio damage.

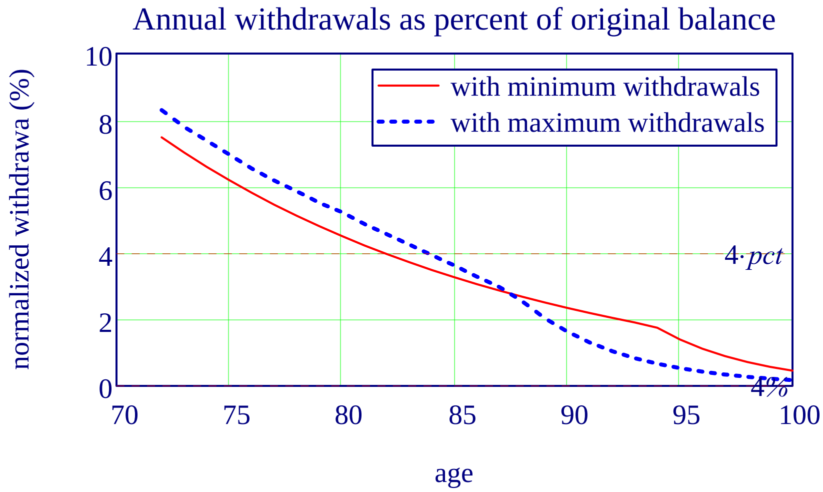


Fig. 3.3. Decline in annual withdrawals with all-minimum (RRIF or LIF) and with all-maximum (LIF) withdrawals. For simplicity, no growth in fund.

Since Fig. 3.3 illustrates withdrawals as percentage of the initial balance in the fund, we can compare them to the flat withdrawals the retiree might have made without the min/max constraints (and in the absence of inflation). Withdrawals at the "prudent 4%" level, shown on the plot, highlight two important points:

- It is only before age 85 that the RRIF and LIF minimum withdrawal rates force the retiree to withdraw more quickly than the 4% guideline. That is still plenty of time for the consequences to accumulate, with lasting effect, but at least the accelerated withdrawals have a finite horizon.
- Another concern is the maximum withdrawal limit on LIFs. If the retiree had been following the all-maximum-withdrawals profile, then the allowable withdrawals would be less than the 4% guideline by age 83. Would it be enough? We'll see further below that LIFs do force a relatively early reduction in spending - which is the intent of LIF legislation, of course, but it may not be appropriate for all.

3.3 A Model Retiree Testbed For Comparisons

In order to compare RRIF withdrawals to unconstrained withdrawals, we have to define a "testbed" - a set of argument values that remain the same in both cases. Our testbed will combine a model retiree and a model market.

Our model retiree has CPP, starting at $11K, and OAS, starting at $6.5K, as the only source of funds outside of the RRIF. The retiree is single, and converts the RRSP to a RRIF at age 71 in 2014. The non-registered savings account starts at zero and earns no interest or market returns. The portfolio is 100% in an ETF for the TSX Composite idex, starting in 1982, and the investment fee is zero. The market extension (i.e., post-2013) consists of repetitions of 1985-1992, which had an unimpressive annualized return of 4.3% - irrelevant, though, since we won't use the extension years in this study. Inflation is a fixed 2%. The argument array is left with just three parameters:

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 1 & 71 & 2014 & NPVflag \\ 100 & Bstart & 0 & 0 \\ 1 & 1982 & 1985 & 1992 \\ 0 & 2 & 0 & 17.5 \\ 0 & 6.5 & 0 & SpendTarget \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

The market that the model retiree experiences is the S&P/TSX Composite in Fig 3.4, where the 1969 level is set to 1. Up to and including 2013, it is the true market record. After that, it would be repetitions of the selected years 1985 to 1992 if we used them. The calculations will launch at age 71, use market returns from age 72, and run up to age 95. Consequently, the latest starting market year that uses historical market records exclusively is 1990. After that, increasing amounts of the market extension would be used, so our results would have to differentiate between true and extended markets, if the latter were used. Inflation is taken as 2% every year.

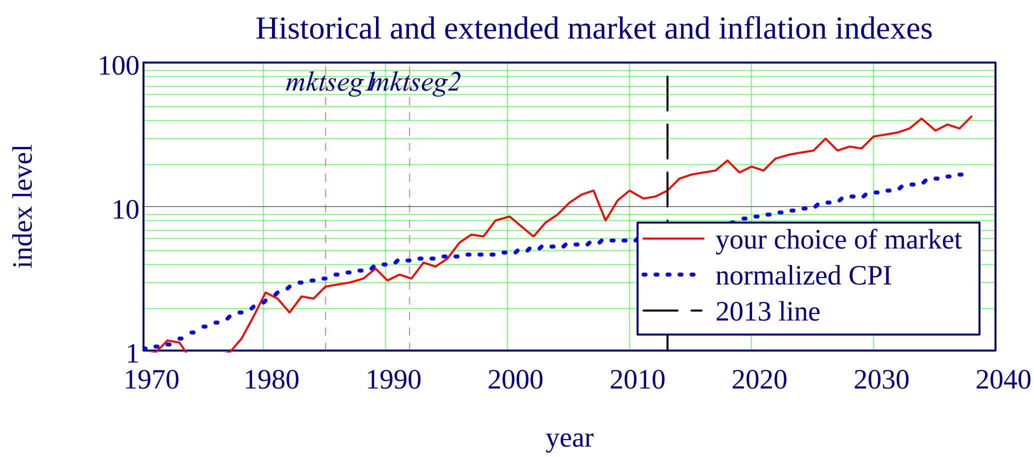
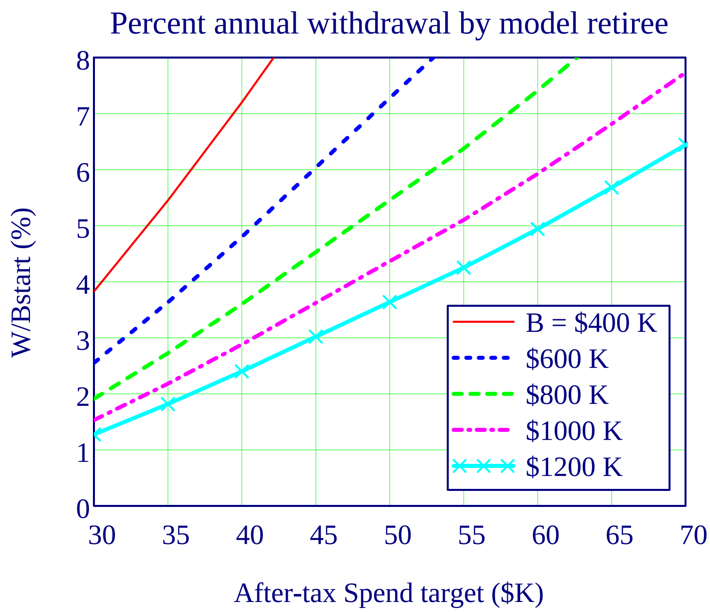


Fig. 3.4. Historical S&P/TSX Composite Index and historical inflation, both normalized by 1969 levels. The post-2013 levels, when used, are repetitions of 1984 to 1995. Our comparisons will use fixed 2% inflation, instead of historical.

We can compare the flat withdrawal scenario and CRA's RRIF/LIF min/max scenario by requiring them both to support the retiree at a given post-tax spending target *SpendTarget* with the same initial balance *Bstart* in the fund. We'll discover, for each scenario, how long that spending can be sustained, how quickly the fund balance declines, how much tax was paid, and how much OAS clawback was inflicted.

But what values of post-tax *SpendTarget* should we choose? For this study, a reasonable approach is to determine the post-tax *SpendTarget* level that gives values of flat withdrawals that are familiar from Trinity (e.g., 4%, 5%); that is, the initial annual withdrawal *W* as a percentage of *Bstart* when withdrawals are unconstrained by CRA min/max requirements. Set *RLNflag* = 2 for this.

Fig. 3.5 shows how the fractional initial withdrawal *W/Bstart* depends on *SpendTarget* level for several values of initial balance *Bstart* when withdrawals are unconstrained by CRA rates. Viewing it another way, it shows how various values of fractional withdrawals, *in conjunction with the model retiree's other sources of income*, affect the achievable post-tax spending level. The plot makes use of the calculator's built-in models of Canadian taxation, including OAS clawback.



The curves all have a slight bend upward, a result of our progressive taxation structure.

Fig. 3.5. Fractional initial withdrawal *W/Bstart* required, in conjunction with model retiree's other sources of income, to achieve the post-tax *SpendTarget* when withdrawals are unconstrained.

Reading directly from Fig. 3.4, we have Table 3.1 below for specific combinations that will be used in the comparisons of unconstrained withdrawals and RRIF withdrawals.

Value of *Spend* target for given fractional withdraw		Fractional withdrawal *W/B* (%)		
		4%	5%	6%
Balance *B*	$400 K	$30.6 K	$33.6 K	$36.7 K
	$800 K	$42 K	$47.5 K	$53 K
	$1200 K	$53 K	$60 K	$67.4 K

Table 3.1. Combinations of initial fund balance *Bstart* and percent withdrawal amount *W/Bstart* to be used in the comparisons. Their corresponding *SpendTarget* values in the table are specific to the model retiree, who also receives CPP and OAS.

3.4 Comments

The model portfolio contains all equities, in order to show the combined effect of CRA's exaggerated initial withdrawals and the timing of market dips and peaks. This is hardly the usual prescription for a retiree's portfolio - but it is not unthinkable, either. An experiment in [4], using market start years from 1970 to 1995, showed that changing the portfolio from a 50/50 mixture of 3% annual return and equities to 100% equities increased the "last age of full spending" (before RRIF collapse) in 23 of the 26 start years. Two remained the same, and only one of the 26 start years adversely affected the "last age," reducing it by just one year.

Table 3.1 has worrisome aspects. At the low end of Table 3.1, our model retiree has managed to save $400 K in a RRIF, which is not particularly easy in middle class jobs or lower. The retiree also receives CPP and OAS at levels not far from the maximum possible. With an eye to making the funds last to age 95 or so, our retiree might try for 4% withdrawals. The result? Only $30.6 K to spend per year (inflation-corrected, of course).

In fact, it is worth viewing that low end of annual expenditures in Table 3.1 through the lens of poverty. Canada does not have an official "poverty line," but Statistics Canada uses a Low Income Cut-off (LICO). This is the point at which an individual or family spends at least 20% more of its income than the average family on food, clothing and shelter (given family and community size) [5]. From [6], that after-tax income for an individual in 2009 was $18.42 K, which we can inflation-correct to $20.2 K in 2014. For the present study, it appears that, unless our retiree has a large RRIF/LIF, reaching a comfortable spend target may mean pushing percent withdrawal into the danger zone, well above Trinity's 4% guideline, and risking early fund collapse.

A further concern, as will be seen in the next two chapters, is that it is the prudent retiree, spending at the flat 4% level, whose finances suffer most from CRA's destructive minimum withdrawal requirements.

4. FLAT VS CRA WITHDRAWALS FOR A SINGLE MARKET START YEAR

This chapter pits flat withdrawals - just enough to meet spending needs - against CRA-mandated minimum withdrawals. The overall objective is to compare their effects on the balances in the RRIF or LIF account, the longevity of these funds, the total tax paid, and the amount left in the estate. Do the CRA requirements *really* make much difference?

This chapter's results are simplified, so that the market experienced by the retiree starts at one specific year in the historical record. Doing so develops detailed understanding of what happens to the retiree's finances. The more comprehensive multi-year tests and their associated statistics are in Chapter 5.

4.1 What Happens - a First Look

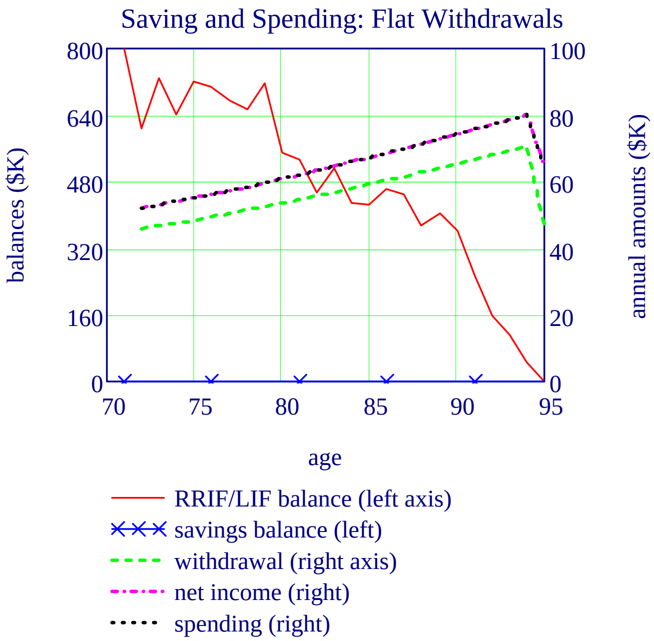
This section compares CRA withdrawals and flat withdrawals for a specific combination of initial RRIF/LIF balance, spending target and starting market year. It does not give statistics of how various markets might affect the retiree, nor can any general conclusions be drawn safely. However, it illustrates the differences in the retiree's finances in the two withdrawal scenarios. That understanding is necessary to interpret the more robust experiments in the next section and the next chapter.

Specifically, the retiree's fortunes will be determined by a RRIF of \$800K and annual spending of \$51K after tax (5.6% withdrawals, if flat). The market starts in 1982. This is off the Table 3.1 grid, in order to illustrate some features. The argument array simplifies to just one free variable

$$\begin{pmatrix} taxmodel & myagenow & thisyear & RLNflag \\ equitysplit & Bstart & InvFee & FIretn \\ whichmkt & mktyear1 & mktseg1 & mktseg2 \\ SaveRetn & InfRate & SaveStart & IPstart \\ NIP & OASstart & dummy & SpendTarget \\ basemkt & g_{pct} & \beta & dummy \end{pmatrix} = \begin{pmatrix} 1 & 71 & 2014 & NPVflag \\ 100 & 800 & 0 & 0 \\ 1 & 1982 & 1985 & 1992 \\ 0 & 2 & 0 & 17.5 \\ 0 & 6.5 & 0 & 51 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

in which *RLNflag* is set to 2 for flat withdrawals and 0 for CRA RRIF withdrawals.

The first scenario is flat withdrawals. Fig. 4.1 shows that it is relatively simple.

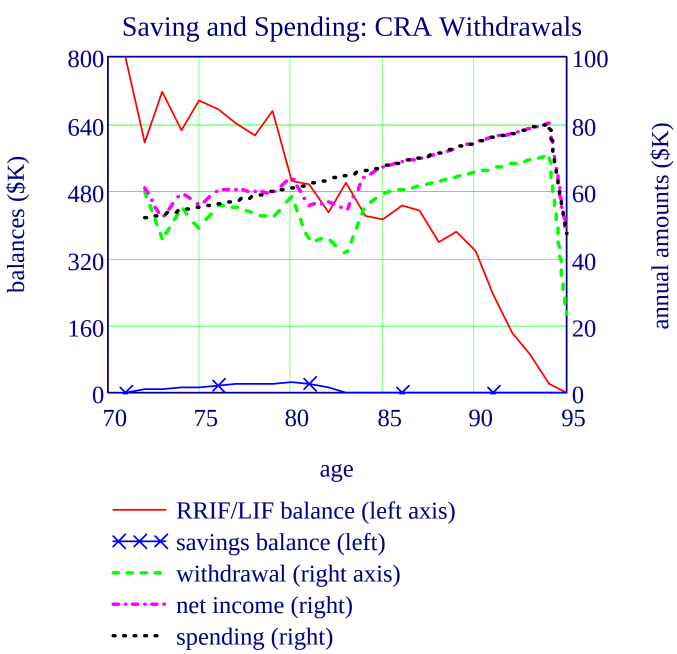


What happens:

- Withdrawals, in conjunction with CPP and OAS, are always just enough after taxes to support spending, with no excess.
- Net income and spending are equal.
- Inflation forces spending up.
- The funds will last to age 94 - the last age of full spending - after which spending collapses to the level supportable by CPP and OAS (off the graph).

Fig. 4.1. Fortunes of the retiree with flat withdrawals. Read the balances (RRIF and savings account) from the left-side axis, and the annual amounts (withdrawal, net income, spending) from the right-side axis.

Next step is the more complicated CRA withdrawal scenario. See Fig. 4.2 and discussion.



What happens:

- Initial withdrawals are larger than needed to maintain spending; excess put into savings account.
- Inflation forces spending up.
- At age 81, savings peak because CRA min withdrawal is just enough to maintain spending, with no excess.
- After 81, min withdrawal (plus CPP, OAS) is insufficient to support spending, but is continued at that rate by drawing on savings.
- By 84, savings gone, now forced to withdraw more than min from RRIF (same amount as flat withdrawals now). Faster decline in balance.
- RRIF lasts until age 94, the same as flat in this example.

Fig. 4.2. Fortunes of the retiree with CRA withdrawals. Read the balances (RRIF and savings account) from the left-side axis, and the annual amounts (withdrawal, net income, spending) from the right-side axis.

In both flat and CRA withdrawals, the retiree receives CPP and OAS, as well as RRIF withdrawals. That amount after taxes (plus some amount from savings, in the case of CRA withdrawals) becomes the amount to be spent. Clearly, determining the size of CRA withdrawals is a complicated interaction of minimum rates, savings account balance, the need to support spending and the desire to minimize the withdrawal. At some age, CRA withdrawals have declined to equal the flat withdrawals, which are just enough to support spending, and the savings account peaks. However, the minimum withdrawals can continue, with help from the savings account, which contains the excess amounts from past years. When the savings account is drained, though, CRA withdrawals jump to equal the flat withdrawals (age 84 above) from then on. The effect of the CRA minimum rates is over at that point, although the fund in the CRA scenario is depleted, relative to flat withdrawals, and full spending ends earlier.

Next, we look at the end game.

We don't want funds to expire before we do - but the flip side of that goal is that we have to expire before they do. When our retiree has reached that grim expiry date, what is the state of the funds? How much tax has been paid during his or her lifetime, how much tax is paid on the lump sum of the RRIF balance, what is the net amount received by beneficiaries of the estate? The amounts are summarized in Table 4.1, as straight sums, and Table 4.2, as net present values, discounted by inflation. The latter is more relevant in a record that spans 25 years and in which the overall numbers are significantly affected by the lump sum left in the account when our favourite retiree is gone.

	retiree expires at age 85		retiree expires at age 95	
	flat	CRA	flat	CRA
lifetimetax	185.4	185.7	344.7	339.1
lumpsum	423.2	411.4	0	0
lumptax	157.7	152.5	0	0
totaltaxpaid	343.1	338.1	344.7	339.1
lumpnet	265.6	258.9	0	0
netestate	265.6	258.9	0	0

Table 4.1. Taxation and estate, seen in total, **as straight sums, not NPV**. Total tax is the sum of tax paid during the retiree's lifetime and tax paid on the lump sum left behind. Net estate is what is left of the lump sum after taxes plus what is left in the savings account. Lump sum values agree with the two graphs above.

Table 4.1 shows that, by 85, CRA's minimum withdrawal regime:

- increases tax the retiree paid during his or her lifetime;
- decreases the total tax collected by CRA;
- decreases the value of the estate.

By 95, flat and CRA properties have reversed by a little - anomalously, as we'll see later.

The equivalent of this table, as net present values, is shown below. The numbers differ, but the pattern is the same.

	retiree expires at age 85		retiree expires at age 95	
	flat	CRA	flat	CRA
lifetimetax	159.3	161.1	267.9	266
lumpsum	320.8	311.8	0	0
lumptax	119.5	115.6	0	0
totaltaxpaid	278.8	276.7	267.9	266
lumpnet	201.3	196.2	0	0
netestate	201.3	196.2	0	0

Table 4.2. Taxation and estate, seen in total, **as NPV, not straight sums**. Total tax is the sum of tax paid during the retiree's lifetime and tax paid on the lump sum left behind. Net estate is what is left of the lump sum after taxes plus what is left in the savings account. Lump sum values do not agree with graphs above, since this table is net present value, discounted by inflation.

These numbers are striking. Remember, though, that this is a single example - a single market start year and a single combination of initial RRIF balance and spending target. Broader results are coming up below and in Chapter 5.

4.2 Retiree Finances in Corner Conditions

Section 4.1 above examined a specific combination of initial RRIF balance and spending target. We need a broader view than that, so the current section examines the results for the four corners of Table 3.1, the combinations of high/low initial balance and high low withdrawal rates. Specifically, we use:

- low balance, low spending (LBLW): $400K balance and $30.6K spending (i.e., 4% withdraw);
- low balance, high spend (LBHS): $400K balance and $36.7K spending (i.e., 6% withdraw);
- high balance, low spending (HBLW): $1,200K balance and $53K spending (i.e., 4% withdraw);
- high balance, high spend (HBHS): $1,200K balance and $67.4K spending (i.e., 6% withdraw).

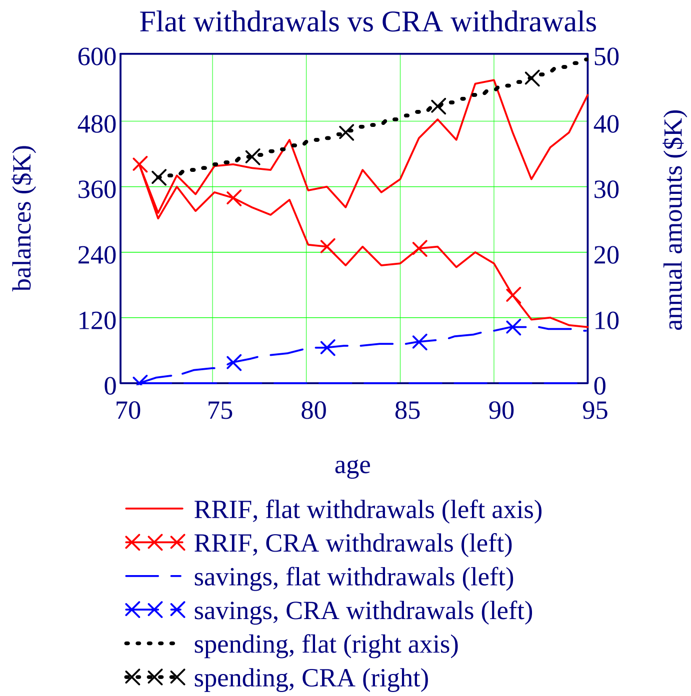
All four combinations are subjected to the same market conditions, since they start in the same market year (1982).

All results in this section start in the same market year - but outcomes can vary significantly from one start year to another in a fund subject to withdrawals. Consequently, full conclusions will have to wait for Chapter 5, which considers all market starting years.

Low balance, low spending

The frugal LBLS retiree's experience is shown below as a graph comparing flat and CRA withdrawals (Fig. 4.3), and as Table 4.3, for lifetime and total taxes and estate.

This comparison is shocking. The flat and CRA withdrawals are set up to support the same relatively modest spending level. Yet 4% flat withdrawals allow the fund to *grow* and CRA withdrawals cause it to *decline* (while force-feeding the savings account). But it did last well, and a slightly higher spend rate would also have worked.



Spending is the same in both cases until the collapse of CRA withdrawals, well past age 95.

Fig. 4.3. Direct comparison of flat and CRA withdrawals for LBLS retiree. Read the balances (RRIF and savings account) from the left-side axis, and the annual amounts (spending) from the right-side axis.

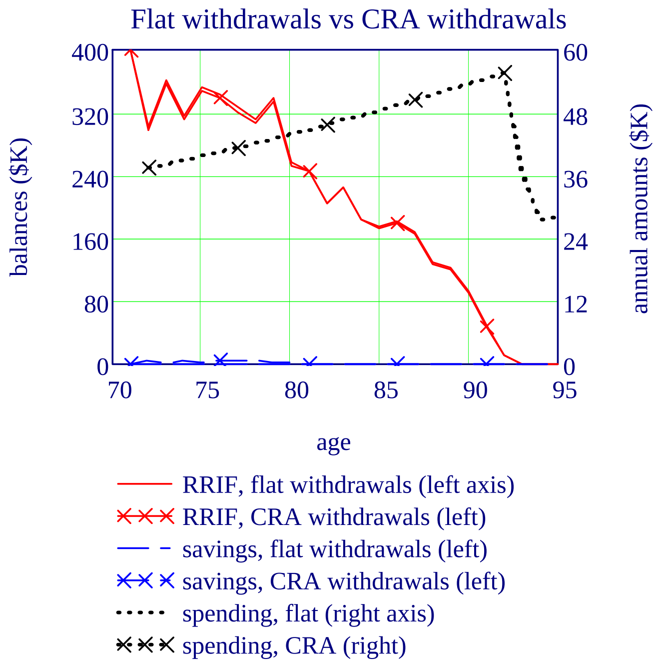
Similarly, Table 4.3 shows that flat withdrawals allow both CRA and the estate to retrieve far more money than with CRA withdrawals if the retiree lives to 95.

	retiree expires at age 85		retiree expires at age 95	
	flat	CRA	flat	CRA
lifetimetax	40	60.1	68.6	93.6
lumpsum	282.2	165	326.5	64.6
lumptax	102.6	51.4	122	12.1
totaltaxpaid	142.6	111.5	190.6	105.6
lumpnet	179.5	113.6	204.5	52.5
netestate	179.5	186.2	204.5	147.6

Table 4.3. Taxation and estate for LBLS **as net present value.** Total tax is the sum of tax paid during the retiree's lifetime and tax paid on the lump sum left behind. Net estate is what is left of the lump sum after taxes plus what is left in the savings account.

Low balance, high spending

The LBHS retiree is high spending only in relative terms. Because the 6% flat withdrawals are not very different from the CRA minimum rates initially, both balances decline in tandem. Not all market years behave like this, as Chapter 5 will show.



By age 81, the savings account used with CRA withdrawals is empty, so the two scenarios make the same withdrawals after that. In both cases, the target spending can be sustained until age 92, after which it drops to the level supportable by CPP and OAS.

Fig. 4.4. Direct comparison of flat and CRA withdrawals for LBHS retiree. Read the balances (RRIF and savings account) from the left-side axis, and the annual amounts (spending) from the right-side axis.

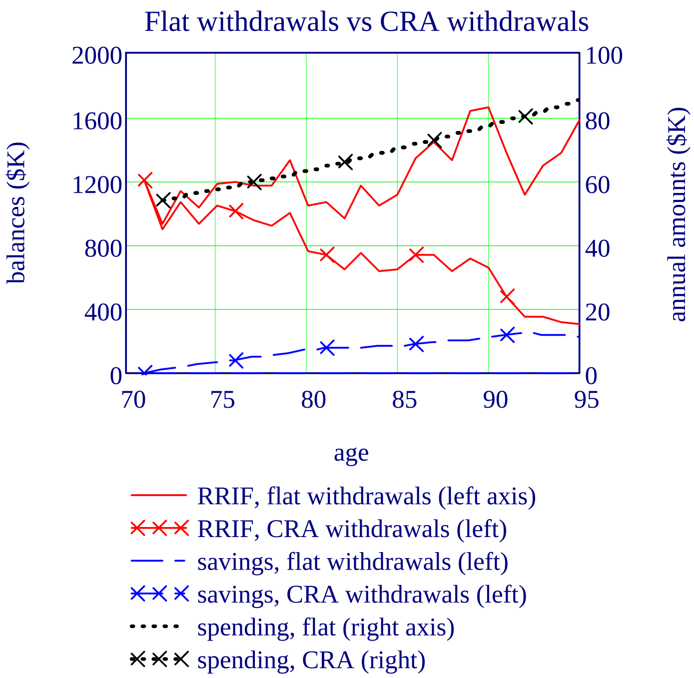
As for total tax and the estate (Table 4.4), disparities between flat and CRA are not as large as they were for LBLS, because the flat rate was not very different from CRA minimum, and because of that particular market start year.

retiree expires at age 85: flat		retiree expires at age 85: CRA	retiree expires at age 95: flat		retiree expires at age 95: CRA
66.4	*lifetimetax*	66.8	100.9	*lifetimetax*	101.1
132.2	*lumpsum*	131.3	0	*lumpsum*	0
37.3	*lumptax*	36.9	0	*lumptax*	0
103.6	*totaltaxpaid*	103.7	100.9	*totaltaxpaid*	101.1
94.9	*lumpnet*	94.4	0	*lumpnet*	0
94.9	*netestate*	94.4	0	*netestate*	0

Table 4.4. Taxation and estate for LBHS as net present value. Total tax is the sum of tax paid during the retiree's lifetime and tax paid on the lump sum left behind. Net estate is what is left of the lump sum after taxes plus what is left in the savings account.

High balance, low spending

RRIF balances for a HBLS retiree (Fig. 4.5) are just scaled versions of LBLS (Fig. 4.3) since flat withdrawals are both 4%, and CRA withdrawals track the minimum until the savings account is empty, then jump to the same as flat. All are proportional to the original balance.



Once again we see great damage inflicted by CRA withdrawals, relative to flat withdrawals, for the retiree who spends at a level that ought to preserve funds.

Fig. 4.5. Direct comparison of flat and CRA withdrawals for HBLS retiree. Read the balances (RRIF and savings account) from the left-side axis, and the annual amounts (spending) from the right-side axis.

Table 4.5 shows that CRA withdrawals have greatly disadvantaged both the estate and CRA itself.

	retiree expires at age 85			retiree expires at age 95	
flat		CRA	flat		CRA
172.7	*lifetimetax*	274.1	296	*lifetimetax*	422
847.2	*lumpsum*	495	981.7	*lumpsum*	193.7
349.6	*lumptax*	195.6	408.3	*lumptax*	64
522.2	*totaltaxpaid*	469.8	704.3	*totaltaxpaid*	486
497.7	*lumpnet*	299.4	573.4	*lumpnet*	129.8
497.7	*netestate*	472.1	573.4	*netestate*	356.5

Table 4.5. Taxation and estate for HBLS as net present value. Total tax is the sum of tax paid during the retiree's lifetime and tax paid on the lump sum left behind. Net estate is what is left of the lump sum after taxes plus what is left in the savings account.

High balance, high spending

From the plot and table below, HBHS is a reprise of LBHS, but scaled up. The reasons are the same as those that noted in HBLS just above.

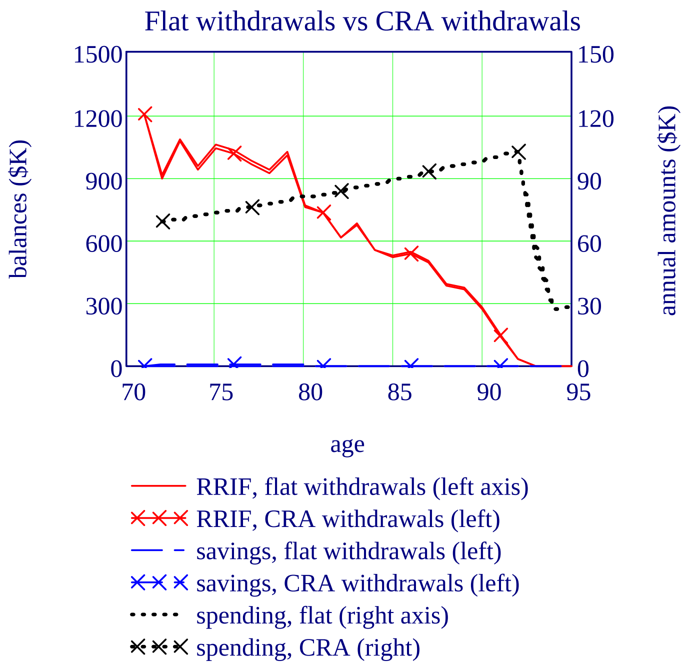


Fig. 4.6. Direct comparison of flat and CRA withdrawals for HBHS retiree. Read the balances (RRIF and savings account) from the left-side axis, and the annual amounts (spending) from the right-side axis.

The wrap-up (Table 4.6) is familiar for this higher spending level: total tax and estate are not dissimilar.

retiree expires at age 85			retiree expires at age 95		
flat		CRA	flat		CRA
306.4	*lifetimetax*	308.8	464.7	*lifetimetax*	466
397.2	*lumpsum*	393.5	0	*lumpsum*	0
152.9	*lumptax*	151.3	0	*lumptax*	0
459.3	*totaltaxpaid*	460.1	464.7	*totaltaxpaid*	466
244.3	*lumpnet*	242.2	0	*lumpnet*	0
244.3	*netestate*	242.2	0	*netestate*	0

Table 4.6. Taxation and estate for HBHS as net present value. Total tax is the sum of tax paid during the retiree's lifetime and tax paid on the lump sum left behind. Net estate is what is left of the lump sum after taxes plus what is left in the savings account.

4.3 What We Found

Many interesting results in this chapter! They were all based on market returns equivalent to starting in the same year (1982), so we could look in comparative detail at the flow of money, to gain some insight. More general conclusions, though, will need the full multi-year analysis coming up in the next chapter.

To compare flat withdrawals with the CRA minimum rate withdrawals, we made both withdrawal schemes support the same after-tax spending level, for the same initial RRIF balance. For four combinations of high/low initial balance and high/low spending level, we examined two types of record: the evolution of fund balances over time; and the financial wrap-up when the retiree dies. In doing so, we also selected the spending levels so that they resulted in familiar ratios (4%, 5%...from Trinity) of initial withdrawals relative to the initial balance in the case of flat (unconstrained) withdrawals. That gave a basis against which to measure the destructive effect of CRA withdrawals.

Because CRA miminum withdrawals are much greater than needed for the first few years, they force the retiree to store the excess in a savings account, which can be drawn on as the minimum rates eventually fall below the flat rate. No savings account is necessary for flat withdrawals.

Comparison of the two regimes - flat and CRA - was shocking when flat withdrawals were at the 4% level, much less than CRA's initial 7.4%. Both supported the same spending level, but flat withdrawals allowed the RRIF to *grow*, while the accelerated CRA withdrawals forced a decline to zero. That's a massive reduction in years of support from retirement savings in a RRIF. With flat withdrawal at 6%, closer to the CRA initial minimum rate, the disparity was lower, if it existed at all - however, that was in part due to the use of 1982 as the market start year. The next chapter will give a broader view.

What determines how long spending can be maintained and the amount of damage inflicted by CRA withdrawals is the flat withdrawal rate as a percentage of initial fund balance, not the fund balance itself. The reason is that most amounts related to the fund are proportional to its initial balance, assuming that CRA keeps adjusting its tax tables according to inflation.

As for the final disposition of funds, questions were the amount of tax paid in the retiree's lifetime and overall, including the lump sum left behind, and the amount left for beneficiaries in the estate. In both cases, both straight sums and net present value were used as measures. In the case of spending levels equivalent to 4% flat withdrawal, CRA withdrawals:

- imposed higher taxes on the retiree during his or her lifetime;
- hastened exhaustion of the RRIF;
- significantly reduced CRA's overall tax recovery (lifetime and lump sum);
- and significantly shrank the estate.

So, everyone lost by this taxation policy.

5. THE BIG PICTURE - FLAT VS CRA IN ALL MARKET YEARS

5.1 What This Chapter Shows

The previous chapter selected one particular year (1982) in the historical market record as the "start year" for the model retiree's market experience. The benefit was ease of tracing the evolution of the retiree's spending and account balances and of comparing the effect of flat RRIF withdrawals against CRA's minimum withdrawals. The drawback was that everything we saw was based on a single market start year - yet market effects on a fund subject to withdrawals can vary significantly from one year to the next, depending on the timing of dips and peaks. Consequently, even after working through all the graphs and numbers in that chapter, we couldn't draw general conclusions.

In contrast, this chapter looks at the results for *all* market starting years and reaches some eyebrow-raising conclusions.

If the reader has just skimmed Chapter 4 to get here, a quick summary of the comparison method should help:

- We defined a model retiree whose financial resources are limited to CPP, OAS and a RRIF.
- We gave the retiree a few combinations of initial RRIF balance and post-tax spending level (inflation-indexed).
- Two models of taking money out of RRIF: (1) flat withdrawals, taking out just enough to support the spending level, rising only through inflation; (2) CRA withdrawals, taking out the larger of the CRA minimum and the amount just enough to meet the spending target, given the savings account balance; any excess in CRA withdrawals to be stashed in the same savings account. Both were set up to allow the retiree to meet the spending target, until the RRIF is exhausted and spending falls to the level supportable by CPP and OAS alone.
- The initial balance and spending combinations were set up so that flat withdrawals were at familiar percentages of the initial balance, e.g., 4%, 5%, 6%, for an easy link to the Trinity study. Of course, CRA minimum withdrawals were different.

A useful finding from Chapter 4 was that, **for a given percent flat withdrawal and given market start year**, the starting balance did not affect the "shape" of any particular quantity over time. Those curves for any particular quantity were all scaled versions of each other, although the scale factor was not proportional to RRIF balance alone, since CPP and OAS played parts. This was true regardless of flat or CRA withdrawals. Consequences:

- The years at which RRIF balances and savings account balances were exhausted do not depend on initial RRIF balance.
- Comparison between flat and CRA withdrawals with respect to years of support do not depend on initial RRIF balance.
- The *relative* amounts of lifetime amount spent, lifetime tax paid, total tax paid and net estate do not depend on initial RRIF balance.

For the comparisons in this chapter, therefore, we can use a single value of initial RRIF balance, which will be $800 K, the middle row in Table 3.1.

5.2 RRIF Longevity: Flat vs CRA Withdrawals

Everyone's first question: how long will it last? We'll take this as "How old will I be in my last year of spending at my target level, before my finances drop to CPP and OAS alone?"

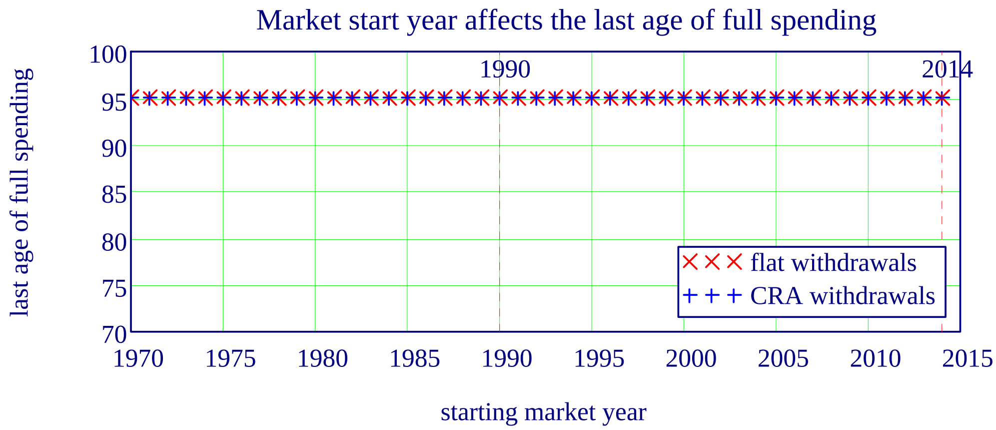


Fig. 5.1. The effect of market start year on last age of full spending, at spending level equivalent to 4% flat withdrawals from initial $800 K (Table 3.1). 1990 and earlier, only historical market values; 1991 to 2014, increasing amounts of the market extension. Age 95 means "95 or older."

Fig. 5.1 shows that, for withdrawals equivalent to 4%, *every* market start year supports full spending to 95 and beyond for both flat and CRA withdrawals, consistent with Trinity for flat withdrawals. Beyond 95, Chapter 4 showed a huge difference in last full spending year between flat and CRA, but few retirees will get there.

Fig. 5.2, for 6% withdrawals, is more interesting. Several years would have been bad times to turn 71 and have an RRSP turned into a RRIF. Interestingly, though, the choice of flat or CRA makes little difference to fund longevity. Chapter 4 showed that, at 6% the two withdrawal schemes tracked each other fairly closely for a 1982 start, and this plot suggests that it is generally true for 6% withdrawals.

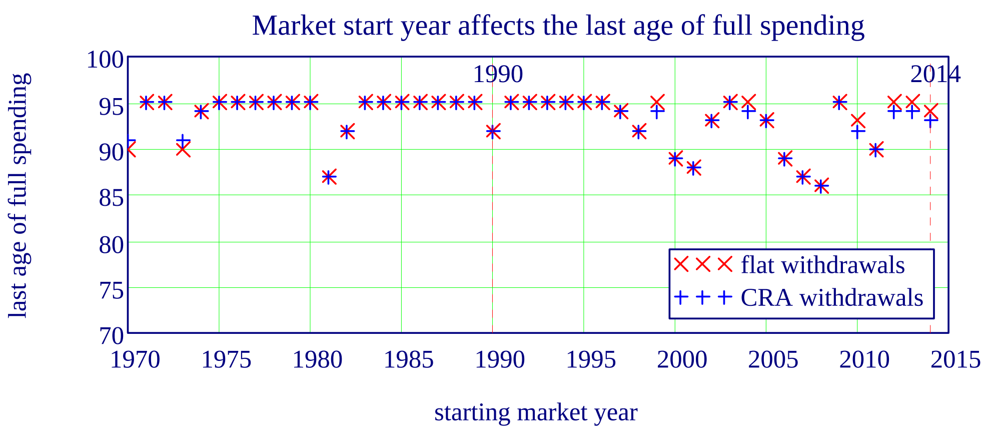


Fig. 5.2. The effect of market start year on last age of full spending, at spending level equivalent to 6% flat withdrawals from initial $800 K (Table 3.1). 1990 and earlier use only historical market values; 1991 to 2014, increasing amounts of the market extension. Age 95 means "95 or older."

5.3 Exit Strategy? The Overall Picture

The next question: Where did the money go? This section looks at the overall picture after the retiree dies - the amount the retiree was able to spend, the amount paid in taxes during his or her lifetime and on the lump sum (if any) left behind in the RRIF, and the post-tax amount left to beneficiaries in the estate (RRIF and savings account). We want to know the effects of the spending level, the age at which the retiree dies, the withdrawal scheme (flat or CRA) and the market start year.

All amounts below are expressed as net present value (NPV), discounted against inflation, since events can span up to 24 years. Also, the analysis keeps to market start years 1990 and earlier, so that only the historical market record is used, with none of the market extension.

Spending equivalent to 4% flat withdrawals

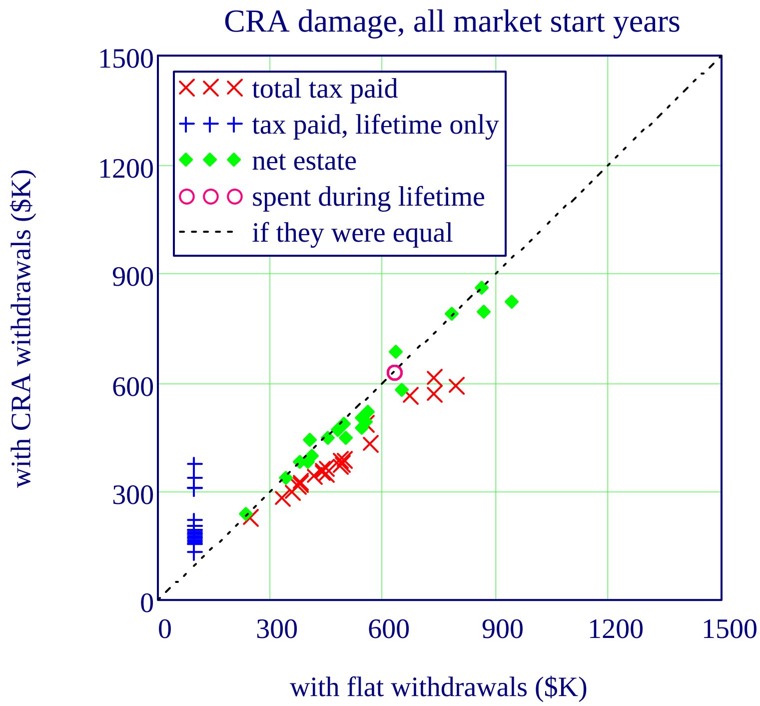
The graphs below answer all these questions. They do need a user's guide, though. Each quantity (e.g., total tax paid) has a point for each market start year. On the horizontal axis is the point's value if flat withdrawals had been used, and on the vertical axis is its value if CRA withdrawals had been used. If the two withdrawal schemes produced the same values, the point would lie on the "if they were equal" line; if CRA withdrawals produced a smaller value than flat, the point would lie below the line; if CRA resulted in a larger value than flat, then above the line.

To get started, recall that Fig. 5.1 above showed that 4% withdrawals allow the retiree to maintain full spending out beyond ages 85 and 95 and (Chapter 4) allowed the RRIF account to grow with flat withdrawals, but shrink with CRA withdrawals. This will affect the next two plots.

Fig. 5.3 shows where the money went if a retiree spends at the 4% level and dies at age 85. It can be interpreted as follows:

- Tax paid during the retiree's lifetime is always higher with CRA withdrawals than with flat - perhaps the original motivation for the minimum withdrawals. But why is that set of points on a vertical line? It's because flat withdrawals are constant (as NPV) and the calculator's income tax model assumes that CRA adjusts its tax table to compensate for inflation - hence the same tax (NPV) every year. Not so with CRA withdrawals!
- But *total* tax paid - lifetime and on the lump sum left behind - is always *lower* with CRA withdrawals than with flat, and the loss to Canada's tax coffers is especially large for those lucky market start years that generated large returns (points farther toward the upper right).
- With two minor exceptions, the size of the net estate is always lower with CRA withdrawals than with flat.
- The amount spent during the retiree's lifetime was the same for all market start years, hence all those points lie on each other; also, flat and CRA schemes supported the same amount, so the point(s) lie on the "equal" line.

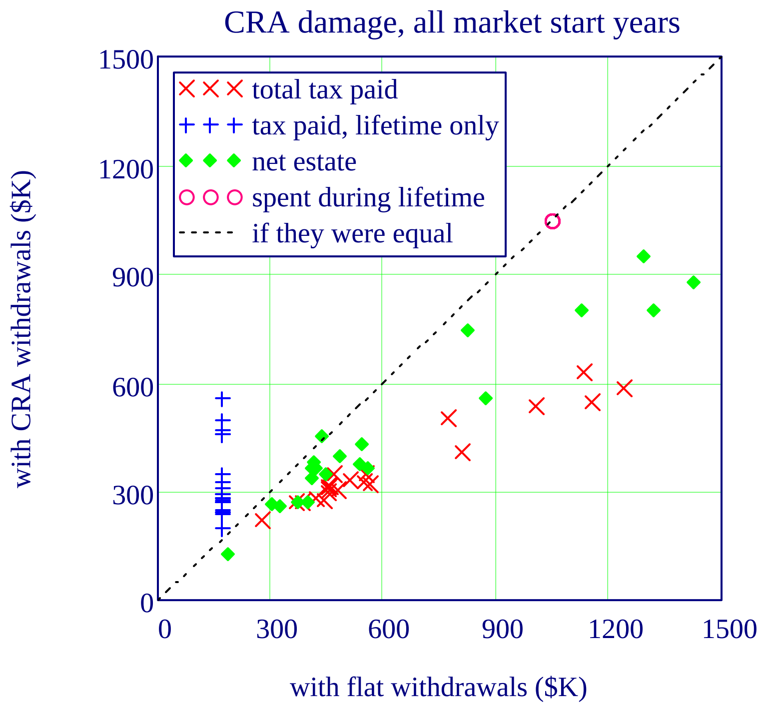
Fig. 5.4, in which the 4%-spending-retiree dies at age 95, shows the same trends as Fig. 5.3, but more pronounced because of the additional ten years. It is particularly striking that, for some years, Canada received less than half as much total tax through use of CRA withdrawals than it would have with flat withdrawals! Beneficiaries also suffered a hit from CRA withdrawals, receiving only about 2/3 as much as with flat withdrawals.



In this case, CRA withdrawals cause:

- more tax paid during lifetime;
- but less *total* tax paid;
- almost always a smaller estate;
- no change in lifetime spending.

Fig. 5.3. Where the money went if retiree spends at 4% level and dies at age 85. Each market start year in 1970 to 1990 has a different point. All quantities NPV.



Here, CRA withdrawals cause:

- much more tax paid during lifetime;
- but far less *total* tax paid;
- almost always a much smaller estate;
- no change in lifetime spending.

Fig. 5.4. Where the money went if retiree spends at 4% level and dies at age 95. Each market start year in 1970 to 1990 has a different point. All quantities NPV.

Spending equivalent to 6% flat withdrawals

The higher spending level equivalent to flat withdrawals of 6% of the initial RRIF balance causes funds to "crash" to CPP and OAS levels in the retiree's later 80s and early 90s for some market start years. See Fig. 5.2. Recall also from Chapter 4 that flat and CRA withdrawals had similar effects - at least for starting in 1982 - since even the initial CRA withdrawals were not much larger than the flat withdrawals. Those factors will affect the next two plots.

Fig. 5.5 shows what happened when the retiree dies at age 85. In comparison with Fig. 5.3, for 4% withdrawals, more tax is paid during the retiree's lifetime, at least with flat withdrawals. However, in comparison with 4% withdrawals, less *total* tax is paid because 6% payouts cause the RRIF to be depleted more quickly. In a familiar theme, less total tax is paid with CRA withdrawals than with flat. The striking feature of the plot, though, is how closely the points (other than lifetime tax) lie to the "equals line," another indication of how similar are the results of flat and CRA schemes when withdrawals are at the 6% level. Yet even this will change when we consider age 95 for the retiree to pass away (coming up next).

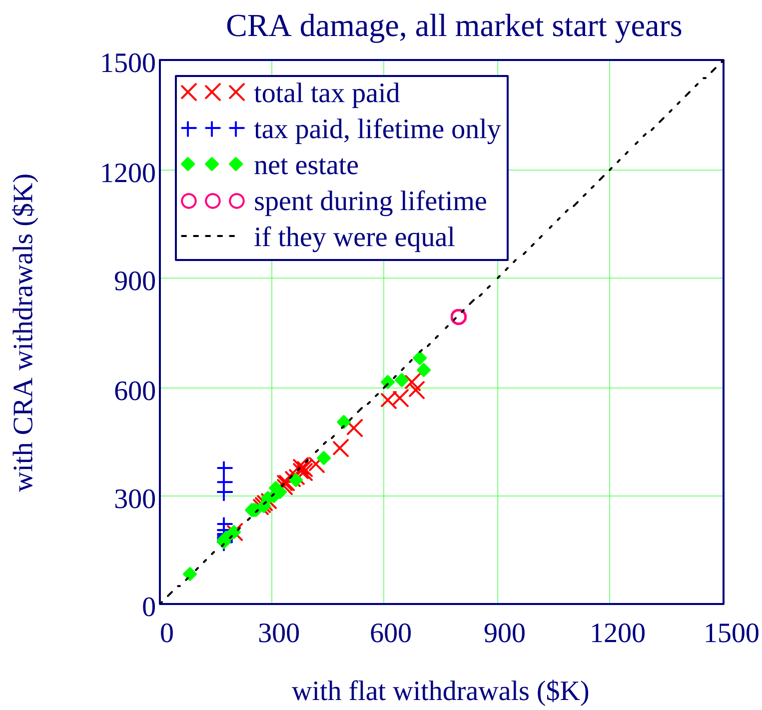


Fig. 5.5. Where the money went if retiree spends at 6% level and dies at age 85. Each market start year in 1970 to 1990 has a different point. All quantities NPV.

One more note about where the money goes. Much of it goes to other investors, when our retiree is forced by minimum withdrawals to sell at a disadvantageous time (buy high, sell low). And, of course, with a different testbed, we would have modeled the fees earned by financial advisors and brokerages.

The last plot, Fig. 5.6, shows what happened when the retiree dies at age 95 after years of 6% withdrawals. With ten more years since Fig. 5.5, the general scale has increased - more of everything, since more time for effects to work through - but some interesting new features have emerged:

- Lifetime tax paid is equal in the two withdrawal schemes for several of the market start years. They correspond to ones in which the RRIF funds collapsed before the retiree's death, leaving only CPP and OAS.
- Total tax paid once again drops below the "equals line" for many market start years - the good ones - in which the total tax (lifetime plus tax on lump sum left behind) Canada receives is substantially less with CRA withdrawals than with flat withdrawals. Could have had $900 K with flat, had only $600 K with the CRA withdrawal scheme.
- The net estate is often small but, in the good market start years where it is larger, it is still substantially less with CRA withdrawals than with flat withdrawals.
- The amount the retiree was able to spend is about the same in the two withdrawal schemes. Unlike the 4% withdrawal rate seen earlier, though, the spent amount has a significant variation with market start year. The reason is that poor start years, ones that cause the RRIF to collapse before age 95, force spending down to the level supportable by CPP and OAS

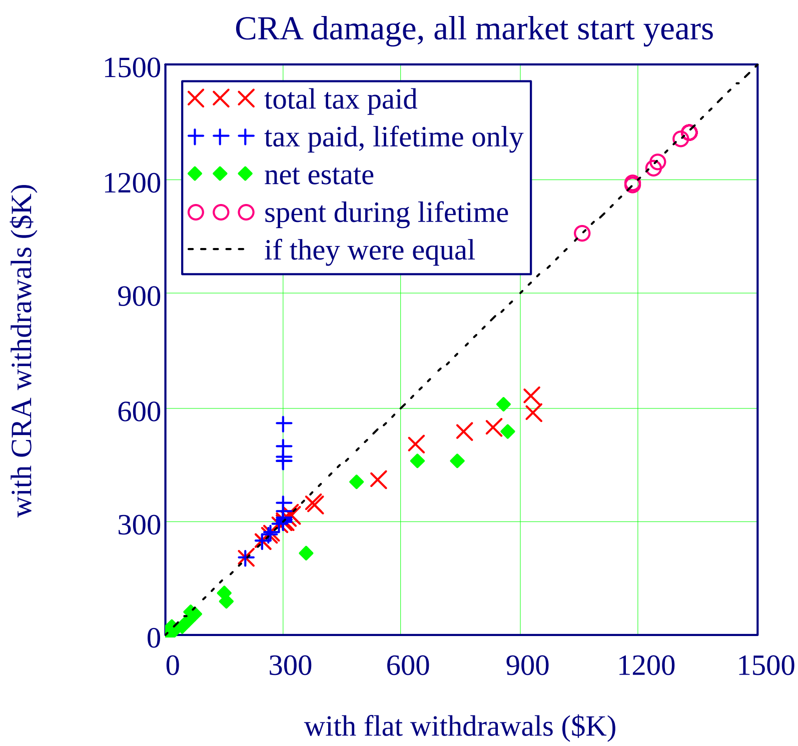


Fig. 5.6. Where the money went if retiree spends at 6% level and dies at age 95. Each market start year in 1970 to 1990 has a different point. All quantities NPV.

5.4 Variations: Other Markets, Savings Account Growth

So far, all the investigations have assumed that: (1) the retiree's RRIF was in an ETF linked to the S&P/TSX Composite index, and (2) the savings account used with CRA withdrawals had no growth. Have we missed anything important by doing so? This section takes a quick look at what happens if the retiree does something else.

A Different Market

The objective is not to examine other markets comprehensively, but to see if the flat vs CRA withdrawal comparisons continue to hold with a different market. The check below uses the S&P/TSX Composite Total Return index, which includes the effect of reinvested dividends. To keep it concise, only withdrawals at the 6% level are considered.

First, Fig. 5.7 shows the balances in the two accounts (RRIF and savings) over time for the Total Return market starting in 1982, as in Chapter 4. It can be compared with Section 4.2's "high spending" (spending causing 6% withdrawal) plots for the S&P/TSX Composite index, in which the two withdrawal schemes ran in parallel and crashed at age 93. In contrast, the Total Return experience below shows the RRIF balance growing (slightly) with flat withdrawals and shrinking with CRA withdrawals. Returns from the better market are not captured well with CRA withdrawals.

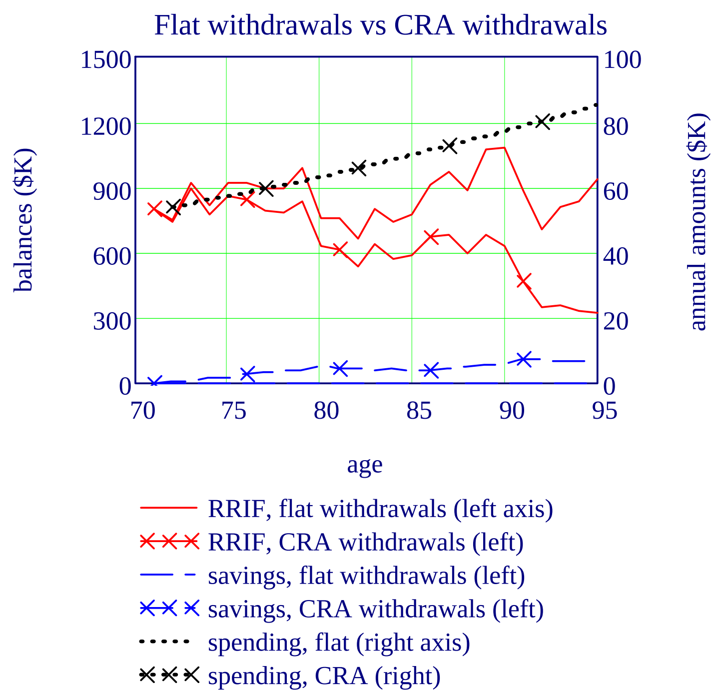


Fig. 5.7. Flat vs. CRA withdrawals for S&P/TSK Composite Total Returns starting in 1982, with $800 K initial RRIF balance, 6% withdrawals. Read the balances (RRIF and savings account) from the left-side axis, and the annual amounts (spending) from the right-side axis.

As for fund longevity, both withdrawal methods always support spending to age 95 and beyond (not shown, to save space). However, the plot above suggests that CRA withdrawals will crash the account earlier in the post-95 years. Only a minority of retirees will experience those years, of course.

Finally, the overall results, lifetime and wrap-up. To save space, only the results for the retiree's involuntary departure at age 95 are shown (Fig. 5.8). Apart from the change in scale (max $2000 K on the axes) resulting from a better market, there is little to distinguish this 6% plot from the *4% plots* in Section 5.3.

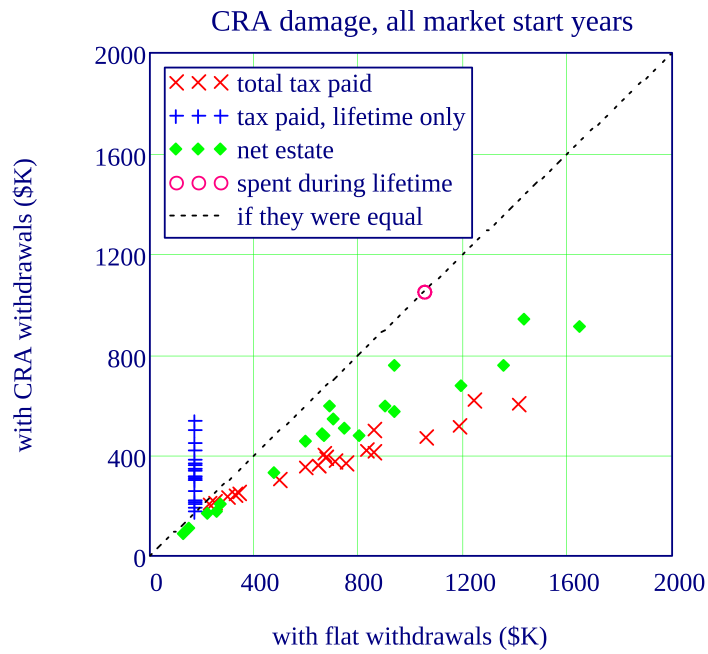


Fig. 5.8. Where the money went if retiree invests in S&P/TSX Composite Total Return, spends at 6% level and dies at age 95. Each market start year in 1970 to 1990 has a different point. All quantities NPV.

Summary: change of market adds nothing new to the flat vs CRA withdrawal comparison, other than the fact that the two withdrawal schemes can diverge, even at the 6% level - the RRIF balance grows with flat withdrawals and shrinks with CRA withdrawals.

Savings Account Growth

The savings account is used to hold amounts from the CRA withdrawals that are excess to current needs. To this point, we have assumed that the retiree keeps them as cash, for liquidity. However, they could be invested in something conservative and relatively short term, like a 3-year GIC. Here we will assume a market-linked GIC that guarantees capital and puts a 3% ceiling on the returns. Typical returns may be close to 2% per year on average, and that is the value we use below.

Growth in the savings account is a second-order effect - the savings account holds the difference between two fractional amounts of the RRIF balance, so it is small, and the growth amount is a small percentage of that. It is not surprising that the single-year (1982) balances and the last year of full spending were unaffected by the savings return at 2%, instead of at zero (neither result shown here).

The overall picture is affected, however. Fig. 5.9 - directly comparable to Fig. 5.6, which had zero savings account growth - shows that some of the net estate points have moved closer to the "equals" line. That is, the savings growth partially mitigated the damage due to CRA withdrawals. The taxes paid, lifetime and overall, are unaffected, of course, and there is negligible change to lifetime spending.

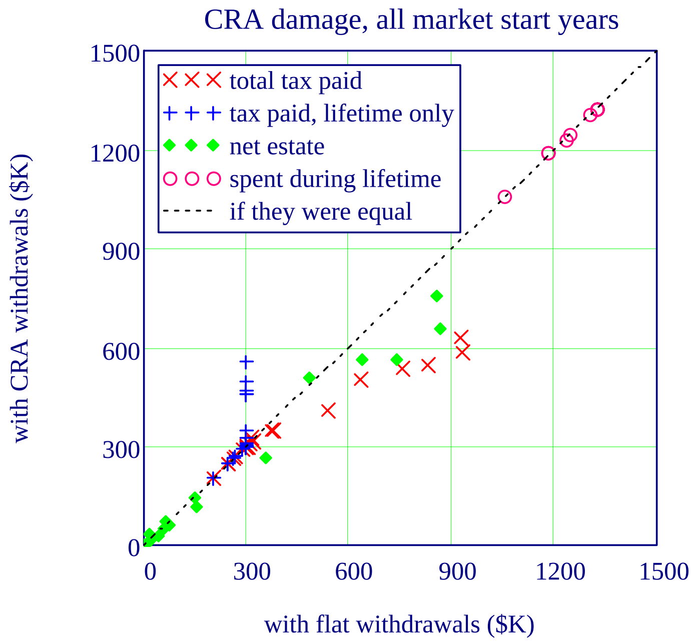


Fig. 5.9. Back to the S&P/TSX Composite index, but savings account has 2% annual growth. Where the money went if retiree spends at 6% level and dies at age 95. Each market start year in 1970 to 1990 has a different point. All quantities NPV.

5.5 Everyone loses

This study examined various sizes of inital RRIF balance, various spending levels, all possible starting market years, two different markets and two different values of savings account growth, and both straight sum and net present value assessments. The comparisons of flat withdrawals and CRA withdrawals were consistent across this range of scenarios.

We now have a clear demonstration of the effect of CRA's minimum withdrawal requirements on RRIFs. Again and again, in comparison to the simpler and more natural flat withdrawal schedule, we have seen that:

- they impose higher taxes on the retiree during his or her lifetime;
- in the case of 4% withdrawals, and even at 6% with the total returns index, they hasten the collapse of the RRIF,
- which significantly reduces CRA's overall tax recovery (lifetime and on the lump sum at the retiree's death), commonly by 1/3 to 1/2;
- they shrink the net estate (i.e., posttax) received by beneficiaries.

So, everyone loses. Strangely, the biggest loser is CRA itself, which is to say the people of Canada, in its under-recovery of total tax. Is this a sensible tax policy?

6. SUMMARY

RRSPs and workplace LIRA are important components in the retirement savings of Canadians, the more so for those who have no workplace pensions or whose workplace supports only defined contribution plans. Retirement planning almost universally considers flat withdrawals - fixed amounts, though inflation-adjusted - with guidelines like "4% annual withdrawals" from the Trinity study [1]. Unfortunately, when a person turns 71, the funds must be converted to RRIFs and LIFs, respectively, which are subject to CRA's minimum withdrawal requirements - and they significantly exceed levels usually considered prudent. The effect of those requirements, relative to the more natural flat withdrawal schedule, was the topic of this study.

The two withdrawal schemes were compared like this:

- We defined a model retiree whose financial resources are limited to CPP, OAS and a RRIF, all taxable.
- We gave the retiree a few combinations of initial RRIF balance and post-tax spending level (inflation-indexed).
- Two models of taking money out of RRIF: (1) flat withdrawals, taking out just enough to support the spending level, rising only through inflation; (2) CRA withdrawals, taking out the larger of the CRA minimum and the amount just enough to meet the spending target, given the savings account balance; any excess in CRA withdrawals to be stashed in the same savings account.
- For proper comparison, both withdrawal methods had to let the retiree meet the spending target, until the RRIF is exhausted and spending falls to the level supportable by CPP and OAS alone.
- The initial balance and spending combinations were set up so that flat withdrawals were at familiar percentages of the initial balance, e.g., 4%, 5%, 6%, for an easy link to the Trinity study. Of course, CRA withdrawals were different, but they still had to support the same spending levels.

With that "test bed," we ran a RRIF/LIF calculator program to generate two types of record: the evolution of fund balances over time; and the financial wrap-up when the retiree dies.

The first set of tests selected one particular year (1982) in the historical market record as the "start year" for the model retiree's market experience. The benefit was the insight gained through ease of tracing the evolution of the retiree's spending and account balances and of comparing the effect of flat RRIF withdrawals against CRA withdrawals. The drawback, though, was that everything we saw was based on that single market start year - yet market effects on a fund subject to withdrawals can vary significantly from one year to the next, depending on the timing of dips and peaks. Consequently, fully general conclusions had to wait for the second set of tests.

In those single-start-year tests, comparison of the two regimes - flat and CRA - summarizes as:

- With flat withdrawals at the 4% level, much less than CRA's initial 7.4%, both regimes supported the same spending level, but flat withdrawals allowed the RRIF to *grow*, while CRA withdrawals forced a decline to zero (Section 4.2). That's a shocking reduction in support from retirement savings in a RRIF, although they both lasted beyond age 95, longer than most retirees do.
- With flat withdrawals at 6%, closer to the CRA initial minimum rate, the RRIF balance declined much more quickly and the disparity was much lower (Section 4.2); however, that was in part due to the use of 1982 as the market start year.

- Even with 6% flat withdrawals, switching to a market with greater returns (TSX Composite Total Returns), once again caused the two withdrawal schemes to diverge (Section 5.4): flat withdrawals allowed the RRIF to grow slightly, but CRA withdrawals forced it to decline.

The second set of tests took the investigation to another level by performing the comparison at all possible starting years in the market record. The results are clear, regarding longevity of the RRIF (Section 5.2):

- For 4% withdrawals, both flat and CRA always supported spending out to age 95. Beyond 95, the Section 4.2 and Section 5.4 results show a massive reduction in RRIF longevity due to CRA withdrawals; however, most RRIF-holders will not get there, so it becomes a moot point.
- For 6% withdrawals, several market start years caused the RRIF to empty in the retiree's later 80s and early 90s. There was little difference between flat and CRA in this case.

As for the final disposition of funds, questions were: the amount of tax paid both in the retiree's lifetime and overall, including the lump sum left behind; the amount left for beneficiaries in the estate; and the lifetime amount the retiree was able to spend (Section 5.3). In the case that the retiree dies at age 95, the CRA minimum withdrawal requirements:

- imposed higher taxes on the retiree during his or her lifetime;
- significantly reduced CRA's overall tax recovery (lifetime and lump sum);
- significantly shrank the estate;
- but had little effect on lifetime spending.

These results were all based on zero growth in the savings account used with CRA withdrawals, so Section 5.4 checked whether this had affected outcomes. Annual growth of 2% growth in the account partially mitigated the reduction in net estate. However, there was no change in last year of full spending and, of course, no change in taxes, either lifetime or total.

The study's general conclusions were robust over various sizes of inital RRIF balance, various spending levels, all possible starting market years, two different markets and two different values of savings account growth, and both straight sum and net present value assessments.

The study has clearly shown the effect of CRA's minimum withdrawal requirements on RRIFs. Again and again, in comparison to the simpler and more natural flat withdrawal schedule, we saw that:

- **they impose higher taxes on the retiree during his or her lifetime;**
- **yet they significantly reduce CRA's overall tax recovery (lifetime and on the lump sum at the retiree's death), commonly by 1/3 to 1/2;**
- **in the case of 4% withdrawals, and even at 6% with a different market, they hasten the collapse of the RRIF;**
- **they shrink the net estate (i.e., post-tax) received by beneficiaries.**

So, everyone loses. Oddly, the biggest loser from the minimum withdrawal regime is CRA itself, in its dramatic under-recovery of overall tax. A very strange tax policy!

APPENDIX A: THUMBNAIL VIEW OF THE TRINITY STUDY

The Trinity study [1] has been cited perhaps a dozen times so far in this document. Time to remind ourselves of its methodology, which guided the present study, and present a small sample of its findings.

The authors made use of historical records: monthly values of the S&P 500 index, for the market, and annual values of the Consumer Price Index, for inflation. They structured the portfolio according to a few defined mixes of stocks and bonds (in some examples, stocks, bonds and cash), and continually rebalanced it to maintain those percentages as the market, inflation and withdrawals did their work. Launching the portfolio in different years let them see how often it would be successful (i.e., still have some money) at the end of 20-year, 25-year and 30-year spans. They expressed it as success *rates* for each span, although we should be careful here, since the time intervals were overlapping, so not independent. Below is the table for a 25 year span, with withdrawal rate *W/Bstart* as a parameter. Withdrawals of 3% are not shown, because they are always 100% successful.

Trinity Study [1]: Portfolio Success Rates over 25 years						
Historical records 1926 - 1997						
	Fixed withdrawals			Inflation-adjusted withdrawals		
Mix stocks/bonds	4%	5%	6%	4%	5%	6%
100%/0%	98%	96%	92%	100%	85%	69%
50%/50%	100%	100%	100%	100%	79%	52%
Historical records 1946 - 1997						
	Fixed withdrawals			Inflation-adjusted withdrawals		
Mix stocks/bonds	4%	5%	6%	4%	5%	6%
100%/0%	100%	100%	100%	100%	82%	61%
50%/50%	100%	100%	100%	100%	64%	36%

Equally interesting is the table on the next page, for a 20-year span. It runs out only to age 91, for an age-71 RRIF beginner, but that may well be enough for most retirees.

Trinity Study [1]: Portfolio Success Rates over 20 years						
Historical records 1926 - 1997						
	Fixed withdrawals			Inflation-adjusted withdrawals		
Mix stocks/bonds	4%	5%	6%	4%	5%	6%
100%/0%	98%	96%	94%	100%	91%	77%
50%/50%	100%	100%	100%	100%	92%	75%
Historical records 1946 - 1997						
	Fixed withdrawals			Inflation-adjusted withdrawals		
Mix stocks/bonds	4%	5%	6%	4%	5%	6%
100%/0%	100%	100%	100%	100%	91%	73%
50%/50%	100%	100%	100%	100%	88%	64%

APPENDIX B: THE WITHDRAWAL AMOUNTS IN DETAIL

The distinction between flat withdrawals and CRA withdrawals was made many times in this study. Although words are often clearer than programs, this may be one of the exceptions. To make it perfectly clear, the subprogram that calculates all types of withdrawal is shown below.

B	current balance in RRIF	*GIother*	gross income from sources other than RRIF
Save	current balance in savings acct	*Spend*	current value of after-tax spending
OAS	current OAS, for clawback calcs	*RLNflag*	0, 1, 2 for RRIF, LIF or no constraints
age	current retiree age	*taxmodel*	1 for individual, 2 for couple
I13	inflation relative to 2013, for tax table correction		
W	amount of the withdrawal		

```
calcW(Wargs) := | "unpack arguments - note they have already gone through argcheck"
                | (B  GIother  Save  Spend  OAS  RLNflag  age  taxmodel  I13) ← Wargs
                | "ideally, take out just enough to meet the after-tax spending target"
                | W_ideal ← GrossIncome(gez(Spend − Save), OAS, taxmodel, I13) − GIother
                | W_min ← B·minrate(age)/100
                | W_max ← B·maxrate(age)/100
                | if  RLNflag = 0
                |    | "it's a RRIF"
                |    | W ← max(W_min, min(W_ideal, B))
                | if  RLNflag = 1
                |    | "it's a LIF"
                |    | W ← max(W_min, min(W_ideal, W_max))
                | if  RLNflag = 2
                |    | "it's neither a RRIF nor a LIF, no withdrawal constraints"
                |    | W ← max(0, min(W_ideal, B))
                | return  W
```

APPENDIX C: THE GRAPH FACTORY

This study was written in Mathcad, so all its plots and tables were generated in the document itself. Those mechanics are in the collapsed area below, if the reader wants to look them over.

REFERENCES

[1] Cooley, Philip L., Carl M. Hubbard, and Daniel T. Walz, "Retirement Savings: Choosing a Withdrawal Rate That Is Sustainable," *American Association of Independent Investors (AAII) Journal*, 10, 3: 16–21, 1998.

[2] Philip L. Cooley, Carl M. Hubbard and Daniel T. Walz, "Sustainable Withdrawal Rates From Your Retirement Portfolio," *Financial Counseling and Planning*, Volume 10 (1), 1999.

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http://www.onefpa.org/journal/Pages/Portfolio%20Success%20Rates%20Where%20to%20Draw%20the%20Line.aspx

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http://www.parl.gc.ca/Content/LOP/ResearchPublications/prb0865-e.htm

[6] UFCW, *By the Numbers: Income distribution and the poverty line*.
http://www.ufcw.ca/index.php?option=com_content&view=article&id=2433:by-the-numbers-income-distribution-and-the-poverty-line&Itemid=306&lang=en

$age := 71 .. 95$

$$age := 72 .. 100$$

$$bmin_{71} := 1 \qquad bmin_{age} := bmin_{age-1} \cdot \left(1 - \frac{minrate(age)}{100}\right)$$

$$bmax_{71} := 1 \qquad bmax_{age} := bmax_{age-1} \cdot \left(1 - \frac{maxrate(age)}{100}\right)$$

$$i := 71 .. 100$$

$$wmin_{age} := bmin_{age-1} \cdot \frac{minrate(age)}{100}$$

$$wmax_{age} := bmax_{age-1} \cdot \frac{maxrate(age)}{100}$$

$$pct := 1$$

Model retirees don't smoke or drink. They do pick up after their dog, and perhaps after other people's dogs, too.

For the graph, Fig. 3.4:

```
fractDr(Spend, B) := | "layout of Wargs, the arguments to calcW:"
                     | "B GIother Save Spend OAS RLNflag age taxmodel I13"
                     | "for the model individual:"
                     | Wargs ← (B  17.5  0  Spend  6.5  2  71  1  1)
                     | W ← calcW(Wargs)
                     | return W/B
```

$Spend := 30, 35 .. 70$

Value of *Spend* target for given fractional withdraw		Fractional withdrawal *W/B* (%)		
		4%	5%	6%
Balance *B*	$400 K	$30.6 K	$33.6 K	$36.7 K
	$800 K	$42 K	$47.5 K	$53 K
	$1200 K	$53 K	$60 K	$67.4 K