Multiuser Transmitter Diversity Through Adaptive Downlink Beamforming

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Abstract – **In adaptive transmitter diversity, the complex weights applied to the downlink antennas are obtained from the instantaneous complex gains in the uplink. This paper provides the first analysis of the effects of decorrelation of gains between the uplink and downlink and of number of antennas, and the first assessment of the effect of fading when the method supports multiple users.**

I. INTRODUCTION

It is common practice to use diversity arrays at a base station receiver for mitigation of fading and for support of multiple users. Use of base station transmitter arrays to produce diversity reception at a mobile with a single antenna is less well understood. Recent approaches to transmitter diversity distinguish the different antennas by relative delays [1], different spreading codes or by space-time codes [2],[3], none of which require or use information about the current channel state. However, if such information is available from uplink measurements that are nearby in time and frequency, then we can develop *adaptive* transmitter diversity methods with potentially better performance. The objective is to apply a set of complex weights to the antennas that produces signal reinforcement at the mobile receiver.

Early descriptions of adaptive transmitter diversity [4]-[6] dealt only with single-user systems. More recently [7]-[10], uplink measurements have been used to select antenna weights that support multiple simultaneous users by nulling or mimimizing the interference experienced at each mobile from transmissions intended for the others. Nevertheless, there are two major deficiencies in our understanding of adaptive transmitter diversity. First, the only analyses [4], [5] of the effects of uplink/downlink decorrelation through delay and frequency separation, and how it plays against the number of antennas, are at best approximate – so, strangely, we still do not know how well the method works in reality. Second, the multiuser variants have not been assessed for their performance in fading, even for perfect channel state information (CSI), let alone decorrelation effects.

This paper provides answers to both of these open issues. It is the first analysis of the critical effect of decorrelation between uplink measurements and downlink transmitter diversity, and shows that the requirement for high correlation is softened as more antennas are employed. It also shows how an existing multiuser antenna weighting algorithm devised for static channels and perfect CSI can be used in a fading and decorrelation environment, and how well it works.

II. SYSTEM MODEL

Figure 1 illustrates the system. The base station has an array of L antennas with which to support M users. The transmitted signal is

$$\mathbf{s} = \sum_{k=1}^{M} c_k \mathbf{w}_k = \mathbf{Wc} \tag{1}$$

where, for user k, $c_k = \pm 1$ is the data symbol and $\mathbf{w}_k$ is the length-L vector of complex weights, $\mathbf{W}$ is LxM with colums $\mathbf{w}_k$ and $\mathbf{c}$ is the data vector. The instantaneous transmitted power of user k and total power are, respectively,

$$P_k = \left|\mathbf{w}_k\right|^2 \text{ and } P = tr\left[\mathbf{W}^H \mathbf{W}\right]. \tag{2}$$

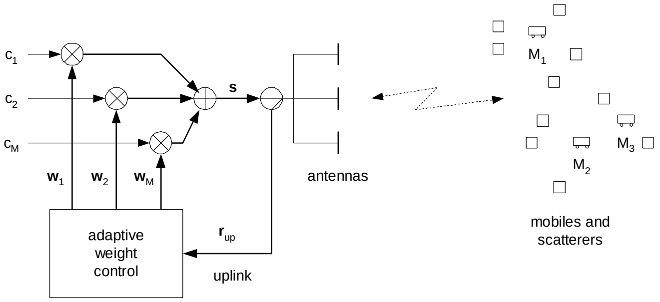

Fig.1. System Configuration

The antenna elements have enough physical separation that the LM complex downlink channel gains to the mobiles are statistically independent with equal variances $\sigma_g^2 = E\left[|g|^2\right]$. The uplink and downlink gains are highly correlated; e.g. in a TDD (time division duplex) system with very short blocks, or in an FDX (full duplex) system where spacing of transmit and receiver frequencies is within the coherence bandwidth of the channel. The base forms an MMSE estimate $\hat{g}$ of each downlink gain g from the uplink signal using multiuser methods like [11], so that $g = \hat{g} + e$, where the estimation error e is uncorrelated with $\hat{g}$. To keep the discussion general, we characterize the estimation by a correlation coefficient ρ between the true gain and its estimate, and assume it to be the same for all gains. The variances are therefore related by

$$\sigma_{\hat{g}}^2 = \rho^2 \sigma_g^2 \text{ and } \sigma_e^2 = \left(1-\rho^2\right)\sigma_g^2 . \tag{3}$$

To model Rayleigh fading, we assume that all gains, their estimates and the estimation errors are complex Gaussian random variables.

The length-M vector of received signals at the mobiles is

$$\mathbf{r} = \mathbf{G}^H \mathbf{W}\mathbf{c} + \mathbf{n} = \hat{\mathbf{G}}^H \mathbf{W}\mathbf{c} + \mathbf{E}^H \mathbf{W}\mathbf{c} + \mathbf{n} \tag{4}$$

where $\mathbf{G}$ is LxM with columns $\mathbf{g}_k$ as the downlink complex gains of user k, and $\hat{\mathbf{G}}$ and $\mathbf{E}$ are the estimates and estimation errors of the gains, respectively. Components of the noise $\mathbf{n}$ have variance σ_n^2. The convention of conjugating channel gains simplifies notation later.

III. SINGLE USER ANALYSIS

To prepare for the multiuser case, we first analyze single-user operation.

A. Optimum Weight Vector

For a single user, (4) simplifies to

$$r = c\mathbf{g}^H \mathbf{w} + n = c\left(\hat{\mathbf{g}} + \mathbf{e}\right)^H \mathbf{w} + n\,. \tag{5}$$

where the user subscripts are omitted for clarity. When conditioned on the gain estimate $\hat{\mathbf{g}}$, the received signal power is

$$\left|\hat{\mathbf{g}}^H \mathbf{w}\right|^2 + \sigma_e^2 \left|\mathbf{w}\right|^2\,. \tag{6}$$

Maximizing received power for a given transmit power P (2) is a standard variational problem, with a solution that makes the weight $\mathbf{w}$ proportional to the estimated gain vector $\hat{\mathbf{g}}$ (recall the conjugate channel convention). Thus diversity is obtained by ensuring that the predictable parts of the gains add coherently in (5). This is the counterpart of perfect CSI optimization [5], [8], [9] that makes $\mathbf{w}$ proportional to the true gain $\mathbf{g}$.

In another interpretation, we note from (5) that the channel is conditionally Rician [12], with K-factor (ratio of direct to scattered power)

$$K = \frac{\left|\hat{\mathbf{g}}^H \mathbf{w}\right|^2}{\sigma_e^2 \left|\mathbf{w}\right|^2}\,. \tag{7}$$

Optimization therefore maximizes the K-factor.

To simplify the analysis, we write the proportional solution

$$\mathbf{w} = \sqrt{\frac{u}{L}}\,\sigma_n \hat{\mathbf{g}} \tag{8}$$

where u, a power control variable, depends on $\hat{\mathbf{g}}$. Define the branch-average estimate of instantaneous power gain as

$$x = \left|\hat{\mathbf{g}}\right|^2 / L\,. \tag{9}$$

The transmit power is then $P = \left|\mathbf{w}\right|^2 = u x \sigma_n^2$ and we have

$$\Gamma = u x \tag{10}$$

where the transmit SNR $\Gamma = P / \sigma_n^2$. In this analysis, we choose to maintain a constant transmit power (constant Γ), so, from (10), u varies inversely with x. The potential problem if x is near zero is not an issue; at that point, the K factor is also close to zero, so the channel is Rayleigh, and any weight vector with the right power gives the same performance.

B. Receiver Performance

The mobile receiver prepares an estimate v of the complex gain its signal experiences, typically through use of pilot symbols. For simplicity, we assume perfect CSI at the receiver, since performance will be dominated by the transmitter decorrelation represented by ρ; thus $v = \mathbf{g}^H \mathbf{w}$. Data decisions $\hat{c}$ are given by the sign of the decision variable $D = \mathrm{Re}\left[r v^*\right]$. Since r and v are correlated Gaussian variates with nonzero mean, the conditional error rate (the probability D is negative when c is positive) is [12], [13]

$$P_e(x) = Q_m(a,b) - C I_0(ab)\exp\left(-\frac{a^2+b^2}{2}\right), \tag{11}$$

where $Q_m(a,b)$ is the Marcum Q-function and the parameters are given by

$$\begin{Bmatrix} a^2 \\ b^2 \end{Bmatrix} = \frac{1}{2}\left|\frac{\mu_r}{\sigma_r} \mp \frac{\mu_v}{\sigma_v}\right|^2 \quad \text{and} \quad C = \frac{1+\eta}{2}, \tag{12}$$

and, in turn, the ratios of means to standard deviations and the r, v correlation coefficient η are, from (5), (8)-(10),

$$\frac{\mu_r}{\sigma_r} = \sqrt{\frac{L\Gamma x}{1+\sigma_e^2\Gamma}}, \quad \frac{\mu_v}{\sigma_v} = \frac{\sqrt{Lx}}{\sigma_e} \quad \text{and} \quad \eta = \frac{1}{\sqrt{1+\dfrac{1}{\sigma_e^2\Gamma}}} \tag{13}$$

where u is determined by (10) for constant Γ.

The conditional error rate (11) must be averaged over x, which (9) shows to have a χ^2 distribution with mean $\sigma_{\hat{g}}^2$ and $2L$ degrees of freedom

$$p_x(x) = \frac{x^{L-1}}{(L-1)!}\,\frac{\exp\left(-xL/\sigma_{\hat{g}}^2\right)}{\left(\sigma_{\hat{g}}^2/L\right)^L}\,. \tag{14}$$

The average error rate is therefore

$$\bar{P}_e = \int_0^\infty P_e(x)\, p_x(x)\, dx\,. \tag{15}$$

which must be evaluated numerically.

Fig. 2 shows the dependence of error rate (15) on transmit SNR for various values of correlation coefficient ρ and number of antennas L. For values of ρ approaching unity, the diversity effect is clearly evident from the slopes of the curves. Performance degrades quickly to Γ^{-1} dependence, even for 4 antennas, as ρ decreases, bringing down the K factor. However, it is still much better than single antenna performance.

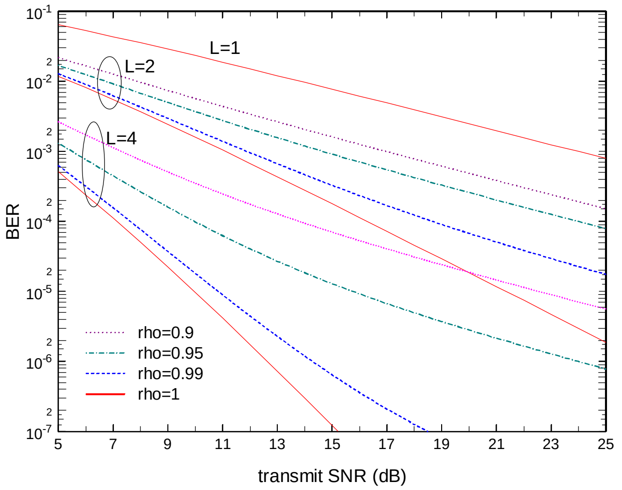

Fig. 2. Error Probability For Single User

We explore a greater range of parameter values in Fig. 3, which presents the dB reduction of required SNR, compared with a single antenna, to achieve an average error rate of 10^{-3}. Evidently, adding more antennas softens the requirement for very high correlation coefficient between uplink and downlink. For the values shown, $r = 0.6$ is roughly the lower limit to make the method attractive.

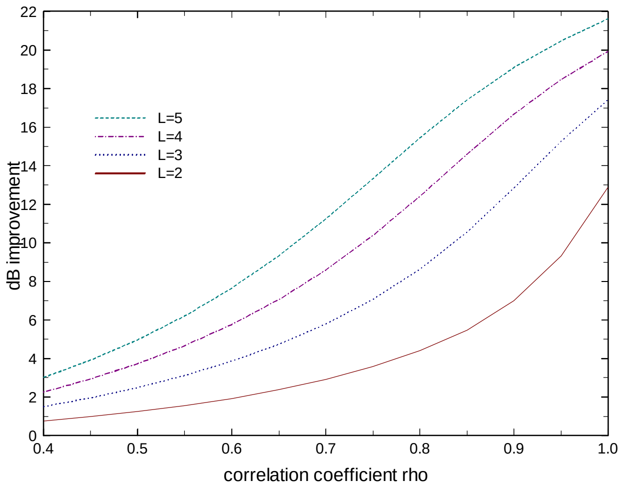

Fig. 3. Power Savings Compared With Single Antenna System For Single User at BER=10^{-3}

Adaptive transmitter diversity shows an interesting behaviour. Traditional frequency diversity saturates at AWGN performance for large numbers of antennas, since transmitter power is diluted among the antennas. In contrast, adaptive transmitter diversity converges for $r = 1$ to the performance of receiver diversity [12], where adding antennas adds signal power as well as diversity. The reason is that our receiver has only a single noise process, which counters the dilution. As a result, performance of adaptive transmitter diversity continues to improve without saturation as we add antennas, as shown clearly in Fig. 3.

IV. MULTIUSER TRANSMITTER DIVERSITY

A. Receiver Peformance

In the multiuser case, we assume that the mobiles employ single-user detectors, as in the previous section, in which the received value and perfect channel estimate of user k are

$$\begin{aligned} r_k &= \hat{\mathbf{g}}_k^H \mathbf{W}\mathbf{c} + \mathbf{e}_k^H \mathbf{W}\mathbf{c} + n_k \\ v_k &= \hat{\mathbf{g}}_k^H \mathbf{w}_k + \mathbf{e}_k^H \mathbf{w}_k \end{aligned} . \qquad (16)$$

An error occurs if the decision variable $D_k = \mathrm{Re}\left[r_k v_k^*\right]$ is negative when c_k is positive. The error rate conditioned on $\hat{\mathbf{G}}$, $\mathbf{W}$ and $\mathbf{c}$ is still given by (11), but a, b and C take a more general form [12], [13], now that $h = h_r + jh_i$ is complex:

$$\left\{ \begin{matrix} a^2 \\ b^2 \end{matrix} \right\} = \frac{ \frac{|m_r|^2}{s_r^2} + \frac{|m_v|^2}{s_v^2} - 2h_i \frac{\mathrm{Im}\left[m_r m_v^*\right]}{s_r s_v} \mp 2\sqrt{1-h_i^2}\, \frac{\mathrm{Re}\left[m_r m_v^*\right]}{s_r s_v} }{2\left(1-h_i^2\right)}$$

$$C = \frac{\sqrt{1-h_i^2} + h_r}{2\sqrt{1-h_i^2}}$$

(17)

From (16) and the statistics of $\mathbf{E}$ and n_k, we obtain the various expressions in (17) as

$$\frac{m_r}{s_r} = \frac{\hat{\mathbf{g}}_k^H \mathbf{W}\mathbf{c}}{\sqrt{s_e^2 \left|\mathbf{W}\mathbf{c}\right|^2 + s_n^2}} \qquad \frac{m_v}{s_v} = \frac{\hat{\mathbf{g}}_k^H \mathbf{w}_k}{s_e \left|\mathbf{w}_k\right|}$$

$$h = \frac{s_e^2 \mathbf{w}_k^H \mathbf{W}\mathbf{c}}{\sqrt{\left(s_e^2 \left|\mathbf{W}\mathbf{c}\right|^2 + s_n^2\right) s_e^2 \left|\mathbf{w}_k\right|^2}} \qquad (18)$$

The conditional error rate given by (11), (17), (18) is as far as analysis will take us easily. It is nevertheless quite useful, since it reduces simulation run times dramatically by incorporating the average over the $\mathbf{E}$ and $\mathbf{n}$ ensembles.

In the special case of perfect uplink/downlink correlation ($r = 1$), singularities occur some of the parameters (18) because $s_e^2 = 0$. Rather than take limits, we simply note from (16) that the vanishing of $\mathbf{E}$ makes all decision variables D_k conditionally Gaussian and real. The conditional error probability for user k (assuming c_k is positive) is then

$$P_{ek} = Q\left(\frac{\sqrt{2}\,\mathrm{Re}\left[\mathbf{w}_k^H \mathbf{g}_k \mathbf{g}_k^H \mathbf{W}\mathbf{c}\right]}{s_n \left|\mathbf{g}_k^H \mathbf{w}_k\right|} \right) \qquad (19)$$

where $Q(x)$ is the usual Gaussian Q-function and $\hat{\mathbf{g}}$ is replaced by $\mathbf{g}$.

B. Selection of Weight Vectors

Next, we consider how to optimize performance with respect to the weights $\mathbf{W}$ when given $\hat{\mathbf{G}}$ and $\mathbf{c}$. From (4), the channel is jointly conditionally Rician, with predictable mutual interference through the direct components $\hat{\mathbf{G}}^H\mathbf{W}$ and unpredicable interference through the scattered components $\mathbf{E}^H\mathbf{W}$. We select the weights $\mathbf{W}$ to balance interference mitigation against the need for strong individual channels.

It is easily seen that the weight vectors $\mathbf{w}_k$ should lie in the column space of $\hat{\mathbf{G}}$ (again recall the conjugate channel convention). From (4), any component of $\mathbf{W}$ orthogonal to $\hat{\mathbf{G}}$ has no effect on the direct component $\hat{\mathbf{G}}^H\mathbf{W}\mathbf{c}$. On the other hand, the covariance matrix of the scattered component $\mathbf{E}^H\mathbf{W}\mathbf{c}$ is proportional to the identity matrix $s_e^2|\mathbf{W}\mathbf{c}|^2\mathbf{I}_L$ so the components of $\mathbf{W}$, orthogonal or not, can affect only the scattered power, not its distribution over the users. This observation simplifies optimization if there are fewer users than antennas, since $\mathbf{W} = \hat{\mathbf{G}}\Omega$ where Ω is only MxM.

Optimization directly with (11), (17) and (18) is clearly not tractable. Instead, we use the single user maximization of K factor as an analogy, and maximize the ratio of direct power to the sum of direct interference, scattered interference and noise, when averaged over the data ensemble. Define the signal to interference and noise ratio of user k as

$$SINR_k = \frac{\left|\hat{\mathbf{g}}_k^H\mathbf{w}_k\right|^2}{\sum\limits_{\substack{m=1\\m\neq k}}^{M}\left|\hat{\mathbf{g}}_m^H\mathbf{w}_m\right|^2 + s_e^2\sum\limits_{m=1}^{M}\left|\mathbf{w}_m\right|^2 + s_n^2}\,. \tag{20}$$

The objective then is to give all users the same SINR, and maximize their common SINR with the constraint on total power (2)

$$\sum_{m=1}^{M}\left|\mathbf{w}_m\right|^2 = P\,. \tag{21}$$

Substitution of (21) into (20) produces

$$SINR_k = \frac{\left|\hat{\mathbf{g}}_k^H\mathbf{w}_k\right|^2}{\sum\limits_{\substack{m=1\\m\neq k}}^{M}\left|\hat{\mathbf{g}}_m^H\mathbf{w}_m\right|^2 + s_e^2P + s_n^2}\,. \tag{22}$$

The objective function now has the format considered in [7]-[9] for perfect CSI, and can be treated similarly.

This constrained maximization is quite difficult, but suboptimum solutions exist. The zero forcing, or pseudo-inverse, method (e.g., [7]) ignores the constant terms in the denominator of (22) and eliminates mutual interference. However, it increases the effect of noise and estimation error, resulting in compromised performance. It is analogous to adaptive diversity in the uplink, where it is well known that MMSE combining [14] trades interference against noise, giving lower error rates than zero forcing.

Here we will adopt the filter bank method devised by Yang and Xu [8]. It requires all users to have the same "footprint" of own gain and cross gains to other users, and optimizes that footprint to trade interference against noise. The method is suboptimal, but good, and is guaranteed to be better than zero forcing. In this study, we demonstrate its performance in fading and decorrelation.

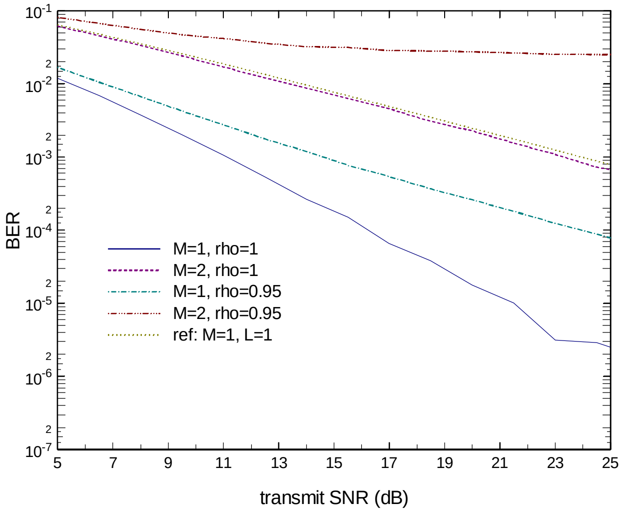

Fig. 4. Error Probability for Multiuser Adaptive Transmitter Diversity, L=2 Antennas

Fig. 4 shows the error probability for adaptive multiuser transmitter diversity for L=2 antennas and M=1 or 2 mobiles, and for correlation coefficient values $r = 1$ (perfect CSI) and $r = 0.95$. The single antenna, single user curve is included for reference. For perfect CSI, the two antennas can support two users, but only with single diversity performance. This is expected, since we have given up a degree of freedom to create the approximate nulls. For $r = 0.95$, itself a large value, performance is disappointing: for a single user, we see the loss of diversity already noted in Fig. 2, and for two users, the system is useless.

The curves were produced by Monte Carlo averaging of the conditional BERs with 100,000 trials. Comparison of some curves with their numerically obtained counterparts in Fig. 2 gives an idea of the quality of this semi-analytic simulation.

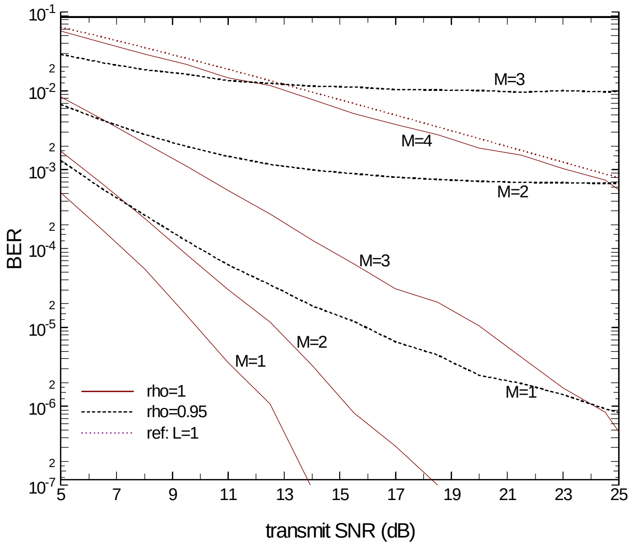

Fig. 5. Error Probability for Multiuser Adaptive Transmitter Diversity, *L*=4 Antennas

Fig. 5 shows performance for *L*=4 antenna with varying numbers of users and the same values of r. From the slope of the perfect CSI curves, it is clear that the method gives up a degree of diversity for each additional user, and cannot support more users than antennas. The phenomenon is well known from the uplink. For $r = 0.95$, the behaviour is again disappointing. The estimation error means that only *M*=2 users can reasonably be supported.

V. CONCLUSIONS

We have quantified the degree of correlation required between instantaneous downlink gains and their estimates obtained from the uplink. As the number of antennas increases, the required correlation decreases. Further, for a given correlation, error rate improves indefinitely as the number of antennas increases. In particular, with four antennas and a BER of 0.001, the correlation in single-user operation can go as low as 0.6 and still provide 7.5 dB better performance than a single antenna. For perfect correlation, the improvement is over 21 dB.

For multiuser operation, the method falls apart rapidly. Even a correlation value of $r = 0.95$ allows very few users to obtain acceptable performance, and it would be a rare system that could so high a correlation. This is a clear warning about applying adaptive transmitter diversity to a multiuser environment.

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