

# Analysis of Polarization Diversity Scheme with Channel Codes

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**Abstract** – A polarization diversity scheme [1] is discussed, to which various space time codes have been applied. Based on the two branch polarization diversity system, a theoretical model is presented for analyzing the system performance under various correlation conditions. Space block codes [2], have been applied to the polarization diversity scheme. It has been found that with space-time codes, the system performance can be improved by 9dB at bit error rate of  $10^{-4}$ .

## I. INTRODUCTION

Various diversity schemes have been proposed to enhance the performance of a communication system without increasing the base station transmitter power. Since diversity schemes can mitigate the multipath fading effects through multiple accesses, the cross-coupling correlation between branches is critical to the system performance. Independent channel characteristics among the multiple accesses is necessary to achieve effective diversity.

The requirement for a low cross-coupling correlation coefficient puts a constraint on the system configuration. Conventional space diversity, requires an antenna separation of  $10-20\lambda$  to keep the cross-coupling correlation below 0.5, for base station reception. The proposed polarization diversity scheme achieves its diversity based on the different propagation characteristics of the vertically and horizontally polarized electromagnetic waves. This scheme does not require a large antenna separation to achieve its diversity.

This paper discusses the polarization diversity scheme of a two-branch system. Space-time coding has been applied to the polarization diversity scheme to mitigate the cross-coupling effects.

A theoretical model of a two branch system is presented in order to analyze the polarization diversity scheme using space-time codes. This model allows for the evaluation of the adverse effects of cross-coupling correlation on the achievable diversity gain.

## II. THEORETICAL MODEL

The basic polarization diversity system configuration is shown in Figure 1. A Polarization diversity scheme consists of vertically and horizontally polarized antennas at both the transmitter and receiver.

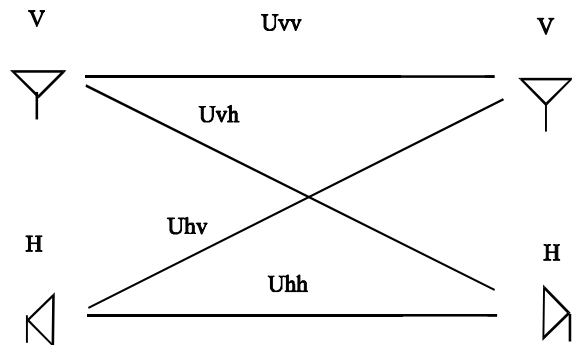


Figure 1 Transmission Links of Polarization Diversity Scheme

Assuming a Rayleigh fading environment. The complex Gaussian random variable  $u_{vv}$  represents the transmission coefficient from the vertically polarized transmitter to the vertically polarized receiver. Similarly  $u_{hh}$  represents the transmission coefficient between the two horizontally polarized antennas. Coefficients  $u_{hv}$  and  $u_{vh}$  represent the cross couplings.

We will assume that the transmitted signal,  $S(t)$ , consists of two PSK signals,  $S_v(t)$  and  $S_h(t)$  for orthogonally polarized antennas. In considering the cross coupling between channels, the received signals can be expressed as:

$$\begin{aligned}
r_v &= u_{vv}S_v + u_{hv}S_h + n_v \\
r_h &= u_{vh}S_v + u_{hh}S_h + n_h
\end{aligned}
\tag{1}$$

where  $n_v$  and  $n_h$  are the additive Gaussian white noise. Equation (1) can also be expressed in matrix form as:

$$\mathbf{r} = \mathbf{S} \cdot \mathbf{u} + \mathbf{n}
\tag{2}$$

where

$$\mathbf{r} = \begin{bmatrix} r_v \\ r_h \end{bmatrix}
\tag{3}$$

$$\mathbf{S} = \begin{bmatrix} S_v & S_h & 0 & 0 \\ 0 & 0 & S_v & S_h \end{bmatrix}
\tag{4}$$

$$\mathbf{u} = \begin{bmatrix} u_{vv} \\ u_{hv} \\ u_{vh} \\ u_{hh} \end{bmatrix}
\tag{5}$$

and

$$\mathbf{n} = \begin{bmatrix} n_v \\ n_h \end{bmatrix}
\tag{6}$$

The receiver can estimate the transmission coefficients by using a pilot tone or embedded pilot symbols.

Suppose  $\mathbf{r}$  is the received signal vector when  $\mathbf{S}_i$  is transmitted, where  $\mathbf{i}$  refers to an event. With the estimated channel gains, we have

$$\begin{aligned}
\mathbf{r} &= \mathbf{S}_i \cdot \mathbf{u} + \mathbf{n} \\
\mathbf{r}_i &= \mathbf{S}_i \cdot \bar{\mathbf{u}} \\
\mathbf{r}_j &= \mathbf{S}_j \cdot \bar{\mathbf{u}}
\end{aligned}
\tag{7}$$

where  $\mathbf{u}$  is the channel gain and  $\bar{\mathbf{u}}$  is the estimated channel gain.  $\mathbf{r}_i$  is the received signal for a transmitted signal  $\mathbf{S}_i$  and  $\mathbf{r}_j$  is the received signal for another transmitted signal  $\mathbf{S}_j$ .  $\mathbf{S}_j$  is called the competing signal of  $\mathbf{S}_i$ . The estimation error of the channel gain is represented as:

$$\mathbf{u} = \bar{\mathbf{u}} + \mathbf{e}
\tag{8}$$

where  $\mathbf{e}$  is the estimation error on the channel gains.

Therefore,

$$\mathbf{r} = \mathbf{S}_i \bar{\mathbf{u}} + \mathbf{S}_i \cdot \mathbf{e} + \mathbf{n} = \mathbf{S}_i \cdot \bar{\mathbf{u}} + \mathbf{e}
\tag{9}$$

where  $\mathbf{e}$  is a random variable:

$$\mathbf{e} = \mathbf{S}_i \cdot \mathbf{e} + \mathbf{n}
\tag{10}$$

The normalized Euclidean distance to the estimated signal is expressed as:

$$M(\mathbf{r}, \mathbf{S}_i, \bar{\mathbf{u}}) = (\mathbf{r} - \mathbf{r}_i)^+ \cdot \mathbf{R}_e^{-1} \cdot (\mathbf{r} - \mathbf{r}_i)
\tag{11}$$

## 2.1 Error Probability

Suppose that the transmitted waveform is  $\mathbf{S}_i$ . Then the probability of the receiver confusing waveform  $\mathbf{S}_i$  with the waveform  $\mathbf{S}_j$  is simply the probability that the random variable  $d(\mathbf{r}, \bar{\mathbf{u}}, \mathbf{S}_i, \mathbf{S}_j)$  is less than zero.

$$d(\mathbf{r}, \bar{\mathbf{u}}, \mathbf{S}_i, \mathbf{S}_j) = M(\mathbf{r}, \mathbf{S}_j, \bar{\mathbf{u}}) - M(\mathbf{r}, \mathbf{S}_i, \bar{\mathbf{u}})
\tag{12}$$

Recall that all the components of  $\mathbf{S}_i$  and  $\bar{\mathbf{u}}$  are zero-mean complex Gaussian variates. It is well known that the probability distribution of a random variate in the quadratic form of complex gaussian variates can be calculated through the Laplace transform [4]. For any matrix  $\mathbf{F} = \{F_{ij}\}$ , a quadratic form in complex variables  $\mathbf{t} = \{t_i\}$ , is defined by:

$$f = \mathbf{t}^+ \cdot \mathbf{F} \cdot \mathbf{t}
\tag{13}$$

and when the Gaussian variates have zero mean, the characteristic function of  $f$  is:

$$G_f(s) = \frac{1}{|I + 2s\mathbf{R}^* \mathbf{F}|}
\tag{14}$$

where  $\mathbf{R}$  is the covariance matrix of  $t$ .

The probability of ( $f < 0$ ) can be obtained by calculating residues for all distinct poles on the right side.

$$P(f < 0) = -\sum_{rp} \text{residues} \left[ \frac{G_f(s)}{s} \right] \quad (15)$$

After rewriting (12) in quadratic form, it can be shown that  $\mathbf{F}$  and  $\mathbf{R}$  matrices are as follows.

$$\mathbf{F} = \begin{bmatrix} (\mathbf{S}_i - \mathbf{S}_j)^+ \cdot \mathbf{R}_e^{-1} \cdot (\mathbf{S}_i - \mathbf{S}_j) & (\mathbf{S}_i - \mathbf{S}_j)^+ \cdot \mathbf{R}_e^{-1} \\ \mathbf{R}_e^{-1} \cdot (\mathbf{S}_i - \mathbf{S}_j) & \mathbf{0} \end{bmatrix} \quad (16)$$

and the covariance matrix of the Gaussian variate  $t$  which has been redefined as  $z$  is :

$$\mathbf{R} = \langle \mathbf{z} \cdot \mathbf{z}^+ \rangle = \mathbf{E} \begin{bmatrix} \bar{\mathbf{u}} \cdot \bar{\mathbf{u}}^+ & \bar{\mathbf{u}} \cdot \mathbf{e}^+ \\ \mathbf{e} \cdot \bar{\mathbf{u}}^+ & \mathbf{e} \cdot \mathbf{e}^+ \end{bmatrix} \quad (17)$$

Since  $\bar{\mathbf{u}}$  and  $\mathbf{e}$  are independent,  $E[\bar{\mathbf{u}} \cdot \mathbf{e}^+] = E[\mathbf{e} \cdot \bar{\mathbf{u}}^+] = 0$ . Therefore, the covariance matrix  $\mathbf{R}$  can be written as:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{\bar{\mathbf{u}}} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{\mathbf{e}} \end{bmatrix} \quad (18)$$

Note elements  $\mathbf{R}_{\bar{\mathbf{u}}}$  and  $\mathbf{R}_{\mathbf{e}}$  in (18) and  $\mathbf{S}_i$ ,  $\mathbf{S}_j$  and  $\mathbf{R}_e^{-1}$  in (16) are matrices.

Using equations (14), (15), (16) and (18), we can now calculate the error probability of a multiple channel system.

### III. SPACE BLOCK CODE

The space block code, proposed by S. M. Alamouti, is based on a simple transmitter diversity scheme. As shown in figure 2, the signal quality at the receivers can be improved by simply block encoding across the multiple antennas at the transmitter. At a given symbol period, two signals are simultaneously transmitted by the two antennas. At the first period, the signal transmitted from antenna 0 is denoted by  $s_0$ , and the signal from antenna 1 by  $s_1$ . The next symbol period signal  $(-s_1^*)$  is transmitted from antenna 0, and signal  $s_0^*$  from antenna 1, where  $*$  is the complex conjugate operation.

Space block codes can be applied to our polarization diversity scheme in the form of a

two Tx antennas and two Rx antennas system. Based on the analytical model developed in the last section, we can analyze the system performance of our polarization diversity scheme using space block coding. The key to calculating error probability with space block codes, for our system, is to accommodate the time diversity. Because it is assumed that the amplitude of fading at each instant is the same, the transmission sequence can be equivalently expressed as a system of simultaneous transmissions by modifying the system configuration.

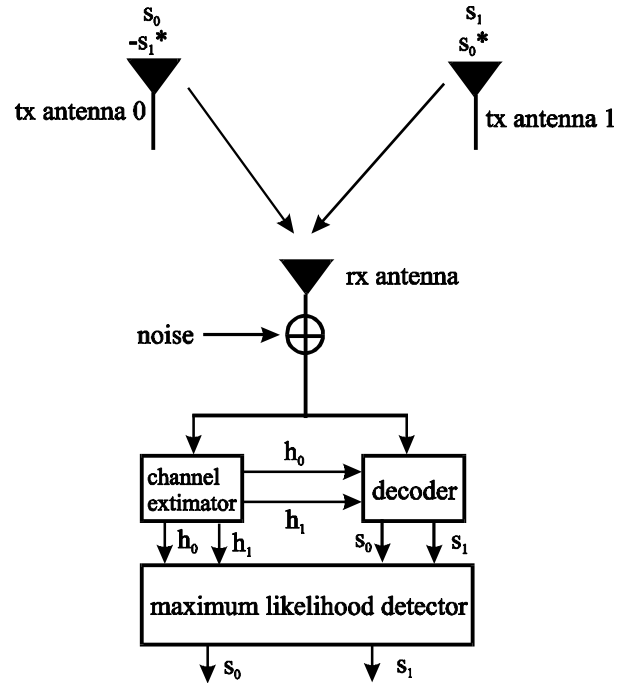


Figure 2 Two branch block oriented space time coding scheme with a single receiver.

The sequential outputs of space block codes are represented with the received signals from different antennas. The signal of the receiving antenna 0 stands for the received signal at time  $t$ , while the output of the receiving antenna 1 stands for the received signal at time  $t+1$ . The transmission signals to each receiving antenna can be separately represented in the different rows of the  $S$  matrix in (4). The encoding and transmitting sequence of the space block code can be easily implemented in the analytical model. With the assumption of slow fading, the channel gain would be same in the transmission sequence. Suppose that the

encoding and transmitting sequence in figure 2 is used, then the matrix of the input signal can be written as:

$$\mathbf{S} = \begin{bmatrix} s_0 & s_1 & 0 & 0 \\ -s_1^* & s_0^* & 0 & 0 \\ 0 & 0 & s_0 & s_1 \\ 0 & 0 & -s_1^* & s_0^* \end{bmatrix} \quad (19)$$

The output of the analytical model is the pairwise error probability. The overall bit error would be [5]:

$$P = \frac{1}{M} \sum_j m_j \cdot p_{ij} \quad (20)$$

where  $p_{ij}$  stands for the pairwise error probability between symbols  $s_i$  and  $s_j$ ,  $m_j$  stands for the hamming distance between  $s_i$  and  $s_j$ , and  $M$  is the bit number of  $s_i$  and  $s_j$ .

For block oriented space time code analysis of our polarization diversity scheme, the overall bit error rate is:

$$\begin{aligned} P &= 0.5 \times p((1,1), (1,-1)) \\ &+ 0.5 \times p((1,1), (-1,1)) \\ &+ 1 \times p((1,1), (-1,-1)) \end{aligned} \quad (21)$$

when (1,1) is selected as the transmitted symbol.

#### IV RESULTS

Cross coupling between the two polarization channels is inevitable. Additionally, the intensity of the two polarized signals could be very different because of the polarization reflection characteristics of the propagation medium. This section will first show the dependence of the diversity gain on the channel cross coupling and signal intensity ratio. Then improvement of system performance with space block codes will be presented.

Figure 3 shows the impact of the signal intensity difference on the diversity gain in

terms of the error rate. Zero channel cross coupling was assumed in this figure.

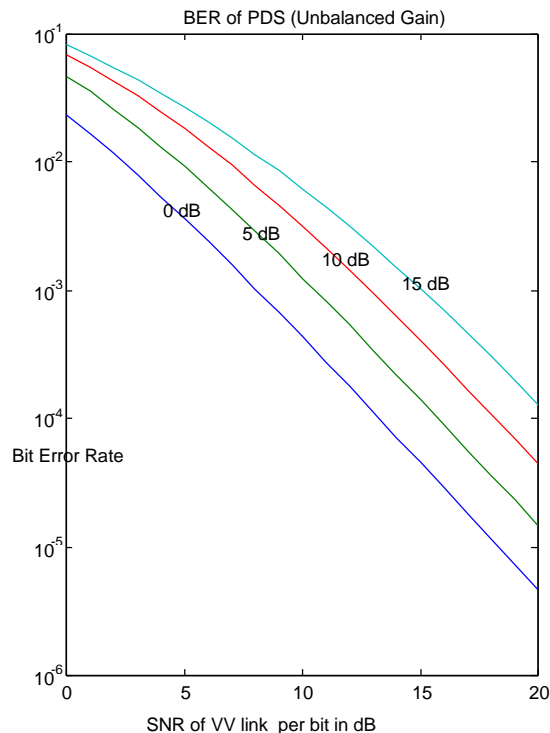


Figure 3 Error rate of a mobile system with polarization diversity. Coupling coefficients  $\langle |u_{vh}|^2 \rangle = \langle |u_{hv}|^2 \rangle = 0$  while  $\langle |u_{vv}|^2 \rangle / \langle |u_{hh}|^2 \rangle =$  (a) 0dB; (b) -5dB; (c) -10dB and (d) -15dB.

From Figure 3, it can be seen that the polarization diversity scheme performs much better with equal signal intensities than with unequal intensities. As the signal intensity difference increases to 15 dB, the system BER is very close to that of a single channel system. The above result confirms the importance of a balanced signal amplitude for achieving high diversity gain. Figure 4 shows the impact of channel cross coupling on the polarization diversity gain in terms of the error rate. It can be seen that the performance of the polarization diversity scheme degrades as the channel cross coupling increases. When the channel cross coupling is as strong as the channel link, the error rate of the system is as bad as a single channel system.

To compare, we calculate the bit error rate for binary modulation. The results are shown in

Figure 5. We have assumed perfect channel state information. The results are consistent with the published simulation results[2]. Also shown in Figure 5, using space coding on a 2Tx and 2Rx antenna system assuming identical transmission coefficients, the coding gain for a BER at  $10^{-4}$  is about 9 dB.

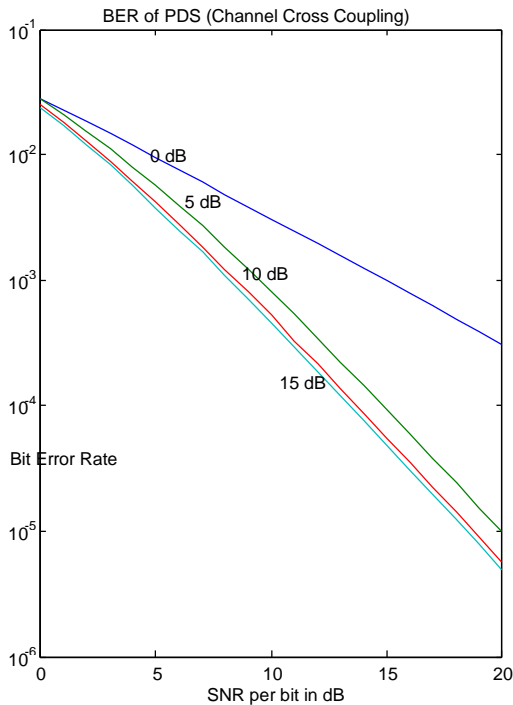


Figure 4 Error rate of a mobile system with polarization diversity. Coupling coefficients  $\langle |u_{vv}|^2 \rangle = \langle |u_{hh}|^2 \rangle = 1$  and  $\langle |u_{vh}|^2 \rangle = \langle |u_{hv}|^2 \rangle$  while  $\langle |u_{vh}|^2 \rangle / \langle |u_{vv}|^2 \rangle =$  (a) 0dB; (b) -5dB; (c) -10dB and (d) -15dB.

We have also investigated the impact of cross coupling on the block oriented space time codes. Figure 6 presents the BER of the polarization diversity scheme with block oriented space time codes for various channel cross coupling coefficients. The result shows that performance of the polarization diversity scheme has no significant degradation when the cross coupling coefficient is less than -6 dB.

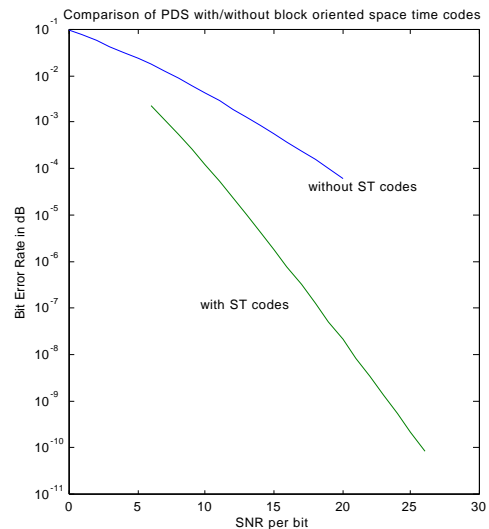


Figure 5 BER Comparison of Polarization diversity scheme (a)space block coding (b)without coding

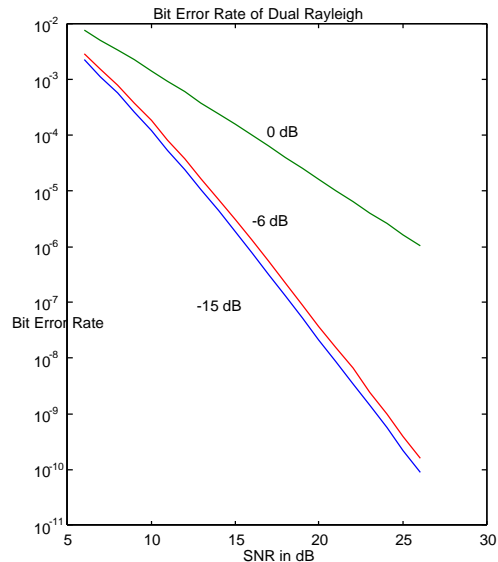


Figure 6 Bit error rate of a mobile system with polarization diversity. Coupling coefficients for  $\langle |u_{vv}|^2 \rangle = \langle |u_{hh}|^2 \rangle = 1$  and  $\langle |u_{vh}|^2 \rangle = \langle |u_{hv}|^2 \rangle$  while  $\langle |u_{vh}|^2 \rangle / \langle |u_{vv}|^2 \rangle$  (a) 0 dB, (b) -6 dB and (c) -15 dB.

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