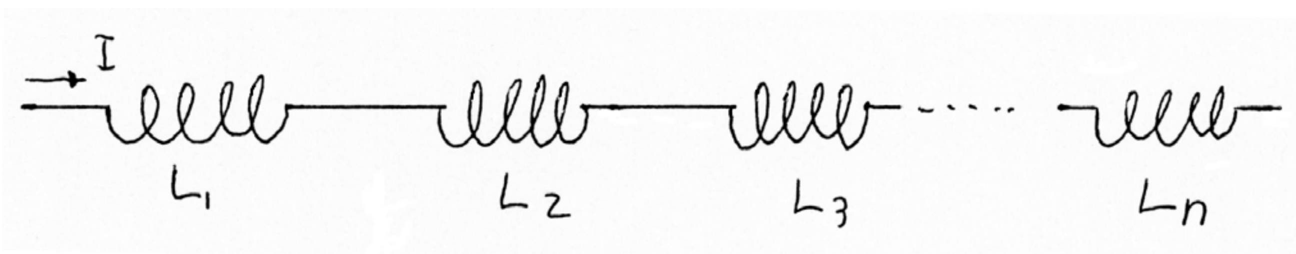
Inductors in Series

- Inductors in series act just like one large inductor
- Inductors in series add to the total inductance
- "Inductors in series act like resistors in Series"

$$L_{total} = \sum_{j=1}^{n} L_j$$

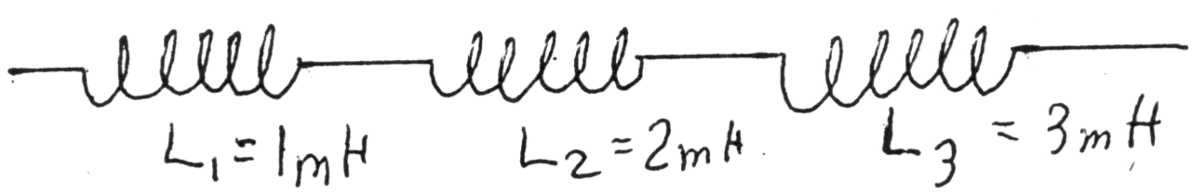
- Warning: this assumes the magnetic fields do not interact
- Thus no mutual inductance



- Example: 1, 2, and 3 mH inductors in series. What is L_{total}

- Thus total is

$$L_{total} = \sum_{j=1}^{n} L_j = L_1 + L_2 + L_3 = 10^{-3} + 2\times10^{-3} + 3\times10^{-3} = 6\ mH$$



Inductors in Parallel

- "Inductors in Parallel act like resistors in Parallel"
- Inverse of the total equals the sum of the inverses

$$\frac{1}{L_{total}} = \sum_{j=1}^{n} \frac{1}{L_j}$$

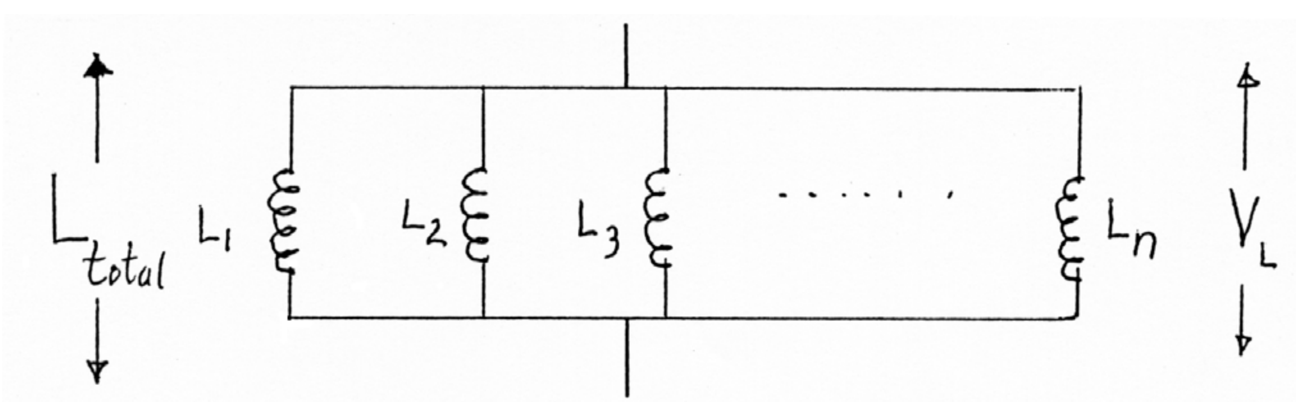
- Again important the magnetic fields do not interact
- Why do Inductors act different than C's?
- Consider parallel L's
- Voltage across each inductor is the same.
- Recall the Current is given by:

$$I = \frac{1}{L} \int V(t)dt$$

- Thus the total current through the inductors is:

$$I_{total} = \sum_{j=1}^{n} I_j = \sum_{j=1}^{n} \left[\frac{1}{L_j} \int V(t)dt \right] = \left[\sum_{j=1}^{n} \frac{1}{L_j} \right] \int V(t)dt$$

- Where t is the time of measurement



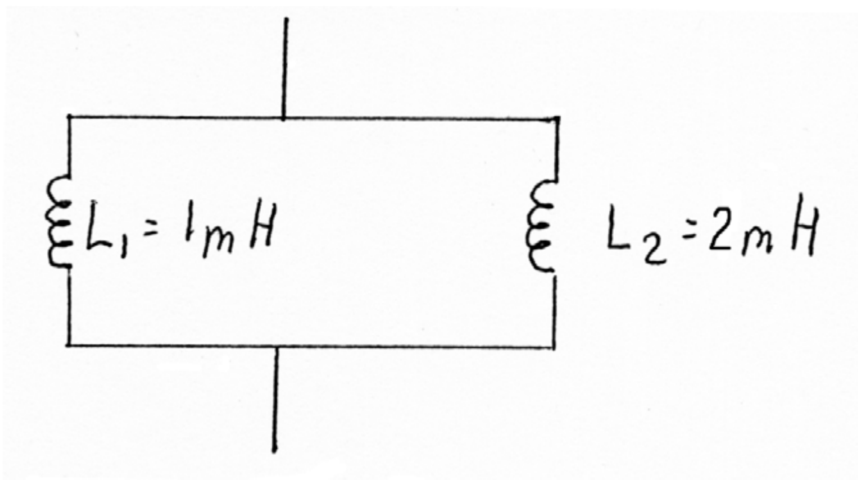
Example Parallel Inductors

Example: two inductors in parallel: L_1 = 1 mH, L_2 = 2 mH

Thus total is

$$\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_2} = \frac{1}{10^{-3}} + \frac{1}{2 \times 10^{-3}} = \frac{3}{2 \times 10^{-3}}\ H$$

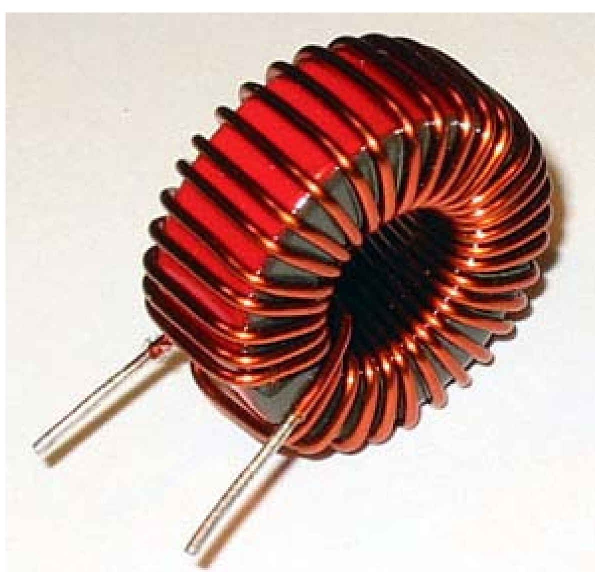
$$L_{total} = \frac{2 \times 10^{-3}}{3} = 0.667\ mH$$



Practical Inductors

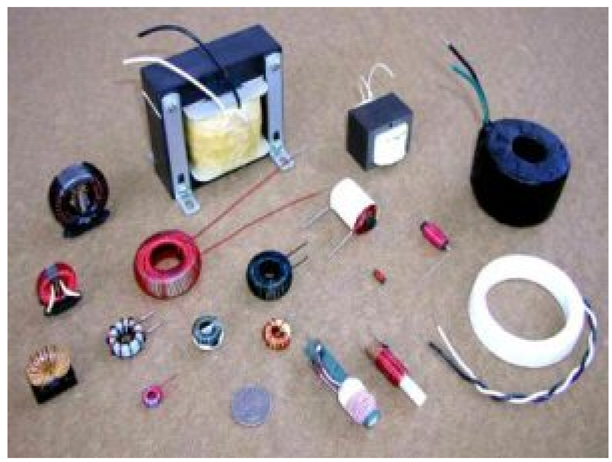
- Real inductors are often air coils: just coiled wire
- Advantage: high speed, very easy to make, calculate and tune
- Disadvantage: lower inductance, high resistance (from wire)
- Needs lots of turns of wire
- Typical values μH to mH

- Often bend coil into a torus (donut core with wire wound around)
- Reason magnetic field confined to the torus core
- Thus ideally no mutual inductance
- Also field is stronger in core, and thus L larger & smaller size



Ferrite Core Inductors

- Ferrite (iron) core inductors are most common
- Advantages:
- Much smaller for given inductance
- Can get very high inductance (henries!) with big ones
- Magnetic field mostly stays within the Ferrite area
- Disadvantage:
- Much harder to calculate (depends on iron type)
- Hence must be measured
- Also iron gets hot (heated by induction)



Natural Response of Resistor Capacitor (EC 7))

- **Natural Response:** behaviour of a circuit to a sudden change
- Consider a Capacitor in series with a resistor R
- At time t=0 a switch connects this to a voltage V_0
- Recall the capacitor current behaviour

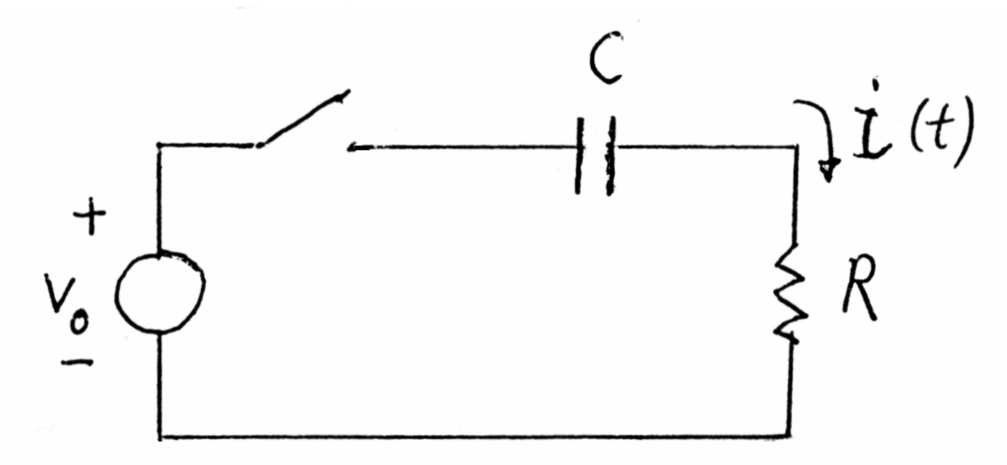
$$i(t) = C\frac{dv}{dt}$$

- Or rewriting this

$$v(t) = \frac{1}{C}\int_0^t i(t)dt$$

- Then writing the KVL for the circuit

$$V_0 - \frac{1}{C}\int_0^t i(t)dt - i(t)R = 0$$



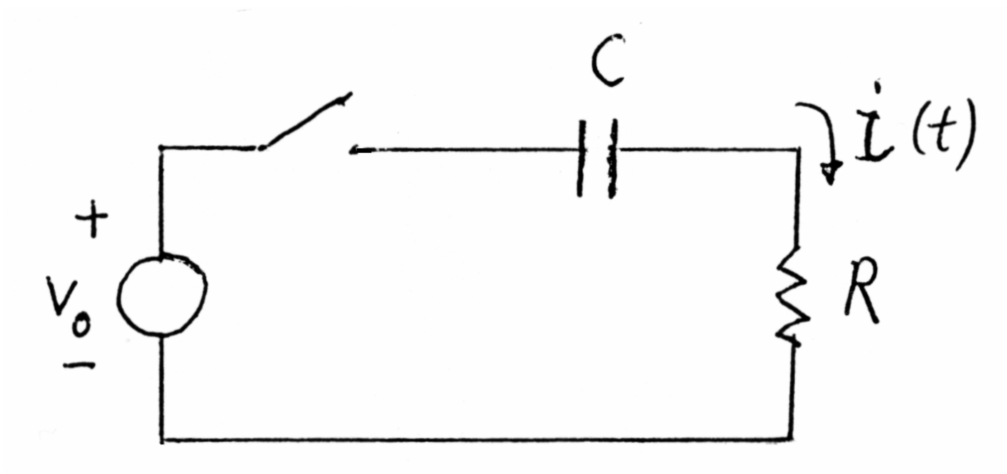
Natural Resistor Capacitor Response: Differential Equations

- Differentiating with respect to time

$$R\frac{di(t)}{dt}+\frac{1}{C}i(t)=0$$

- This is the "Differential Equation" form of the KVL equation
- Using the integration solution method for DE's
- To solve get into integral form

$$\int\frac{1}{i(t)}di=-\frac{1}{RC}\int dt$$



Natural Resistor Capacitor Response: Solutions

$$\int \frac{1}{i(t)} di = -\frac{1}{RC} \int dt$$

- Integrating both sides to solve

$$ln(i(t)) = -\frac{t}{RC} + A$$

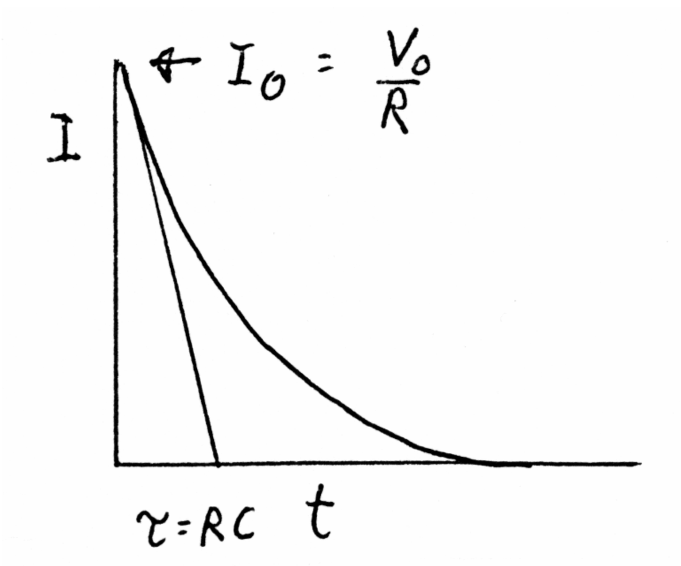
- Where A is a constant of integration
- Taking the exponential of each side
- Then setting at time t=0 call the current I_0 then

$$i(t) = I_0 \, exp\left(-\frac{t}{RC} \right)$$

- The time constant is

$$\tau = RC$$

- This is an exponential decay of current
- Thus at as time goes to infinity Capacitors act as opens
- Current goes to zero



Natural Resistor Capacitor Response: Initial Conditions

- With the voltage suddenly applied
- Eventually C becomes charge to voltage source level
- At time zero Capacitors act as short circuits
- Thus for t=0 can ignore capacitor, so

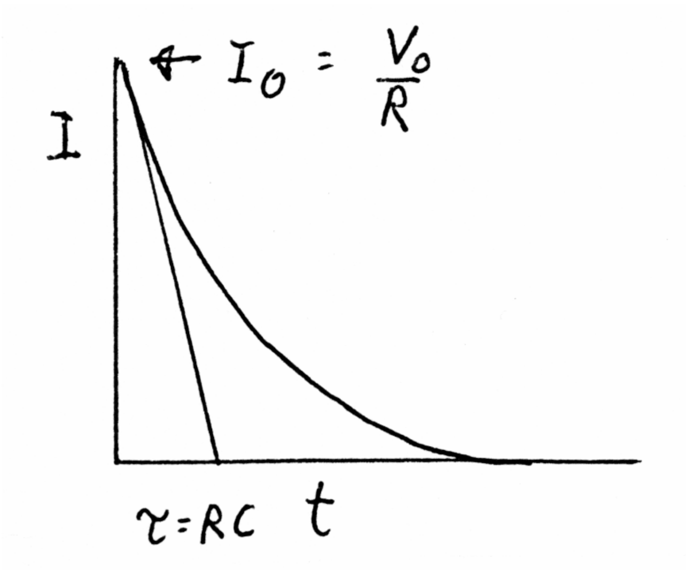
$$I_0 = i(t=0) = \frac{V_0}{R}$$

- Thus the current equation is

$$i(t) = \frac{V_0}{R} exp\left(-\frac{t}{RC}\right)$$

- Initial slope of line of line is

$$\frac{di(t=0)}{dt} = -\frac{I_0}{\tau} = \frac{V_0}{R^2 C}$$



Natural Resistor Capacitor Response: Voltage

- Final voltage on Capacitor matches the V source

$$V_C(t \to \infty) = V_0$$

- Voltage across the capacitor from the KVL

$$v_C(t) = V_0 - i(t)R = V_0 - \frac{V_0}{R} exp\left(-\frac{t}{RC}\right)R = V_0\left(1 - exp\left(-\frac{t}{RC}\right)\right)$$

- Thus voltage rises to equal the voltage source expodentially
- Same equations for C charge to initial voltage plus R with no V
- Only difference is final voltage on Capacitor now zero
- Both called "RC circuits"