General Exponential

- Many circuits respond to step change with an exponential.
- General form of an exponential decay

$$a(t) = A_0 \exp\left(-\frac{t}{\tau}\right)$$

Where

a(t) = signal at time t
A_0 = max signal (t=0 value)
exp = natural exponential, e = 2.718
t = time in seconds
τ = "time constant" in seconds

- Time constant: time to decay to 1/e = 0.368 of max
- System response measured in terms of time constant
- τ = projection of initial slope on t axis

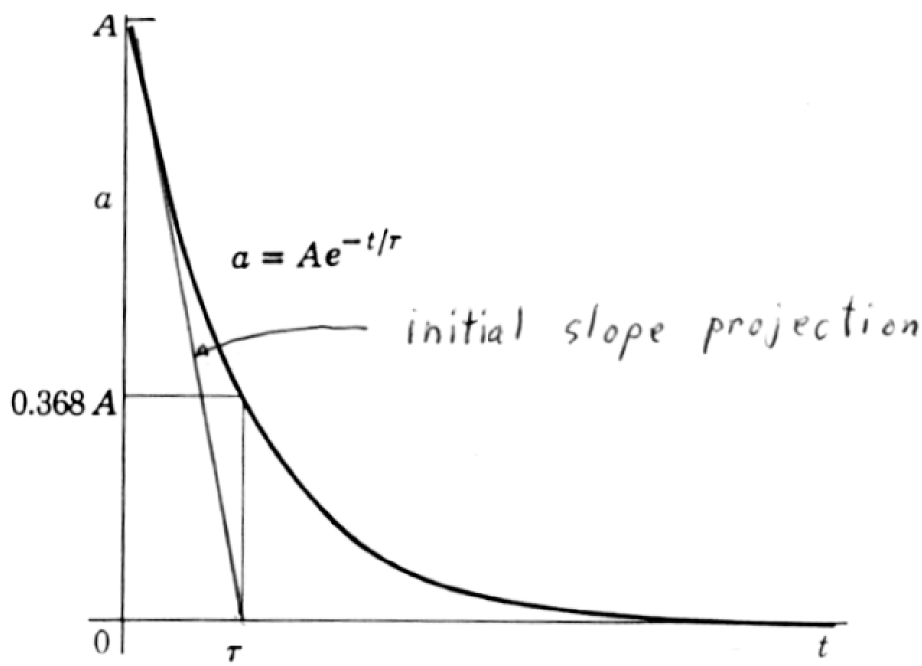


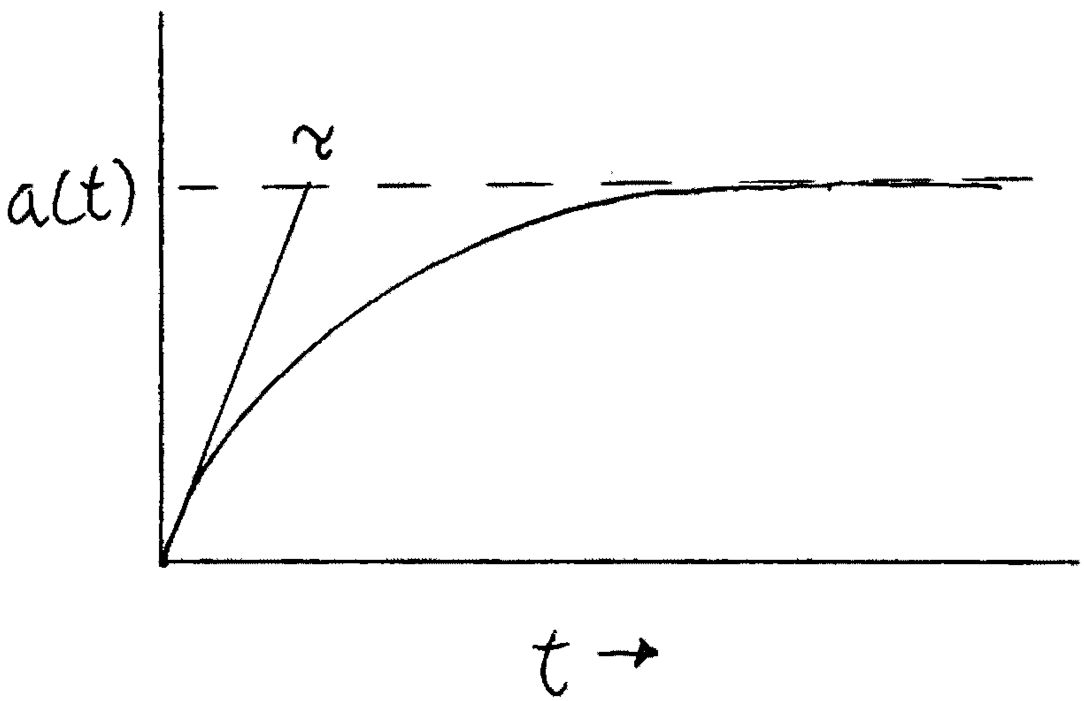
Figure 3.3 The decaying exponential.

Exponential Increase

- Cannot have a true exponential increase: goes forever
- Response to a sudden increase often an "exponential change"

$$a(t) = A_0\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$$

- This is an exponential "decay" to new steady state value
- Very common behaviour in electronic circuits



Exponential Importance

- Exponentials importance: derivative & integral are exponentials

$$a(t) = A_0 \exp\left(-\frac{t}{\tau}\right)$$

$$\frac{da(t)}{dt} = -\frac{A_0}{\tau} \exp\left(-\frac{t}{\tau}\right)$$

$$\int a(t)\, dt = -\tau\, A_0 \exp\left(-\frac{t}{\tau}\right)$$

- For circuits with inductors and capacitors:
  voltage/current related by derivative/integrals
- Thus exponential voltages will generate exponential currents

Example Exponential Voltages and Currents

- Eg. RC circuit with capacitor initially charged
- Charged capacitor will decay exp. through the resistor
- C= 2 μF initially charged to $V_c(t=0) = 10$ V,
- Current discharges through R = 1000 Ω which removes energy
- Thus max current at t=0 is

$$I_0 = i(t=0) = \frac{V_0}{R} = \frac{10}{1000} = 0.01\,A = 10\,mA$$

- Time constant is

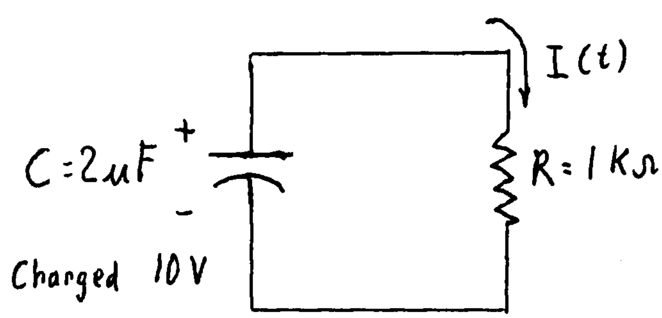
$$\tau = RC = 1000(2 \times 10^{-6}) = 0.002 \text{ sec} = 2\ ms$$

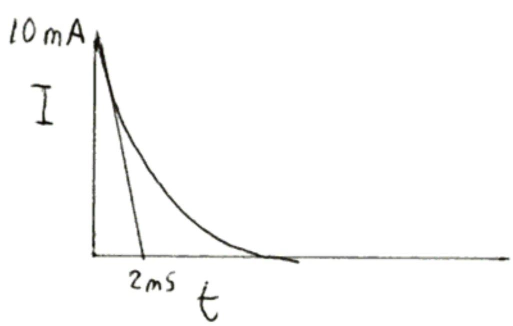
- Thus the current i(t) at time t is

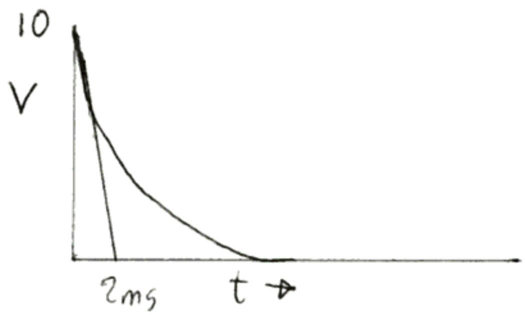
$$i(t) = \frac{V_0}{R}\exp\left(-\frac{t}{RC}\right) = 0.01\exp\left(-\frac{t}{0.002}\right) Amp$$

- By KVL the Voltage across C = V across R

$$i(t)R = 0.01(1000)\exp\left(-\frac{t}{0.002}\right) = 10\exp\left(-\frac{t}{0.002}\right) V$$

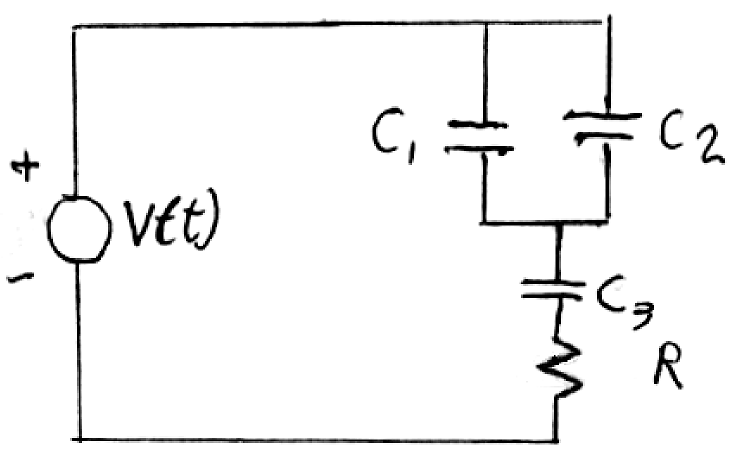


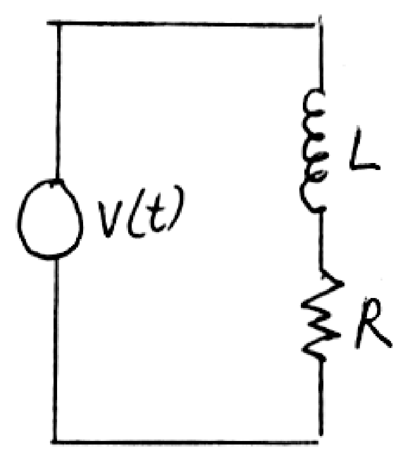




Natural Response and First Order Circuits

- Looking at response to a sudden change.
- The applied V or I is called the Forcing Function
- Basic Type: first order circuits
- Has only one energy storage device: C or L
- The RC circuit called a First order circuit
- Can have capacitors or inductors if combinable to a single
- Use simple C or L parallel/series combinations
- However for 1st order cannot mix C and L
- 1st order: only 1st derivative in equations
- Thus creates first order differential equations
- 1st order general behaviour exponential in form for sudden change





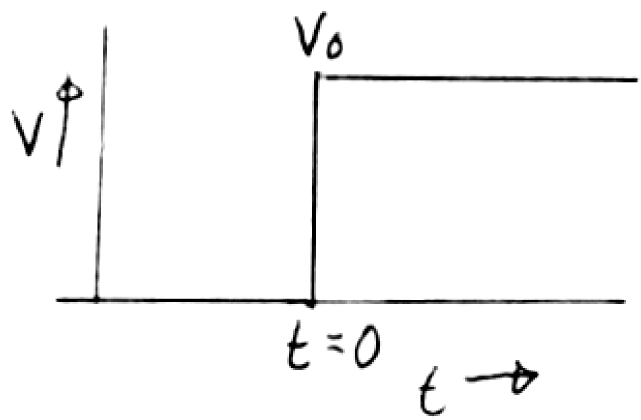
Initial and Final Response to a "Step Function"

- Often want to know circuit responce to a sudden V or I change
- Called a **Step Function**:
- Sudden instantaneous change in voltage or current
- Has two regions:

$$V(t) = 0 \qquad t < 0$$

$$V(t) = V_0 \qquad t > 0$$

- Often call time just prior to step 0-
- Time just after step 0+
- Step function in current similar
- In practice an ideal step is impossible to create:
- Real systems do not have an instantaneous change

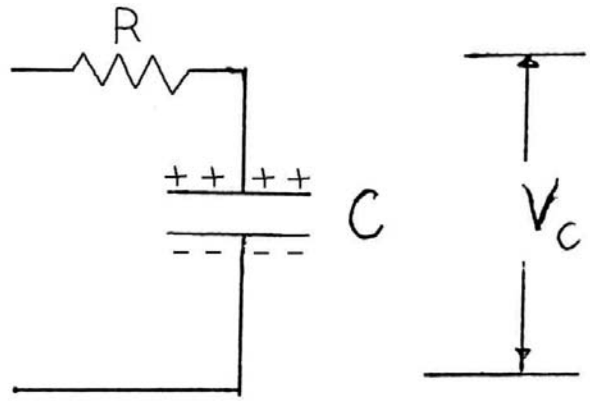


C with Initial Energy Storage

- Capacitors or Inductors may start with stored energy
- Capacitors with a stored charge has an initial voltage across it from

$$V = \frac{Q}{C}$$

- Charged capacitors can exist in an open circuit



L with Initial Energy Storage

- Inductors can only have stored energy with flowing current
- Thus a voltage/current source present before any change in circuit
- Then use a switch to remove (short out) initial source
- But still keep L as part of a circuit

- Sometimes have both applied step function & initial stored energy
- Then voltage on C or current in L when new source applied