Time Average

- For periodic waves time average important
- What is the function averaged over its period

$$f_{av} = \frac{1}{T}\int_{t}^{t+T} f(t)dt$$

- For current

$$I_{av} = \frac{1}{T}\int_{t}^{t+T} i(t)dt$$

- Note that the total charge transferred is

$$Q = I_{av}T$$

- Often need the time average voltage

$$V_{av} = \frac{1}{T}\int_{t}^{t+T} v(t)dt$$

Sine wave time average

- Note: for sin/cos waves the time average is zero

$$f_{av} = \frac{1}{T}\int_{t}^{t+T} F_0 \cos(\omega t + \alpha)dt = 0$$

- However has value for a half cycle

$$f_{av} = \frac{2}{T}\int_{t-T/4}^{t+T/4} F_0 \cos\left[\frac{2\pi t}{T}\right]dt$$

$$= \left[F_0 \frac{2}{T}\left[\frac{T}{2\pi}\right]\sin\left[\left[\frac{2\pi t}{T}\right]\right]\right]_{-T/4}^{+T/4}$$

$$= \frac{F_0}{\pi}\left[\sin\left[\frac{\pi}{2}\right] - \sin\left[-\frac{\pi}{2}\right]\right]$$

$$= F_0\frac{2}{\pi} = 0.637F_0$$

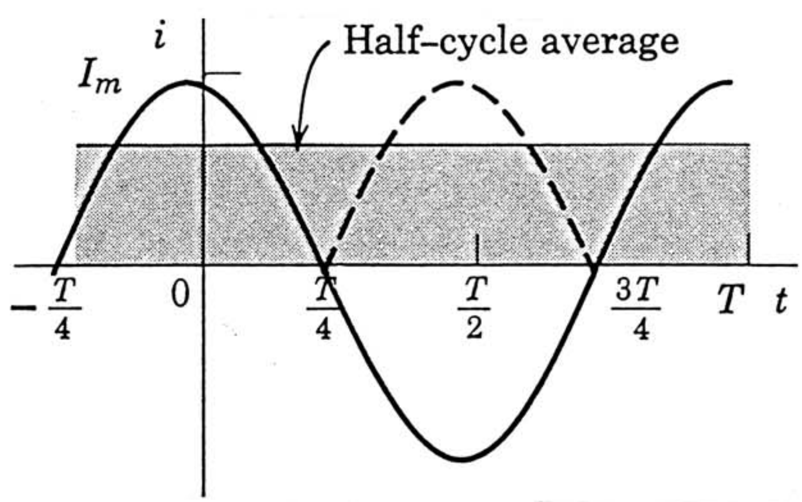


Figure 3.10 The half-cycle average.

Effective Power in AC signals

- While a sin wave has zero average over cycle
- the power dissipated does not.

$$P = I^2 R$$

if the current a sin wave

$$I(t) = I_0 \cos(\omega t)$$

$$P_{av} = \frac{1}{T}\int_0^T I(t)^2 R dt = \frac{1}{T}\int_0^T [I_0 \cos(\omega t)]^2 R dt$$

$$= \frac{I_0^2 R}{T}\left[\frac{t}{2} + \sin(2\omega \frac{t)}{4\omega}\right]_0^T$$

Since

$$\left[\sin(2\omega \frac{t)}{4\omega}\right]_0^T = \frac{[\sin(4\pi) - \sin(0)]T}{4\pi} = 0$$

Thus

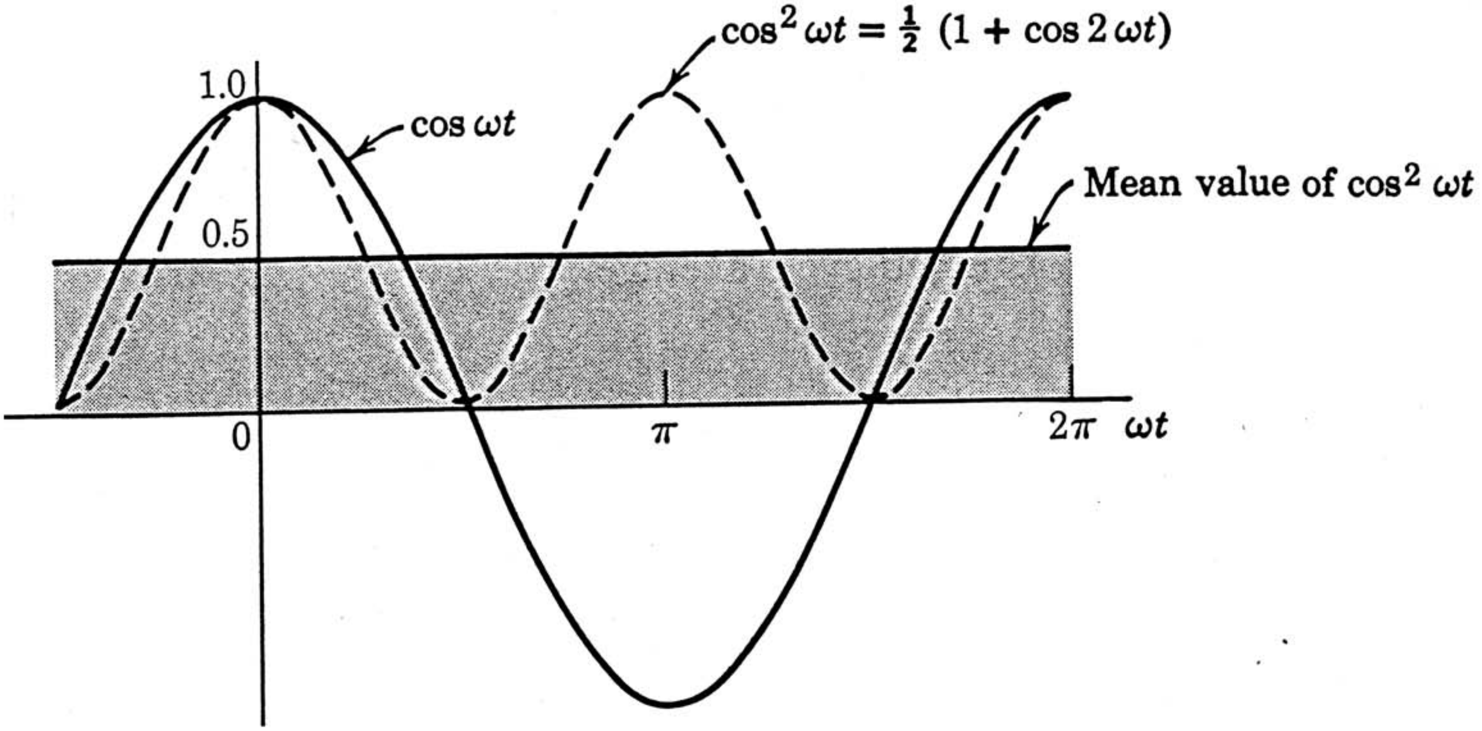
$$P_{av} = \frac{I_0^2 R}{T}\,\frac{T}{2} = \frac{I_0^2 R}{2}$$

Effective Current and voltage in AC signals

- For AC signals want the effective I or V
- used for calculating power lost/gained
- Power is dependent on I^2 or V^2
- Thus want "square root of the time average squared"

$$I_{eff} = \left[\frac{1}{T}\int_0^T I(t)^2 dt\right]^{1/2}$$

- called "root mean square" or rms
- RMS value is related to max value by a simple relations
- relationship changes for each type of signal



Calculating the root-mean-square value of a sinusoid.

RMS Current and voltage in AC signals

- Since for sin waves:

$$I_{mean} = \frac{1}{T}\int_0^T [I_{max}\cos(\omega t)]^2 dt = \frac{I_{max}^2}{2}$$

$$I_{rms} = \left[\frac{I_{max}^2}{2}\right]^{1/2} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

- The same relationship holds for voltage

$$V_{rms} = 0.707\, V_{max}$$

- Thus the average power is obtained using rms values

$$P_{av} = I_{rms}^2 R = \frac{V_{rms}^2}{R} = I_{rms} V_{rms}$$

- warning: the IV relationship sometimes is wrong:
- depends on the phase relationship (discussed later)

- Standard line AC voltage is given as 120 V_{rms} thus

$$V_{max} = \frac{120}{0.707} = 170\ V$$

RMS non Sine waves

- The relationship changes with the wave shape
- For a sawtooth wave the factor is 0.693

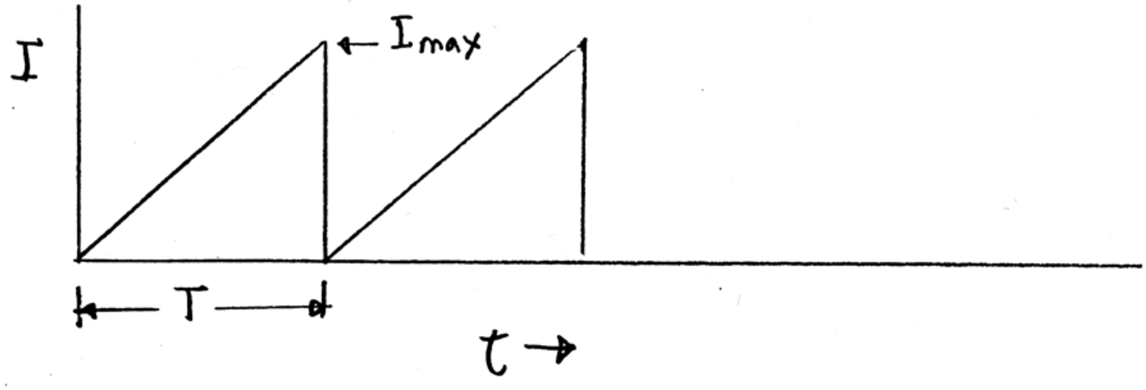
$$I(t) = \frac{I_{\max}}{T} t$$

Thus

$$I_{rms} = \left[\frac{1}{T} \int_0^T \left(\frac{I_{\max}}{T} t \right)^2 dt \right]^{1/2}$$

$$= \left[\left(\frac{I_{\max}^2 t^3}{3T^3} \right)_0^T \right]^{1/2} = \frac{I_{\max}}{\sqrt{3}}$$

- other waves different
- Older type meters used moving coils
- that measured RMS for a sin wave only
- Newer meters do the rms electronically



Impedance, Phasors, and Resistors

- Impedance: the resistance to sine AC current flow
- Define impedance (Z) the same way as resistance:

$$Z = \frac{V}{I}$$

- However now use phasors and complex numbers
- Consider a Resistor attached to an AC source

$$V_1 = V_0 \exp(j\omega t) = V_0[\cos(\omega t) + j \sin(\omega t)$$

Thus

$$V_R = V_1 = V_0 \exp(j\omega t)$$

$$V_R = V_0 \underline{/0}$$

- The current

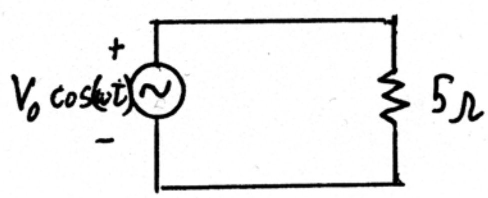
$$\vec{I}_R = \frac{V_R}{R} = \frac{V_0}{R} \exp(j\omega t)$$

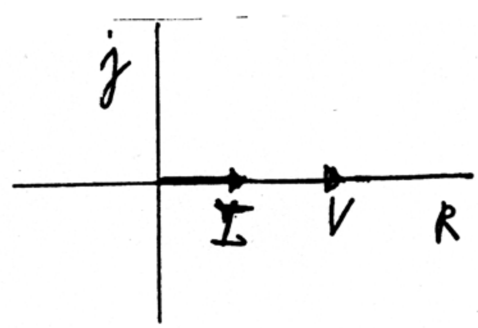
$$\vec{I}_R = \frac{V_0}{R} \underline{/0}$$

- Thus the Impedance of a resistor is only its R

$$Z_R = \frac{V}{I} = R$$

- Thus the impedance is real here.





Voltage and Current from Impedance

- The Impedance acts just like the resistance

$$V = IZ \qquad Power = I^2 Z = \frac{V^2}{Z}$$

- However, the multiplication/divisions are complex
- Eg. If a pure 10 V sin wave is applied to a R=5 ohms
- A sin is a cos wave at a -90 degree phase angle

$$V_1 = 10 \exp(j[\omega t - \frac{\pi}{2}]) = 10 \underline{/-90^o}$$

- And the current is

$$I_R = \frac{10}{5} \exp(-j[\omega t - \frac{\pi}{2}]) = 2 \underline{/-90^o}$$

- i.e. a pure real impedance does not change the phasor
- using phasors called working in the "Frequency Domain"

Impedance and Inductors

- Recall for an inductor

$$V_L = L\frac{di}{dt}$$

- Thus for the phasor applied sinewave

$$I_1 = V_0 \exp(j\omega t)$$

The resulting voltage across the inductor is

$$V_L = L\frac{di}{dt} = V_0\, j\omega L \exp(j\omega t)$$

- Since

$$j = \exp(j\frac{\pi}{2}) = 1 \underline{/90^o}$$

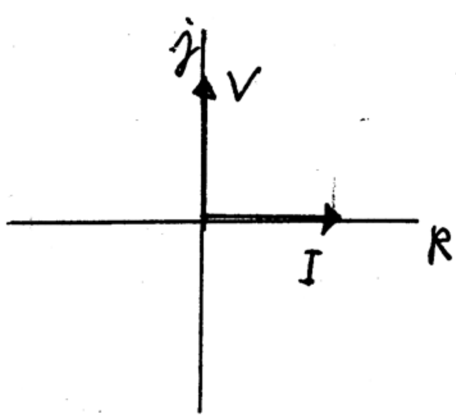
- voltage across an inductor leads the current by 90^o
- Get the same effect by defining the complex impedance

$$Z_L = \frac{V_L}{I_L} = j\omega L \;\; ohms$$

- A purely imaginary impedance is called an Reactance X

$$X_L = Z_L = j\omega L = \omega L \underline{/90^o}$$

- the unites of impedance are still ohms



Complex Impedance and V, I calculations

- if we want to find the Voltage using impedance

$$V = IZ_L = jI\omega L$$

- thus the voltage is a purely imaginary phasor
- Another way using the vector form

- Recall the Multiplication of complex numbers in Polar

$$R_1\exp(-j\theta_1)R_2\exp(-j\theta_2) = R_1R_2\exp(-j[\theta_1 + \theta_2])$$

- Thus just multiply the magnitudes and add the angles

$$\vec{C}_3 = \vec{C}_1 \, x \, \vec{C}_2 = C_1 \underline{/\theta_1} \, C_2 \underline{/\theta_2}$$

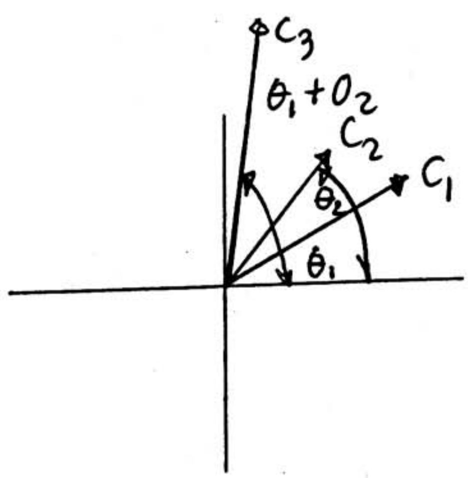
$$C_3 = C_1C_2$$

$$\theta_3 = \theta_1 + \theta_2$$

$$\vec{C}_3 = C_1C_2 \underline{/\theta_1 + \theta_2}$$

- Thus for the inductance

$$V = I \underline{/0} \, \omega L \, \underline{/90^o} = \omega LI \, \underline{/90^o}$$



Example Impedance of an Inductor

- Example: what is the impedance of a L = 5 mH
- and Voltage from a current source of
- (a) 10 mA, 60 Hz and (b) 10 mA, 1000 Hz
- The inductive reactance is

$$Z_L = j\omega L$$

- (a) thus at 60 Hz

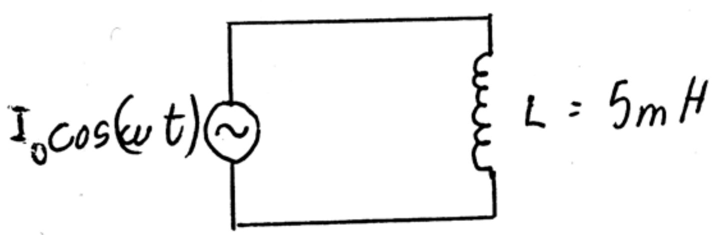
$$\omega = 2\pi f = 377$$

$$Z_L = j377x0.005 = j1885 = 1.89 \underline{/90^o} \ ohms$$

- For the voltage is:

$$I = 10 \underline{/0} \ mA$$

$$V_L = IZ_L = 0.01 \underline{/0} \ 1.89 \underline{/90^o} = 18.8 \underline{/90^o} \ mV$$



Example Impedance of an Inductor con'd

- (b) at 1000 Hz

$$Z_L = j2\pi fL = j6283x0.005 = 31.4\ ohms$$

- thus the voltage is

$$V_L = IZ_L = 0.01\ \underline{/0}\ 31.4\ \underline{/90^o} = 31.4\ \underline{/90^o}\ V$$

- Note the large increase in the reactance and reactive voltage
- Also note the large voltage
- thus as frequency increase

takes much larger power supply to drive L to same I

Power and Inductors (EC 10)

- Power is given by

$$P = I^2 Z = jI^2 \omega L$$

- Thus power is purely imaginary
- Only real power is important
- Thus inductors take larger power supplies to drive
- But consume no real power

Impedance and Capacitors

- Recall for an capacitor

$$V_C = \frac{1}{C}\int idt$$

- Thus for the phasor applied sinewave

$$I_1 = I_0 \exp(j\omega t)$$

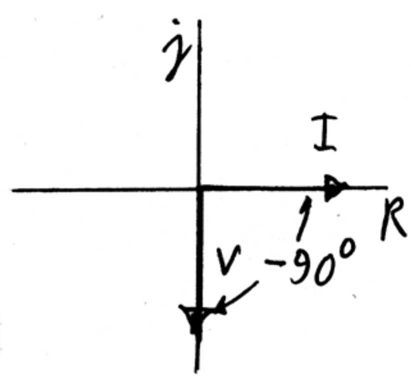
The resulting voltage across the capacitor is

$$V_C = \frac{1}{C}\int idt = I_0 \frac{1}{j\omega C}\exp(j\omega t)$$

- Since

$$\frac{1}{j} = -j = \exp(-j\frac{\pi}{2}) = 1 \angle\underline{-90^o}$$

- voltage across an capacitor lags the current by 90^{o}



Impedance and Capacitors Con'd

- Get the same effect by defining the complex impedance

$$Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C} \; ohms$$

- Thus the reactance is

$$X_C = Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C} \angle -90^o$$

- thus the capacitive reactance declines with frequency
- Capacitors act as high frequency shorts!
- Z becomes infinite at DC, thus blocks DC

Example Impedance of an Capacitor

- Example: what is the impedance of a C = 2 μF
- and Voltage from a current source of
- (a) 10 mA, 60 Hz and (b) 10 mA, 1000 Hz
- The capacitive reactance is

$$Z_C = \frac{1}{j\omega C}$$

- (a) thus at 60 Hz

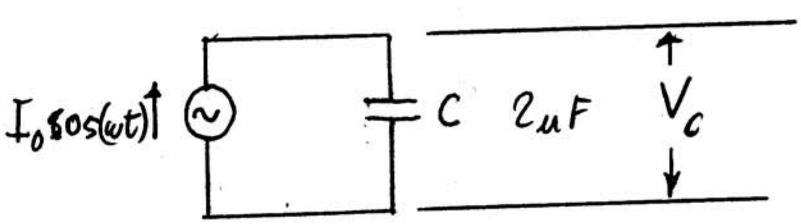
$$\omega = 2\pi f = 377$$

$$Z_C = \frac{1}{j377x2x10^{-6}} = -j1326 = 1326 \underline{/-90^o} \ ohms$$

- For the voltage is:

$$I = 10 \underline{/0} \ mA$$

$$V_C = IZ_C = 0.01 \underline{/0} \ 1326 \underline{/-90^o} = 13.26 \underline{/-90^o} \ V$$



Example Impedance of an Capacitor con'd

- (b) at 1000 Hz

$$Z_C = \frac{1}{j6283 \ x \ 2x10^{-6}} = -j79.6 = 79.6 \ \underline{/-90^o} \ ohms$$

- thus the capacitive reactance declines with frequency

- the voltage is

$$V_C = IZ_C = 0.01 \ \underline{/0} \ 79.6 \ \underline{/-90^o} = 0.79 \ \underline{/-90^o} \ V$$

- Note: large decrease in reactance and reactive voltage
- Also note the large voltage
- thus as frequency increase

takes much lower power supply to drive C to same I