How Impedances Combine

- Impedances Combine just like resistors
- However, must use complex numbers and math
- Impedances in series add to the total impedance

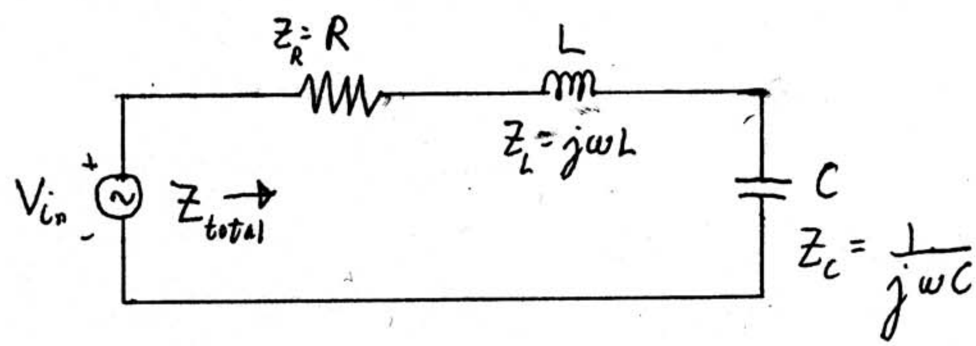
$$Z_{total} = \sum_{j=1}^{n} Z_j$$

- All these additions are using complex numbers

- Thus in the RLC example

$$Z_{total} = Z_R + Z_L + Z_C$$

$$Z_{RLC} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$



Impedances in Parallel

- Impedances in parallel:
- inverse of the total equals the sum of the inverses

$$\frac{1}{Z_{total}} = \sum_{j=0}^{n} \frac{1}{Z_j}$$

- note how this generates the laws for Capacitors
- thus for capacitors in parallel:

$$\left[\frac{1}{j\omega C_{total}}\right]^{-1} = \sum_{k=1}^{n} \left[\frac{1}{j\omega C_k}\right]^{-1}$$

Thus

$$C_{total} = \sum_{j=1}^{n} C_j$$

- Unites are mhos or Siemans (S)
- The complex conductance has real and imaginary parts

$$Y = \frac{1}{Z} = G + jB = Y\angle\theta_y$$

Where

G = conductance

B = susceptance

- Thus For circuit in parallel just add admittances

$$Y_{parallel} = \sum_{j=1}^{n} Y_j$$

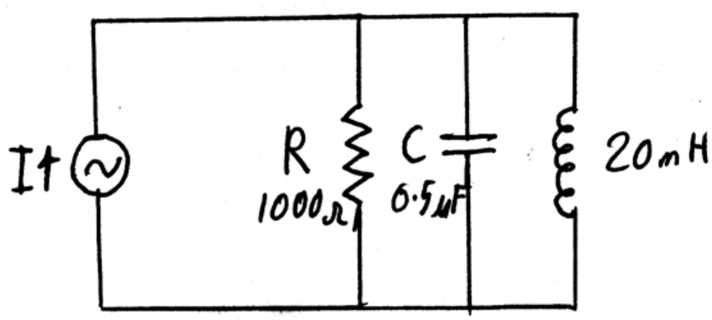
Parallel RLC circuit

- Consider a R, L and C in parallel
- Then their admittance is:

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C = \frac{1}{R} + j\left[\omega C - \frac{1}{\omega L}\right]$$

- If driven by an AC current source
- Then acts as a filter of the voltage

$$I = VY$$



Example Parallel RLC circuit

- eg. R=1000 ohms, L=20 mH and C=0.5 μF in parallel
- Then the admittance is:

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C = \frac{1}{R} + j\left[\omega C - \frac{1}{\omega L}\right]$$

- The frequency of natural response is

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.02x\,5x\,10^{-7}}} = 10^4\ rad/s$$

- At the natural frequency admittance is minimum

KVL, KCL and Complex impedances

- Kirchoff's laws, KVL and KCL work in complex Z
- Voltage dividers, current dividers, Thevenin/Norton
  Mesh/Node analysis all work as with resistances
- just use the complex impedances Z did resistances
- However all solutions must use complex math
- Apply a sin wave input use phasor form
- will get a phase shift in the V and I

RL and RC reactance and phase angle

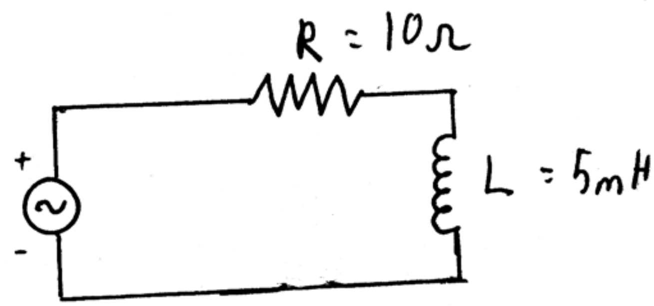
- Adding a resistor to L or C changes the impedance
- Eg. For an RL system

$$Z = R + j\omega L$$

$$\theta = arctan\left[\frac{\omega L}{R}\right]$$

- the phase angle is thus decreased
- the larger the resistance, the smaller the phase angle

- the larger the frequency, the closer to 90^{o}



Example RL reactance and phase angle

- eg. For R = 10 ohms, L= 5 mH
- applied voltage 10 V at 60 Hz and 1000 Hz

$$\vec{Z}_{60} = 10 + j377x0.005 = 10 + j1.89$$

$$Z_{60} = (10^2 + 1.89^2)^{1/2} = 10.18\ ohms$$

$$\theta = arctan\left[\frac{1.9}{10}\right] = 10.6^o$$

$$\vec{Z}_{60} = 10.18\ \underline{/10.7^o}$$

- Thus the is mostly real

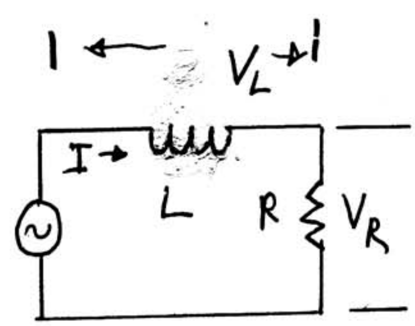
- For the circuit

$$I = \frac{V}{Z_{60}} = \frac{10}{10.18\ \underline{/10.7^o}} = 0.937\ \underline{/-10.7^o}\ Amp$$

$$V_R = IZ_R = 0.937\ \underline{/-10.7^o}\,10 = 9.37\ \underline{/-10.7^o}\ V$$

$$V_L = IZ_L = 0.937\ \underline{/-10.7^o}\ x\ 1.89\ \underline{/90^o} = 1.77\ \underline{/79.3^o}\ V$$

- Note: This is a complex voltage divider
- But output voltages have different phase relationship



Example RL reactance and phase angle

- At 1000 Hz then

$$\vec{Z}_{1000} = 10 + j6283x0.005 = 10 + j31.4 \; ohms$$

$$\vec{Z}_{1000} = 33.0 \,\underline{/72.3^o}$$

- For the circuit

$$I = \frac{V}{Z_{1000}} = \frac{10}{33.0 \,\underline{/72.3^o}} = 0.303 \,\underline{/-72.3^o} \; Amp$$

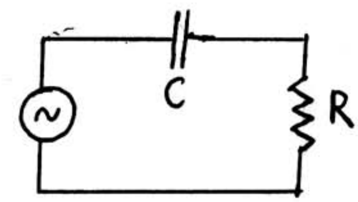
$$V_R = IZ_R = 0.303 \,\underline{/-72.3^o}\, 10 = 3.03 \,\underline{/-72.3^o} \; V$$

$$V_L = IZ_L = 0.303 \,\underline{/-72.3^o} \; x \; 31.4 \,\underline{/90^o} = 9.52 \,\underline{/17.7^o} \; V$$

- Thus acting closer to an inductive reactance

RC reactance and phase angle

- For an RC system



$$Z = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C}$$

- the phase angle is thus reduced
- the large the resistance, the smaller the phase angle
- the smaller the frequency, the closer to -90^{o}
- eg. R=10 ohms, C=2 μF, at 60 Hz, 1 KHz, 10 KHz

$$\vec{Z}_{60} = 10 - j\frac{1}{377x\,2x\,10^{-6}} = 10 - j\,1326$$

$$\vec{Z}_{60} = 1329 \,/\underline{-89.6^{o}}$$

- Thus the is mostly capacitive reactive

- At 1000 Hz then

$$\vec{Z}_{1000} = 10 - j\frac{1}{6283x\,2x\,10^{-6}} = 10 + j\,79.6\ ohms$$

$$\vec{Z}_{1000} = 80.2 \,/\underline{-82.8^{o}}$$

- At 10 KHz then

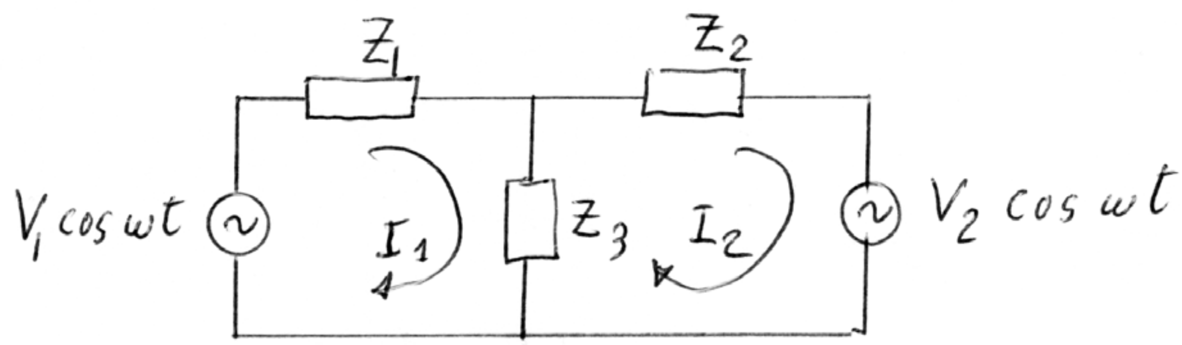
$$\vec{Z}_{10K} = 10 - j\frac{1}{62830x\,2x\,10^{-6}} = 10 + j\,7.95\ ohms$$

$$\vec{Z}_{10K} = 12.8 \,/\underline{-38.5^{o}}$$

- Thus acting closer to an real resistance

Impedance and Circuit Analysis

- Can use the Z's to analysis frequency domain response
- Z's work just as resistors before
- All current/voltage dividers, mesh and node analysis
- Only requires that you use complex numbers



Impedance Divider Example

- Example Consider an impedance divider
- For input waves can find the phase delay
- e.g. RL circuit: what is voltage across capacitor

$$V_L = \frac{Z_L}{Z_{RL}} V_{in}$$

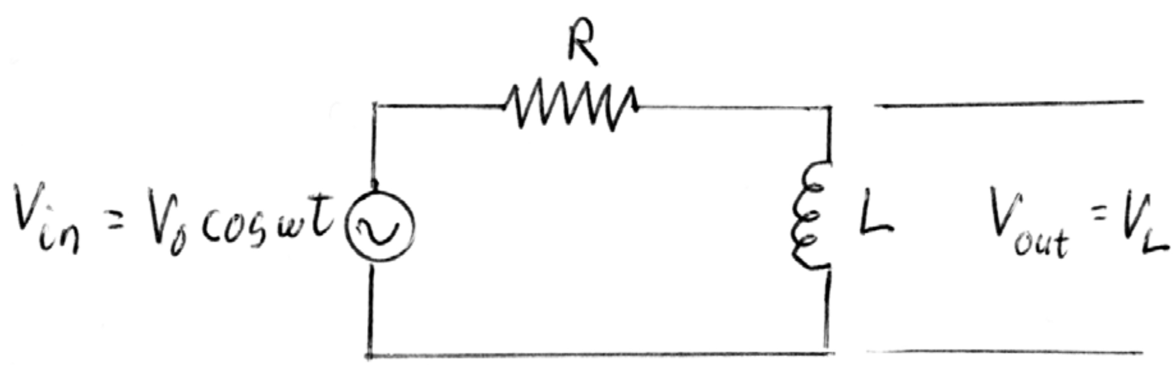
$$Z_L = j\omega L = \omega L \angle 90°$$

$$Z_{RL} = R + j\omega L$$

- The voltage divider equation is

$$V_L = \frac{Z_L}{Z_{RL}} V_{in} = \frac{j\omega L}{R + j\omega L} V_{in}$$

- Note the sensitivity to frequency



Example RL Divider

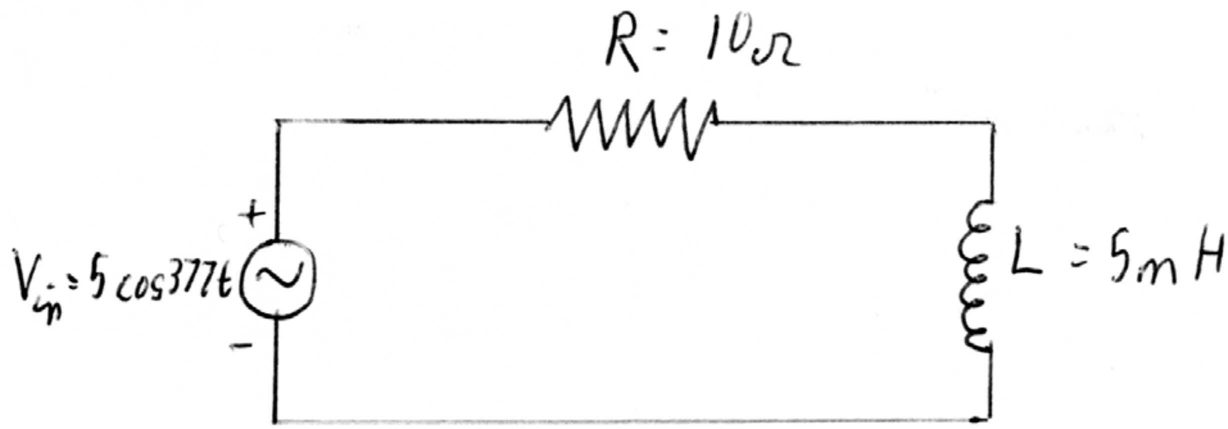
- Consider the RL divider voltage across L
- For R = 10 ohms, L= 5 mH,
- Driven by a 5 V, 60 Hz with a phase of 45¶o¶
- As before the "Input Impedance" seen by the source

$$Z_{RL60} = R + j\omega L = 10 + j377 \times 0.005 = 10 + j1.89 = 10.18\angle 10.6$$

Thus the output is

$$V_L = \frac{Z_L}{Z_{RL}} V_{in} = \frac{1.89\angle 90^\circ}{10.18\angle 10.6^\circ} 5\angle 45^\circ$$

$$V_L = \frac{Z_L}{Z_{RL}} V_{in} = \frac{1.89}{10.18} 5\angle \underline{90 - 10.6 + 45}^\circ = 0.93\angle 124.5^\circ$$



Dual Circuits and Complex Impedance

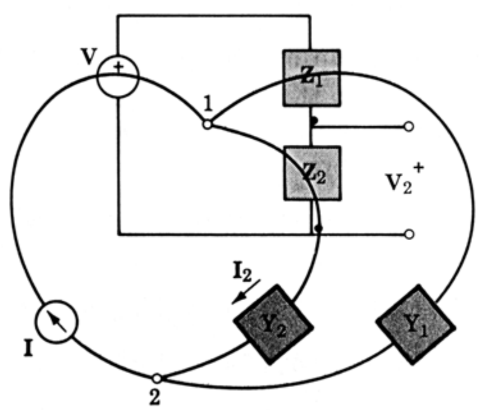
- Just as with resistors can create a dual Z circuit
- Dual circuit behaves same in I as first circuit does in V
- Replace Z with Y's and V with I's
- Also replace L with C and C with L
- The table of replacements are:

Table 5-2 **Dual Quantities**

L	R	C	v	i	Z	X
C	G	L	i	v	Y	B

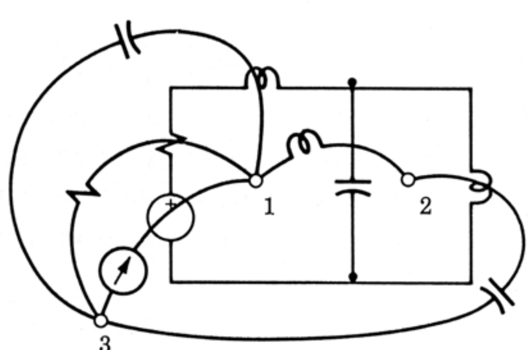
Table 5-3 **Dual Relations**

Loop current	Node voltage
Kirchhoff's voltage law	Kirchhoff's current law
Series connection	Parallel connection
Current source	Voltage source
Short circuit	Open circuit



(a) Voltage divider

Figure 5.18 Deriving the dual of a circuit.



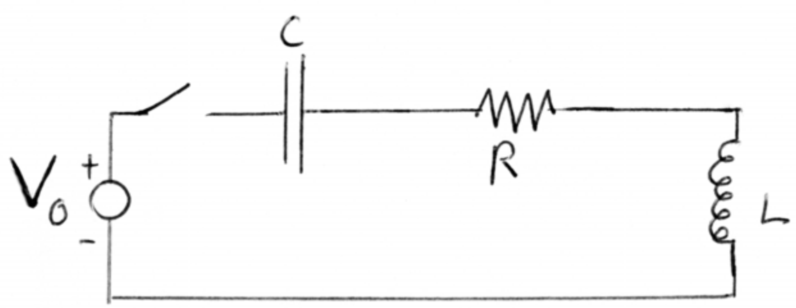
(a)

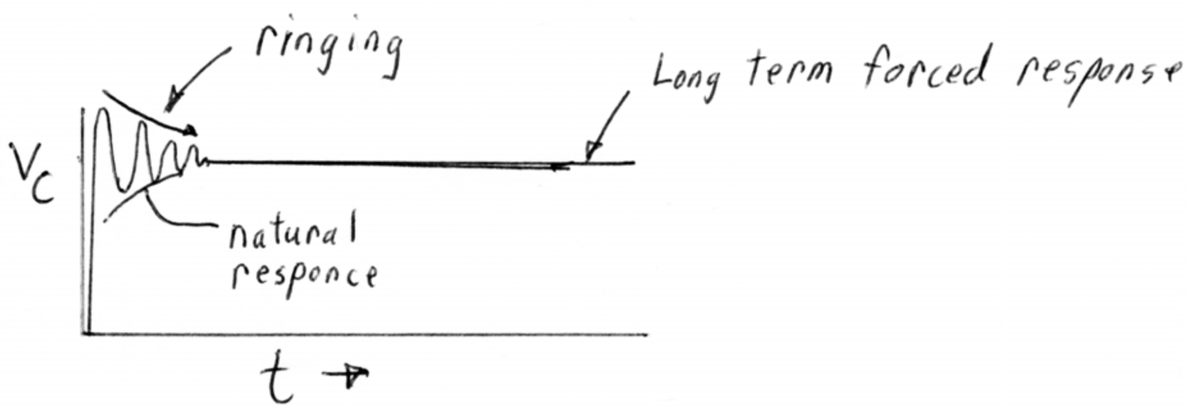
Figure 5.17 Construction of a dual circuit.

Complete Response

- Combines both natural and forced response
- Complete response: what happens to a sudden change
- e.g. Suddenly close a switch
- Response is:

$$V_{complete} = V_{natural} + V_{forced}$$





Initial Underdamped Second Order Systems Con'd

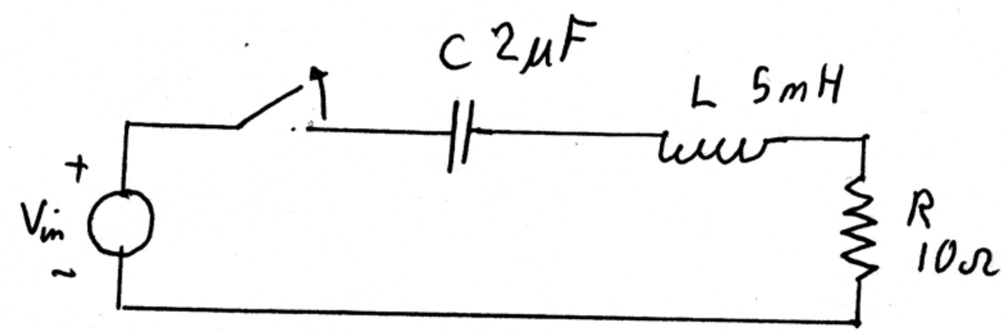
- for the example case L = 5 mH, C = 2 µF, R = 10 ohms
- Exposed to a square wave: 0 to 1 V changes.
- as solved in the RCL underdamped example

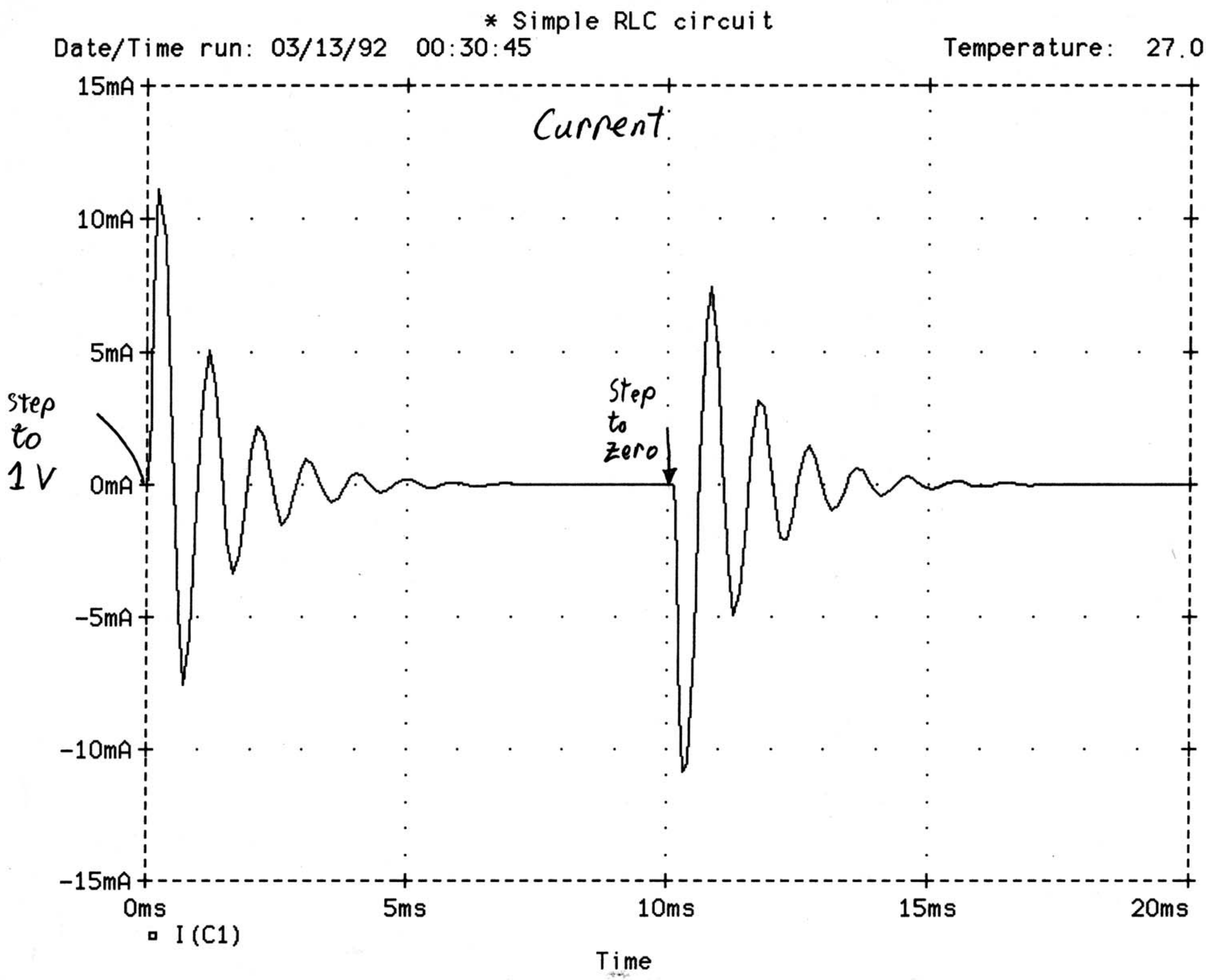
$$i(t) = A_1 \exp(-\alpha t) 2j \sin(\omega t)$$

$$A_1 = \frac{V_c}{2j\omega L} = \frac{1}{2j x 9.95 x 10^3 x 0.005} = \frac{20}{2j}\ mA$$

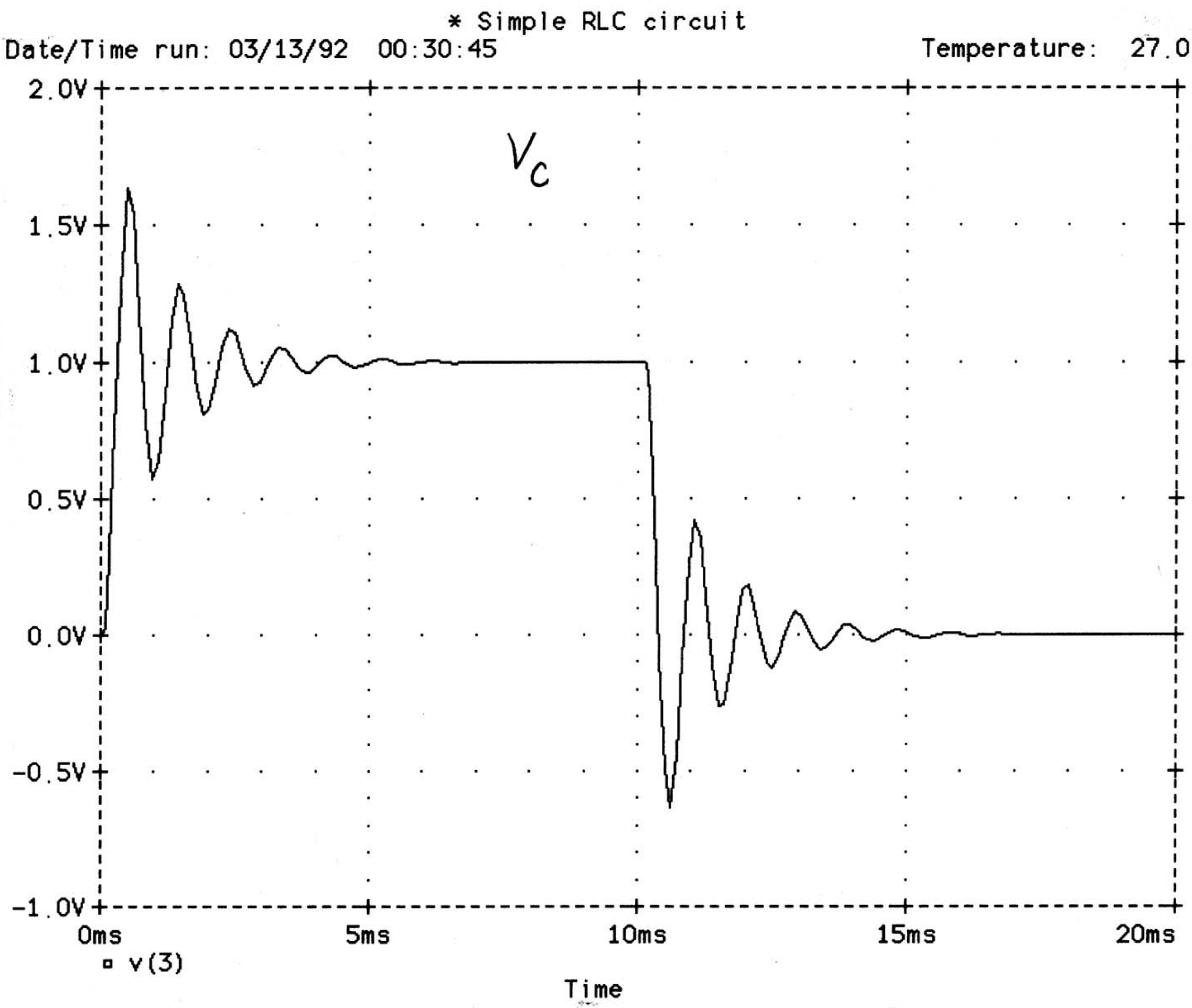
The 2j term is eliminates that from the sin function

$$i(t) = 20 \exp(-10^3 t) \sin(9.95 x 10^3 t)\ mA$$





* Simple RLC circuit
Date/Time run: 03/13/92 00:30:45
Temperature: 27.0
Current
15mA
10mA
5mA
0mA
-5mA
-10mA
-15mA
step to 1V
Step to zero
0ms
5ms
10ms
15ms
20ms
I(C1)
Time



* Simple RLC circuit
Date/Time run: 03/13/92 00:30:45
Temperature: 27.0
Vc
2.0V
1.5V
1.0V
0.5V
0.0V
-0.5V
-1.0V
0ms
5ms
10ms
15ms
20ms
v(3)
Time

Complete Response: Sudden AC changes

- Use same complete response procedure for AC
- Long term AC response given by Z impedance
- procedure the same as sudden DC changes

$$V_{complete} = V_{natural} + V_{ACforced}$$

- in long term get a phase shifted AC wave
- phase is same as Z caluations give after time
- but near switching variations

Complete Response: Sudden AC changes on RL

- Consider an RL circuit with a switched AC voltage
- at t=0 an AC voltage is switched on so

$$V(t \geq 0)_{in} = V_0 \cos(\omega t) \qquad V(t<0) = 0$$

- This is called an AC voltage step

(1) From previous results the natural response is:

$$I(t)_{nat} = A \exp\left[-\frac{Rt}{L}\right]$$

(2) In the long term the complex impedance applies

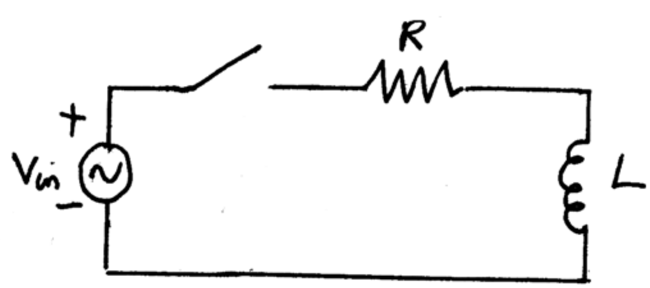
- Input Impedance of the circuit is

$$Z = R + j\omega L$$

$$I_{AC} = \frac{V}{Z} = \frac{V_0}{R + j\omega L} = V_0 \frac{R - j\omega L}{R^2 + (\omega L)^2}$$

$$\theta_Z = \arctan\left(\frac{\omega L}{R}\right)$$

$$I_{AC} = |I_{AC}| \cos(\omega t - \theta_Z)$$



Sudden AC changes on RL Con'd

(3) Combining the equations

$$I(t) = A \ \exp\left[-\frac{Rt}{L}\right] + I_{AC}\cos(\omega t - \theta_Z)$$

(4) Solve for inital conditions

At t=0 L is open so no current must flow

$$0 = I(t) = A \ \exp\left[-\frac{R\,0}{L}\right] + |I_{AC}| \cos(\omega 0 - \theta_Z)$$

$$0 = A + |I_{AC}| \cos(-\theta_Z)$$

$$A = -|I_{AC}| \cos(-\theta_Z)$$

- Note: as this becomes purely inductive:

$$\theta_Z \rightarrow 90^o \qquad A \rightarrow 0$$

Example of sudden AC changes on RL

- Example: RL circuit subjected to a sudden AC signal
- R = 4 ohms, L = 10 mH, and 2 V, 60 Hz cos wave

- For the natural response the decay factor is

$$\frac{R}{L} = \frac{4}{0.01} = 400 \text{ sec}^{-1}$$

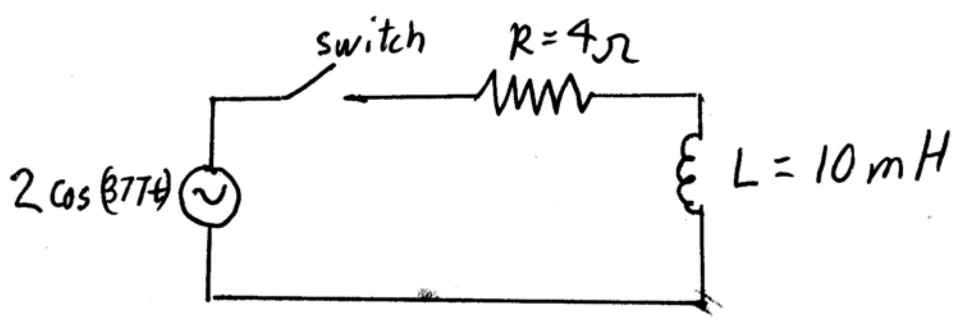
- and Time constant

$$\tau = \frac{L}{R} = 2.5\ msec$$

- The AC response at long time has an impedance

$$Z = R + j\omega L = 4 + j377x0.01 = 4 + j3.77 = 5.50/\underline{43.3^o}$$

$$I_{AC} = \frac{V}{Z} = \frac{2/0}{5.50/\underline{43.3^o}} = 0.364/\underline{-43.3^o}\ Amp$$



Example Sudden AC changes on RL Con'd

(3) Combining the equations

$$I(t) = A\ \exp[-400t] + I_0 \cos(377t - 43.3^o)$$

(4) Solving for inital conditions

At t=0 L is open so no current must flow

$$A = -|I_{AC}| \cos(-\theta_Z) = 0.364 \cos(-43.3^o) = -0.264\ Amp$$

$$I(t) = 0.364 \cos(377t - 43.3^o) - 0.264\exp(-400t)]$$

- NOTE: with phasors there are two used values
- work with Peak V or I (best for cases like above)
- work with RMS V or I (best for power calculations)
- either are correct
- Do not mix the two types in one problem

* Simple RL to sudden AC switch on

Date/Time run: 03/16/92 11:35:13 Temperature: 27.0