# Filters (EC 14)

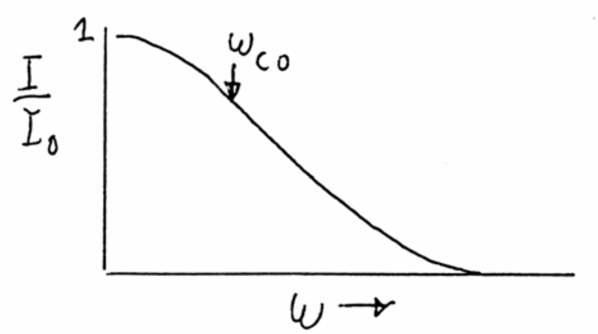
- Often need is to remove some frequencies
- This is done using Frequency Filters
- 4 basic types, each with a critical frequency $\omega_{co}$ or $\omega_o$

## Low Pass Filter:

- Filter passes frequencies well below the critical frequency:

$$\omega << \omega_{co}$$

- Used in detectors to remove high frequency signals
- Perfect filter would cut off sharply at **Critical Frequency** $\omega_{co}$
- Simplest are First order circuits
- Plots these in the **Frequency Domain**
- Plot the relative response($I/I_0$) vs Frequency $\omega$
- $I_0$ is the peak response (may not be at 0 frequency in all cases)
- Usually is in current, but may be in voltage

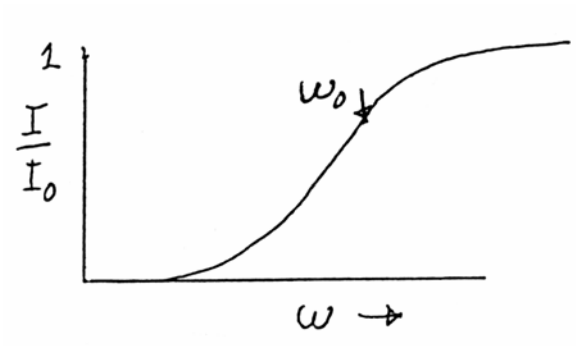


## High Pass Filter:

- Passes frequencies well above critical frequency $\omega_{co}$:

$$\omega >> \omega_{co}$$

- Used to remove low frequency noise (eg. 60 Hz line noise)
- Blocks DC signals
- Simplest are First order circuits



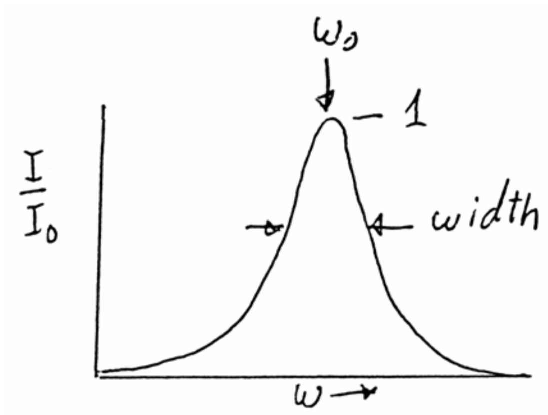
# Filter Types: Band Pass/Reject

## Band pass filter:

- Passes frequencies near the critical frequency $\omega_{co}$

$$\omega \sim \omega_{co}$$

- The width of the passed area is the **Bandwidth**
- Used in tuning circuits eg. radio, TV
- These require second order circuits

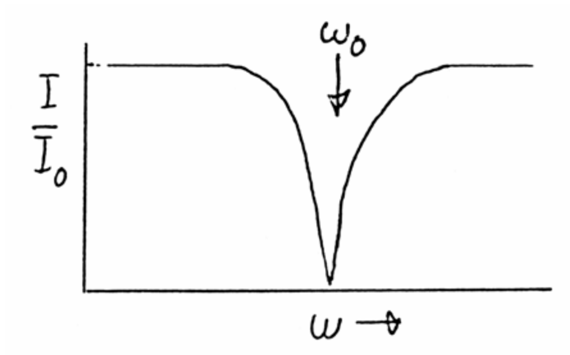


# Band Reject or Notch filter:

- Rejects frequencies near the critical frequency $\omega_{co}$

$$\omega \neq \omega_{co}$$

- Used to remove specific freq. eg 60 Hz line noise
- The width of the rejected frequency is the **Bandwidth**
- These require second order circuits



# Low Pass RL Filters

- Consider a series RL circuit:
- Recall Inductors reject high frequencies
- The current varies with frequency using impedances

$$I(\omega) = \frac{V}{Z} = \frac{V}{R + j\omega L}$$

- At DC the inductor acts as a short so

$$I(\omega = 0) = I_{lo} = \frac{V}{R + j0L} = \frac{V}{R}$$

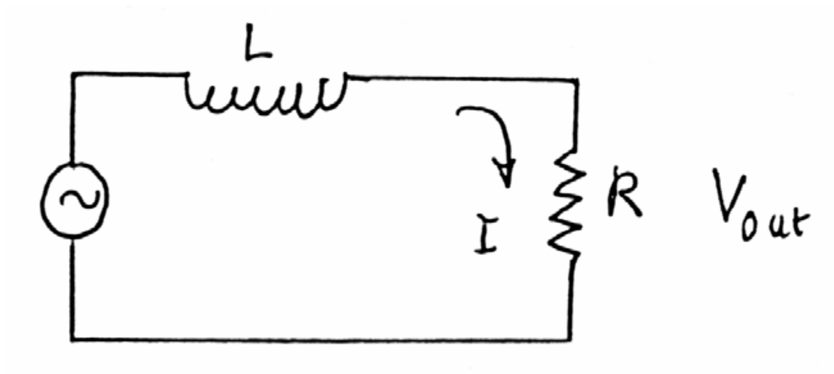
- This is the maximum current point
- Thus the filter behaviour is

$$\frac{I(\omega)}{I_{lo}} = \frac{V}{R + j\omega L}\left[\frac{R}{V}\right] = \frac{1}{1 + j\dfrac{\omega L}{R}} = \frac{|I(\omega)|}{I_{lo}} \angle\theta$$

where

$$\theta = \arctan\left(-\frac{\omega L}{R}\right)$$

$$\frac{|I(\omega)|}{I_{lo}} = \frac{1}{\sqrt{1 + \left[\dfrac{\omega L}{R}\right]^2}}$$



# Low Pass RL Filters & Cutoff Frequency

- Because response goes to zero as $\omega \to \infty$ called **Low Pass Filter**
- Note RL only one example of Low Pass Filter
- Frequency begins to fall rapidly after $X_L = R$
- Called the Cutoff Frequency

$$\omega_{co} L = R \qquad \omega_{co} = \frac{R}{L}$$

- At the cutoff frequency the phase angle is

$$\frac{I(\omega_{co})}{I_{lo}} = \frac{1}{1 + j\dfrac{\omega_{co} L}{R}} = \frac{1}{1 + j1} = \frac{1}{\sqrt{2}} \angle 45^o$$

- This is the 70% signal point
- Also the half power frequency
- Because power supplied to resistor 1/2 max.

$$P_R(\omega_{co}) = I(\omega_{co})^2 R = \frac{I_{lo}^2}{2} R$$

$$P_{\max} = P_R(0) = I_{lo}^2 R$$

# Example of Low Pass RL Filters

- Problem: Create a filter so that 120 Hz signals are reduced by 80%
- Reduced relative to DC signals
- Want a RL with L = 100 mH
- The RL response is

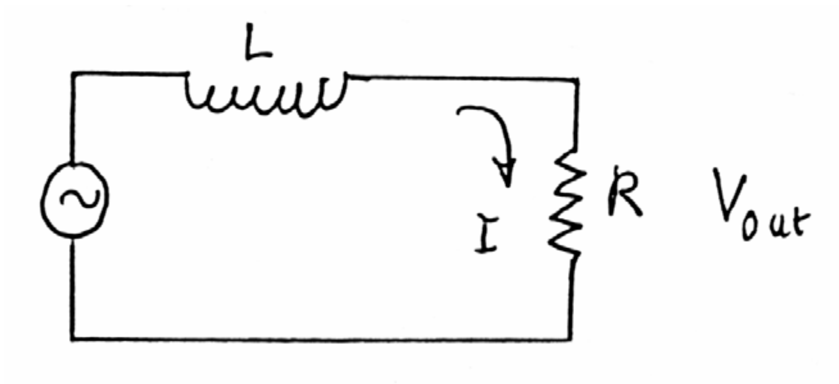
$$\frac{|I(\omega)|}{I_{lo}} = \frac{1}{\sqrt{1+\left[\frac{\omega L}{R}\right]^2}}$$

- Thus

$$\left[\frac{I_{lo}}{|I(\omega)|}\right]^2 = \left[1+\left[\frac{\omega L}{R}\right]^2\right]$$

- Thus the change at a give frequency is

$$\frac{\omega L}{R} = \frac{\omega}{\omega_{co}} = \sqrt{\left(\left[\frac{I_{lo}}{|I(\omega)|}\right]^2 - 1\right)}$$



# Example of Low Pass RL Filters Cutoff Frequency

- At the desired frequency of 120 Hz

$$\omega = 2\pi 120 = 754\ rad/s$$

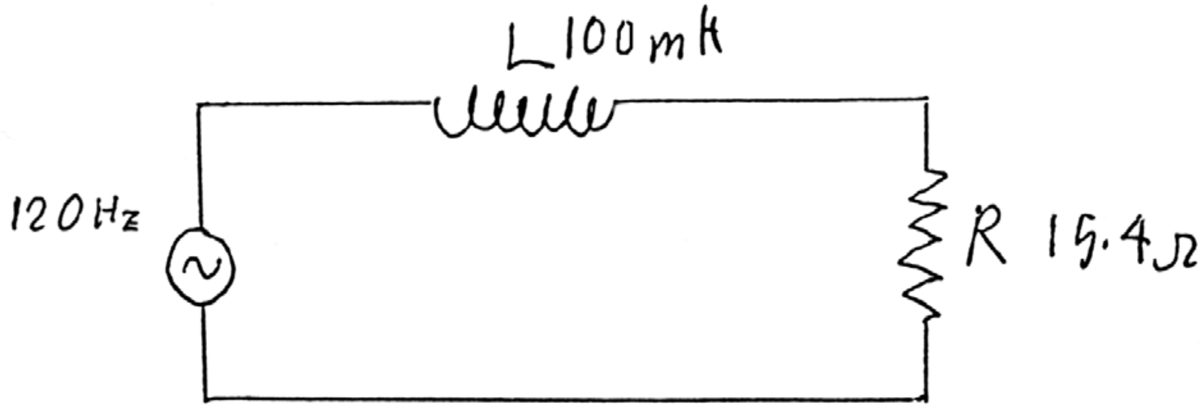
$$\frac{|I(\omega)|}{I_{lo}} = 0.2$$

- Thus the cutoff frequency is

$$\omega_{co} = \frac{\omega}{\sqrt{\left(\left[\frac{I_{lo}}{|I(\omega)|}\right]^2 - 1\right)}} = \frac{754}{\sqrt{\left(\left[\frac{1}{0.2}\right]^2 - 1\right)}} = \frac{754}{\sqrt{25-1}} = 153.9\ rad/s$$

- Thus the cutoff frequency required = 153.9 rad/s = 24.5 Hz
- Let L = 100 mH then the required resistance is

$$\omega_{co} = \frac{R}{L} \qquad R = \omega_{co} L = 153.9(0.1) = 15.4\ \Omega$$



## High Pass RC Filters

- Recall Capacitor pass high frequencies
- Thus RC is a high pass filter

$$I(\omega) = \frac{V}{Z} = \frac{V}{R - j\dfrac{1}{\omega C}}$$

- At very high frequencies the C acts as a short so

$$I_{hi} = \frac{V}{R}$$

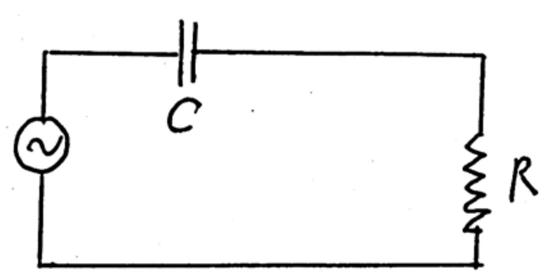
- thus the filter behaviour is

$$\frac{I(\omega)}{I_{hi}} = \frac{1}{1 - j\dfrac{1}{\omega CR}} = \frac{|I(\omega)|}{I_{hi}} \underline{/\theta}$$

where

$$\theta = arctan\left[\frac{1}{\omega CR}\right]$$

$$\frac{|I(\omega)|}{I_{hi}} = \frac{1}{\left[1 + \left(\dfrac{1}{\omega CR}\right)^2\right]^{1/2}}$$



## High Pass RC Filters Cont'd

- Recall C reject low frequency
- Frequency begins to fall rapidly after $X_C = R$

$$\omega_{co} = \frac{1}{RC}$$

- again this is the cutoff frequency
- where the power is down by 1/2

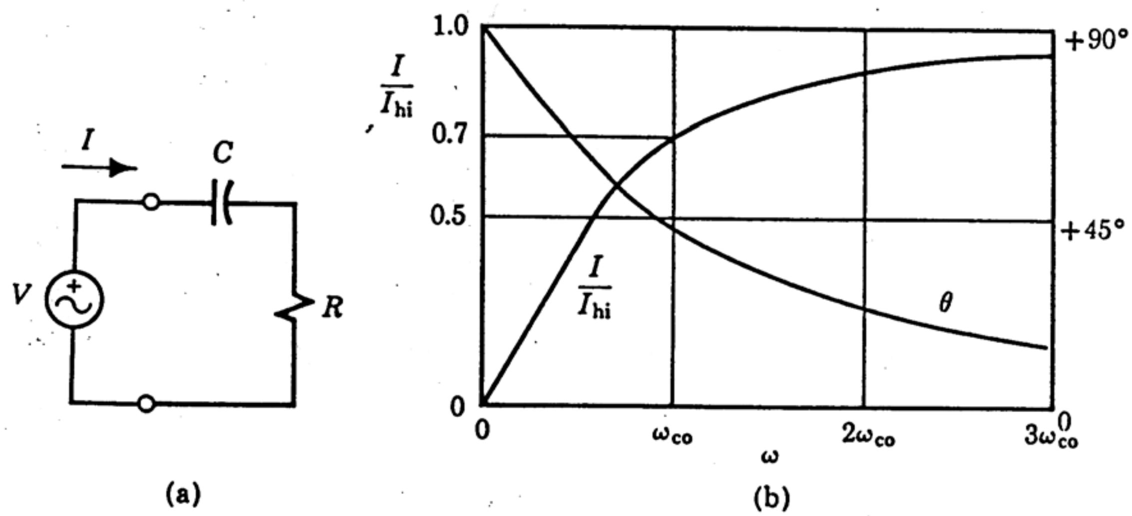



Figure 7.10 A simple high-pass filter.

# RLC Resonance Filters

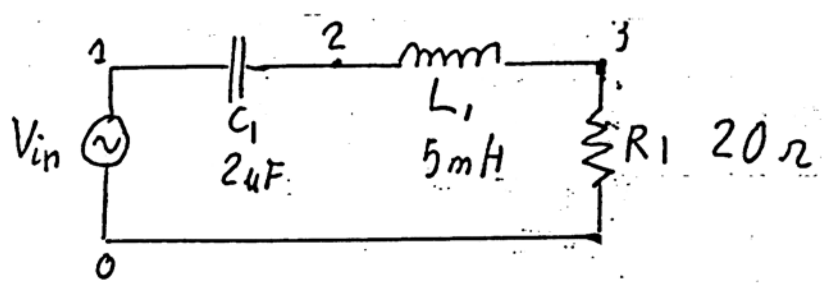
- Recall the RLC filter has impedance of

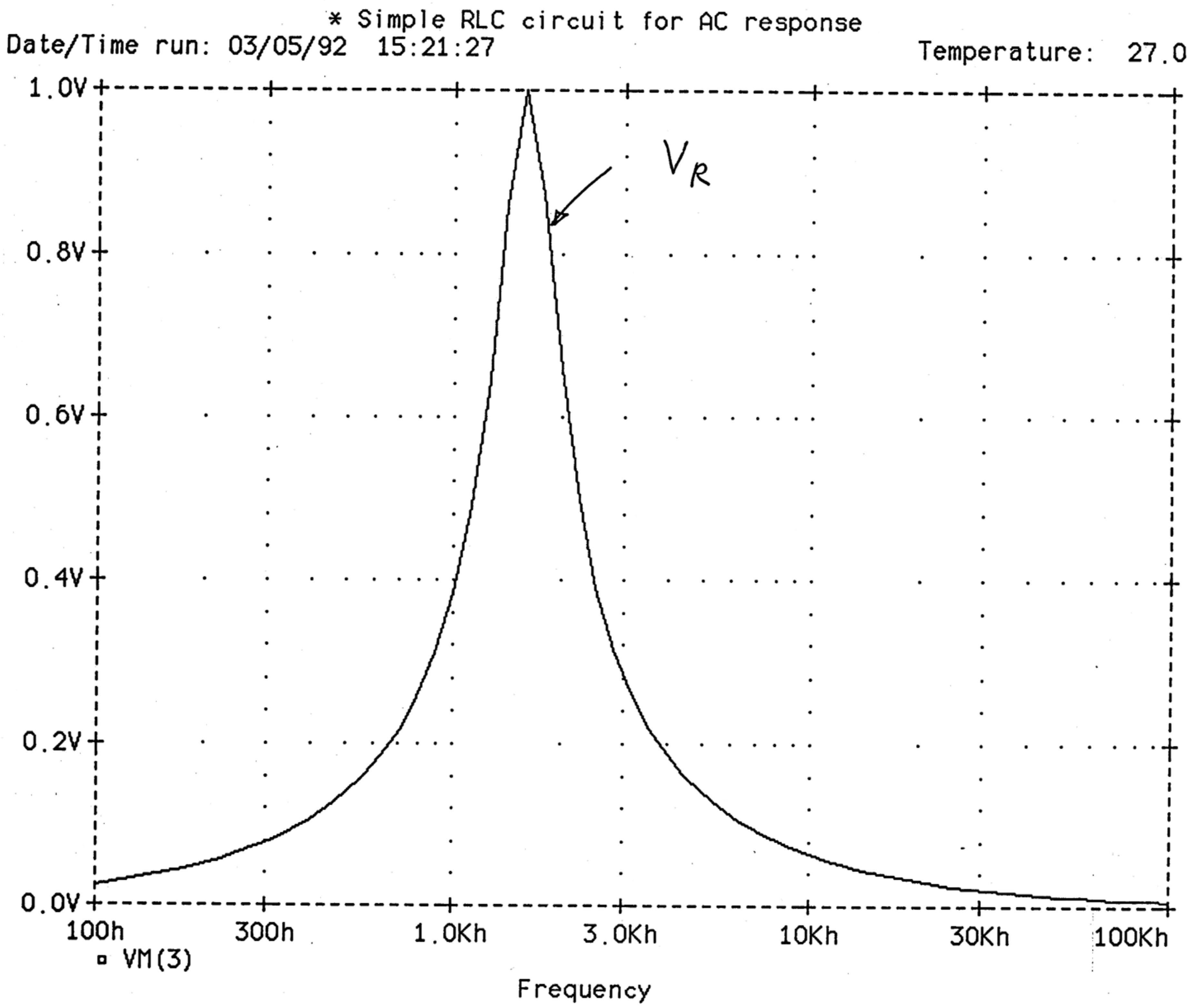
$$Z = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

- and the natural or resonance frequency of

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

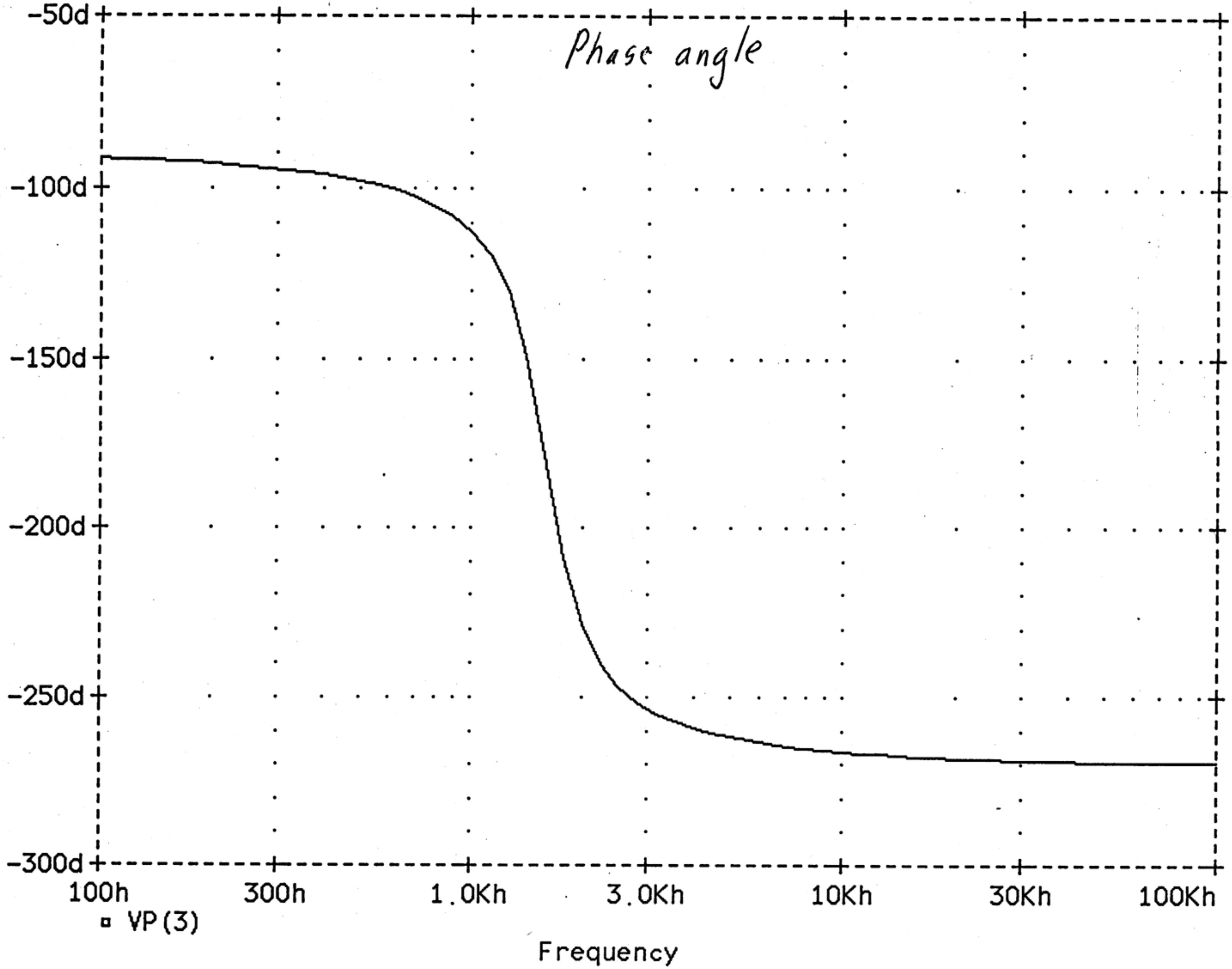
- This acts as a Band Pass filter for current
- Voltage output across resistor reflects this

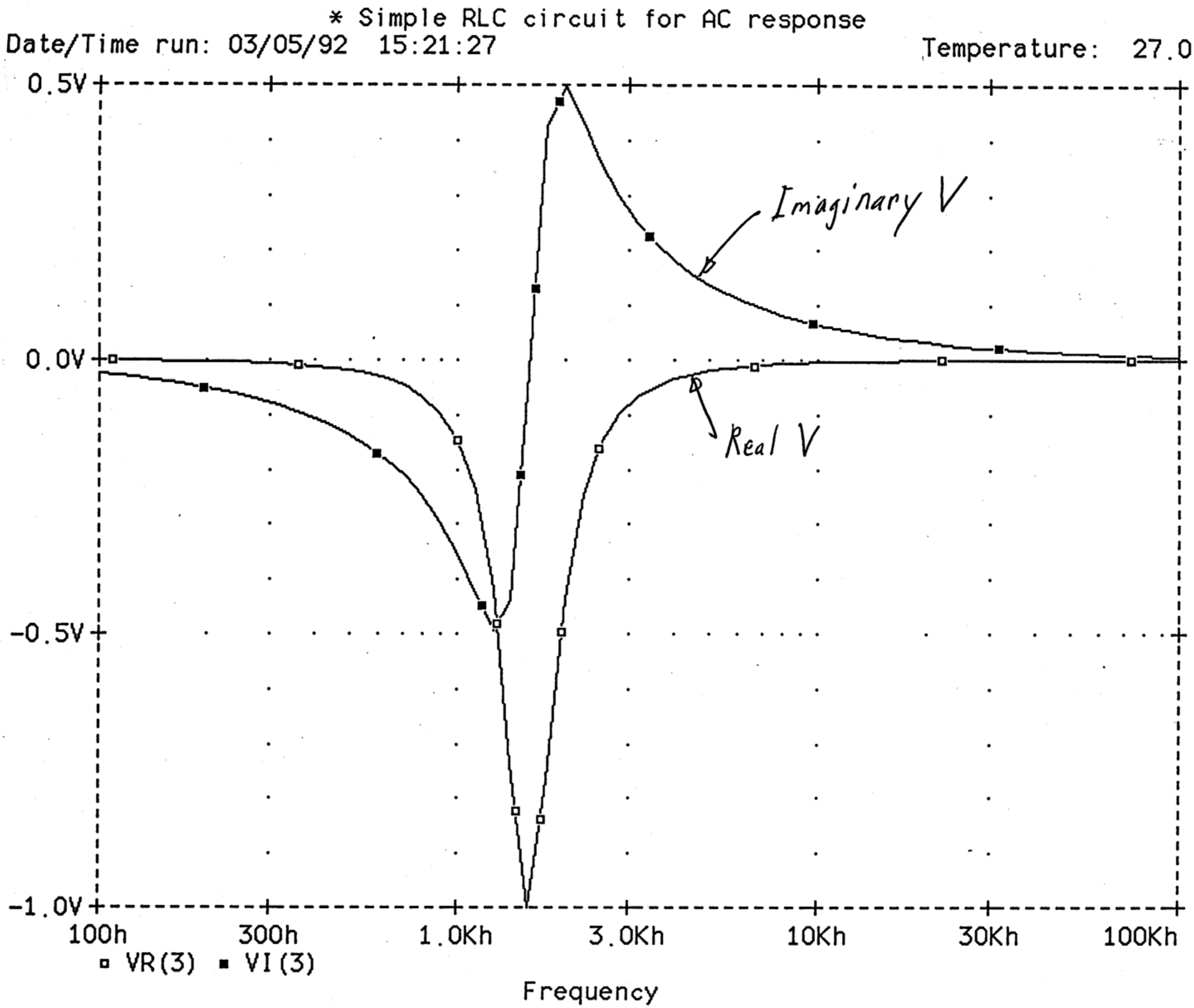



* Simple RLC circuit for AC response
Date/Time run: 03/05/92 15:21:27
Temperature: 27.0
1.0V
0.8V
0.6V
0.4V
0.2V
0.0V
$V_R$
100h
300h
1.0Kh
3.0Kh
10Kh
30Kh
100Kh
▫ VM(3)
Frequency

* Simple RLC circuit for AC response

Date/Time run: 03/05/92 15:21:27 Temperature: 27.0

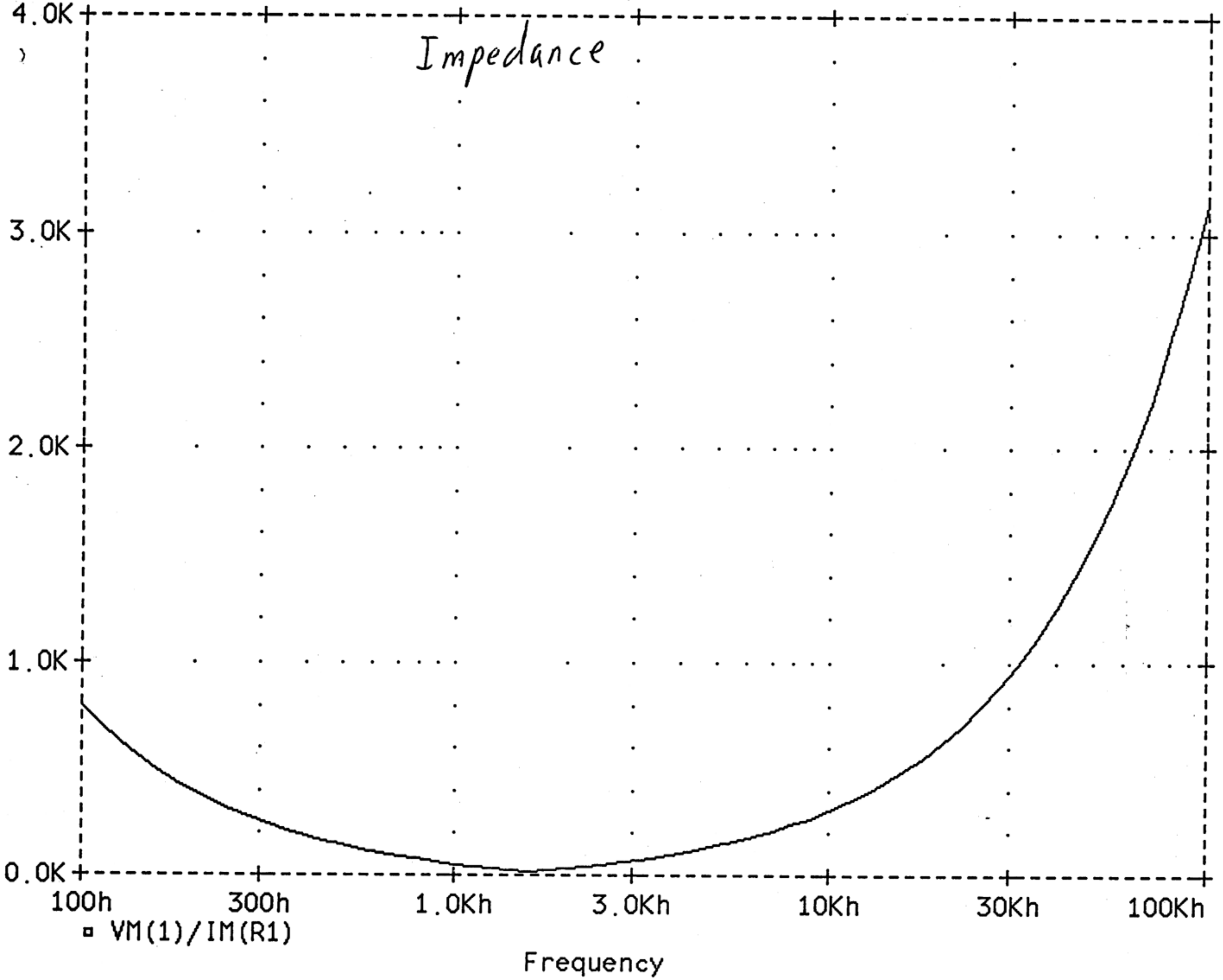




* Simple RLC circuit for AC response
Date/Time run: 03/05/92 15:21:27
Temperature: 27.0
0.5V
0.0V
-0.5V
-1.0V
100h
300h
1.0Kh
3.0Kh
10Kh
30Kh
100Kh
VR(3)
VI(3)
Frequency
Imaginary V
Real V

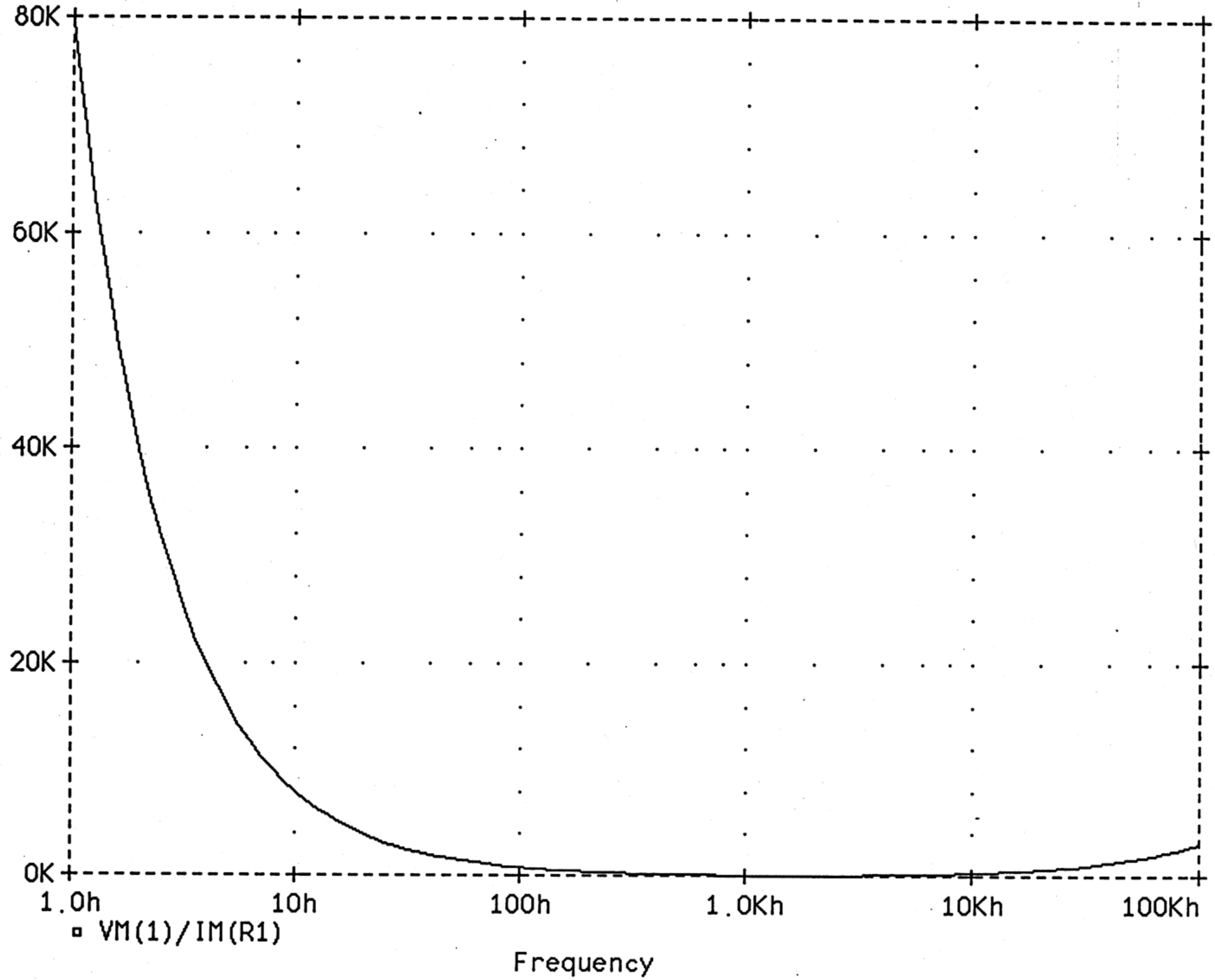
* Simple RLC circuit for AC response

Date/Time run: 03/05/92 15:21:27 Temperature: 27.0



* Simple RLC circuit for AC response

Date/Time run: 03/05/92 15:41:46 Temperature: 27.0

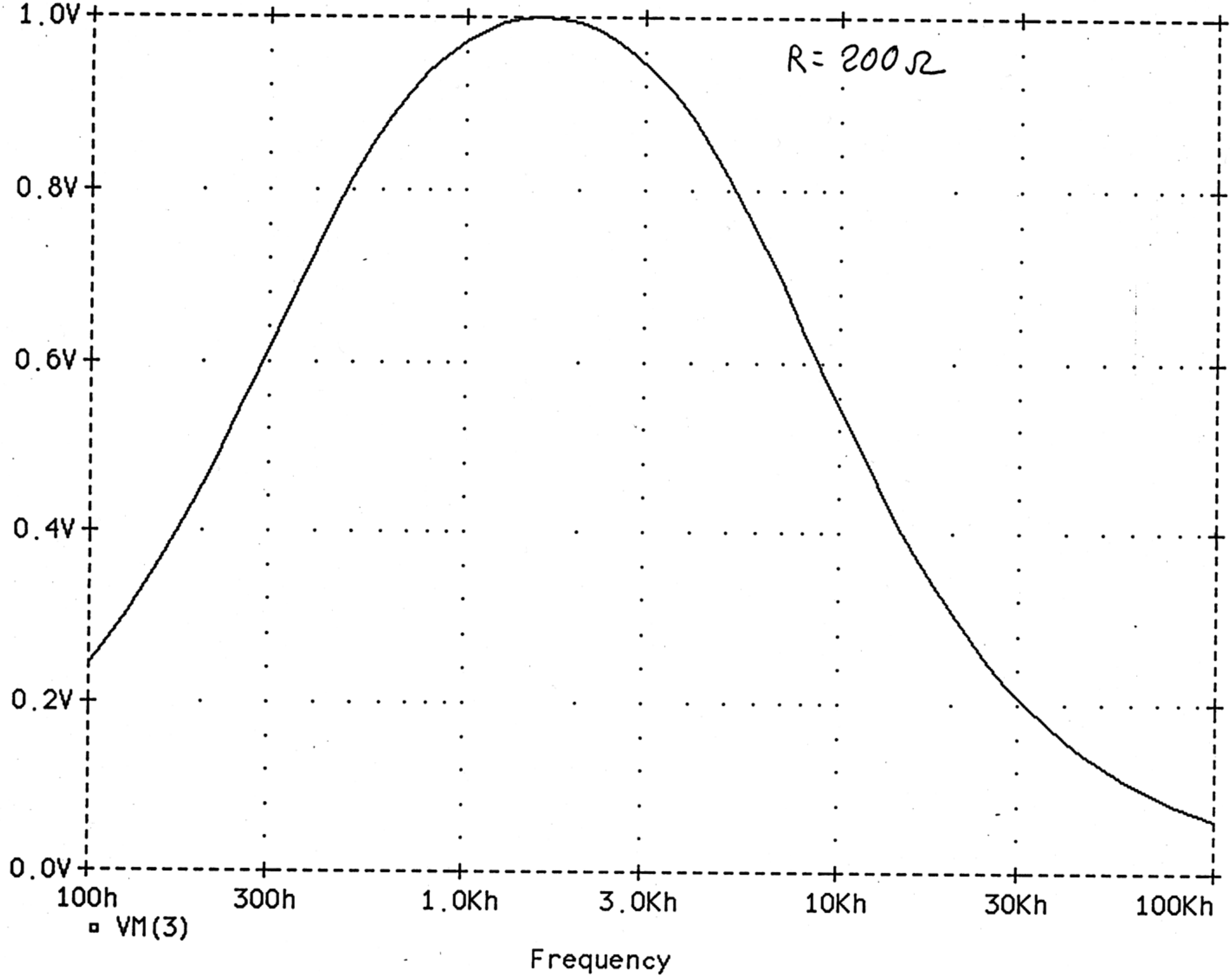


## Width of the Frequency Response

- As R is increased the width of the reponse increases
- Thus can use this to create a filter
- The width of the frequency is set by R
- The resonance frequency by C and L
- This is called a Band Pass filter

* Simple RLC overdamped circuit for AC response

Date/Time run: 03/05/92 15:59:59 Temperature: 27.0



## Quality factor Q and Filters

- Changing R changes the resonance width
- Controlled by the energy loss per cycle
- called the damping factor
- Define the Quality Factor Q as

$$Q = 2\pi \frac{\max \textit{ energy stored}}{\textit{energy lost per cycle}}$$

- the Q measures how good a circuit is
- the higher the Q, the sharper the peak

## Quality factor Q in Filters (

- Energy loss is in the resistor

$$W_{cycle} = \frac{I_{rms}^2 R}{f_0} = \frac{2\pi I_{rms}^2 R}{\omega_0}$$

- Max energy stored in the Inductor is

$$W_{Lmax} = \frac{i^2 L}{2} = I_{rms}^2 L$$

- Thus the Q factor is

$$Q = 2\pi \frac{I_{rms}^2 L}{\frac{2\pi I_{rms}^2 R}{\omega_0}} = \frac{\omega_0 L}{R}$$

- Similarly for the capacitor

$$Q = \frac{1}{\omega_0 C R}$$