Quality factor Q and Filters

- Changing R changes the resonance width
- Controlled by the energy loss per cycle
- called the damping factor
- Define the Quality Factor Q as

$$Q = 2\pi \frac{\max\, \textit{energy stored}}{\textit{energy lost per cycle}}$$

- the Q measures how good a circuit is
- the higher the Q, the sharper the peak

Quality factor Q in Filters (

- Energy loss is in the resistor

$$W_{cycle} = \frac{I_{rms}^2 R}{f_0} = \frac{2\pi I_{rms}^2 R}{\omega_0}$$

- Max energy stored in the Inductor is

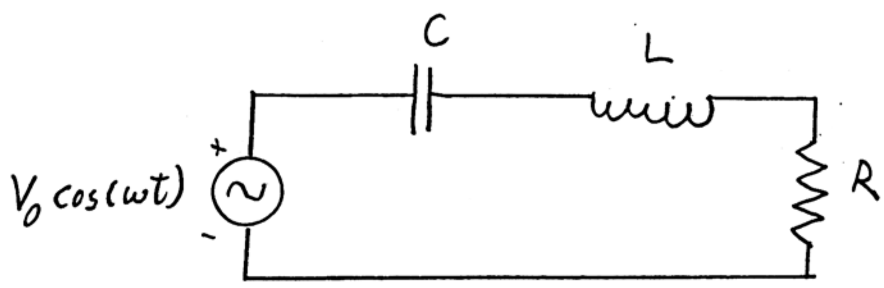
$$W_{Lmax} = \frac{i^2 L}{2} = I_{rms}^2 L$$

- Thus the Q factor is

$$Q = 2\pi \frac{I_{rms}^2 L}{\frac{2\pi I_{rms}^2 R}{\omega_0}} = \frac{\omega_0 L}{R}$$

- Similarly for the capacitor

$$Q = \frac{1}{\omega_0 CR}$$



Normalized Filter Response (EC 14.2)

- Filter equations can be expressed in Q
- consider the series RLC circuit

$$Z = \frac{1}{Y} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

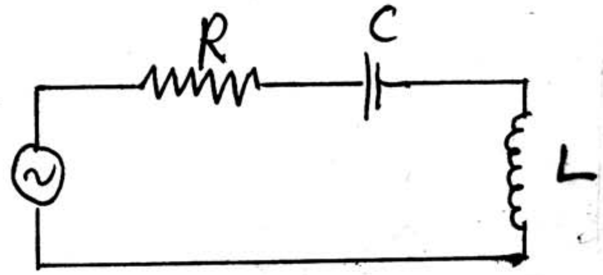
- at the resonance frequency

$$Z_0 = R = \frac{1}{Y_0}$$

- relative to the resonance values

$$\frac{Y}{Y_0} = \frac{Z_0}{Z} = \frac{1}{1 + j\left[\omega\frac{L}{R} - \frac{1}{\omega CR}\right]}$$

- called the Normalized Response



Quality factor Q & Filters

- Since the Quality Factor is

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

- thus

$$\frac{L}{R} = \frac{Q}{\omega_0}$$

$$\frac{1}{CR} = Q\,\omega_0$$

- Thus can write the Normalized Response

$$\frac{Y}{Y_0} = \frac{Z_0}{Z} = \frac{1}{1 + jQ\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]}$$

- this same equation works for parallel RLC circuit or any other simple resonance circuit
- only Q's and ω_0 change

Bandwidth and Q in Filters

- Want to measure the Bandwidth
- Bandwidth: frequency range between the 70% points
- in the form

$$\frac{Y}{Y_0} = \frac{Z_0}{Z} = \frac{1}{1 + jQ\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]}$$

- thus want the point where

$$\frac{Y}{Y_0} = \frac{1}{1 \pm j1}$$

- thus

$$\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right] = \pm\frac{1}{Q}$$

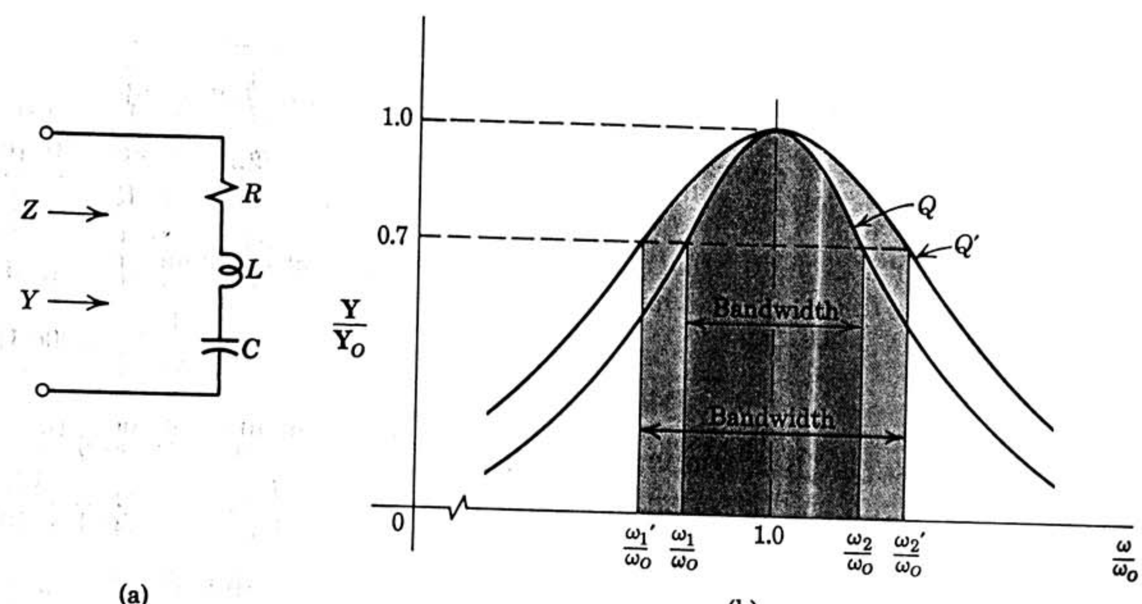


Figure 7.22 The normalized response of a series *RLC* circuit.

Bandwidth and Q in Filters

- Define 2 half power or 70% points frequencies
- ω_1 = lower frequency

$$\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1} = -\frac{1}{Q}$$

- and for ω_2 = upper frequency

$$\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2} = +\frac{1}{Q}$$

Bandwidth and Q in Filters Cont'd

- Solving for these at the lower frequency

$$\frac{\omega_1^2 - \omega_0^2}{\omega_1 \omega_0} = -\frac{1}{Q}$$

- solving the quadratic gives

$$\omega_1 = \omega_0 \left[1 + \left(\frac{1}{2Q} \right)^2 \right]^{1/2} - \frac{\omega_0}{2Q}$$

- similarly for the upper frequency

$$\omega_2 = \omega_0 \left[1 + \left(\frac{1}{2Q} \right)^2 \right]^{1/2} + \frac{\omega_0}{2Q}$$

- Define bandwidth from the 70% points as

$$Bandwidth = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$$Bandwidth = f_2 - f_1 = \frac{f_0}{Q}$$

Bandwidth and Q in Filters

- thus bandwidth determined by the Quality Factor

- One approximation useful
- When Q ≥ 10 then with < 2% error

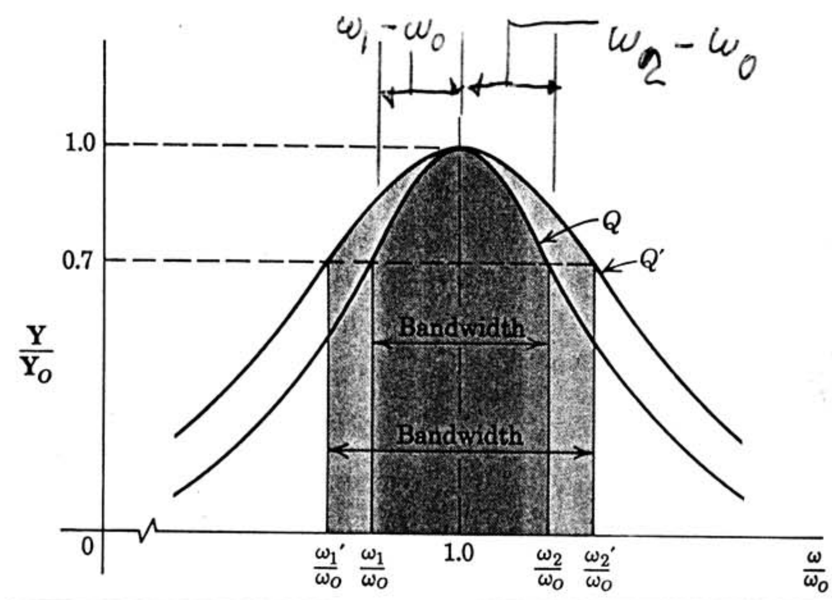
$$\left[1 + \left(\frac{1}{2Q}\right)^2\right]^{1/2} \approx 1$$

- Thus can approximate

$$\omega_1 \approx \omega_0\left[1 - \frac{1}{2Q}\right] \qquad \omega_2 \approx \omega_0\left[1 + \frac{1}{2Q}\right]$$

- and the curve is symmetric about ω_0

$$\omega_2 - \omega_0 = \omega_0 - \omega_1 = \frac{\omega_0}{2}Q$$



Example Bandwidth and Q in Filters

- Design a parallel RLC circuit to select the 1000 KHz frequency of AM radio, with a bandwidth of 5 KHz. Find the C & R needed for the circuit when
- L = 20 μH

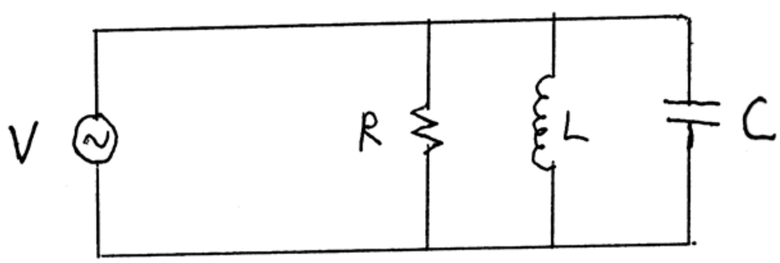
- Since for the parallel RLC

$$Y = G + j\left[\omega C - \frac{1}{\omega L}\right]$$

- thus

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi 10^6)^2 x\, 2x\, 10^{-5}} = 1.27x\,10^{-9} = 1.27\ nF$$



Example Bandwidth and Q in Filters Cont'd

- to get the desired resistance note

$$Bandwidth = f_2 - f_1 = \frac{f_0}{Q}$$

$$Q = \frac{f_0}{f_2 - f_1} = \frac{10^6}{5x10^3} = 200$$

- Since for the parallel RLC

$$Q = \frac{1}{\omega_0 LG}$$

$$R = Q\omega_0 L = 200x6.14x10^6x2x10^{-5} = 25.1\ Kohms$$

Phasors: Peak and RMS values (EC 10)

- NOTE: with phasors there are two used values
- work with Peak V or I (best for where plotting is needed)
- work with RMS V or I (best for power calculations)
- either are correct
- Do not mix the two types in one problem

- example

$$V = 10\cos(\omega t + 45^o)$$

- In peak value phasor form:

$$\vec{V} = 10\underline{/45^o}$$

- In RMS value phasor form:

$$\vec{V} = \frac{10}{\sqrt{2}}\underline{/10^o} = 7.07\underline{/45^o}$$

Power Calculation in AC

- Recall Power is always given by

$$P = I^2 Z = \frac{V^2}{Z}$$

- NOTE: for this must use RMS phasors
- For AC waves the L and C produce only imaginary
- For Inductor

$$i(t) = I_0 \cos(\omega t) = \frac{I_0}{\sqrt{2}} \underline{/0}$$

$$V_L(t) = I j \omega L = \frac{I_0 \omega L}{\sqrt{2}} \underline{/90^o}$$

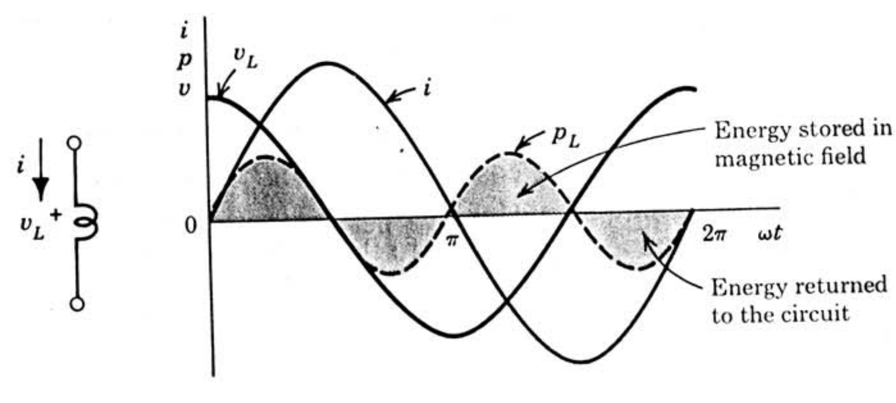
$$v_L(t) = \frac{I_0 \omega L}{\sqrt{2}} \sin(\omega t - \pi)$$

- thus the instantaneous power is

$$P = i(t) v(t) = \frac{2 I_0^2 \omega L}{2} \sin(\omega t) \cos(\omega t)$$

where

$$\sin(\omega t) \cos(\omega t) = 2 \sin(2\omega t)$$



Reactive Power Calculation in AC

- since a sin wave averaged over two periods is

$$P_{avg} = \int_0^T \frac{2I_0^2 \omega L}{2} \sin(\omega t)\cos(\omega t) dt = 0$$

- Thus the average power is zero
- However the reactive power stored is

$$P_{peak} = \frac{I_0^2 \omega L}{2} = I_{rms}^2 X$$

Where

$$I_{rms} = \frac{I}{\sqrt{2}} \qquad X = \omega L = reactive\ \ impedance$$

- Reactive power must be supplied
- but is returned over the cycle

AC Power Factor

- Real AC Power is affected by the V phase angle

$$P(t) = V_{rms}\sqrt{2}cos(\omega t + \theta)\ I_{rms}\sqrt{2}cos(\omega t)$$

$$= 2V_{rms}I_{rms}cos(\omega t + \theta)\cos(\omega t)$$

$$= V_{rms}I_{rms}cos(\theta) + V_{rms}I_{rms}cos(\omega t + \theta)$$

because

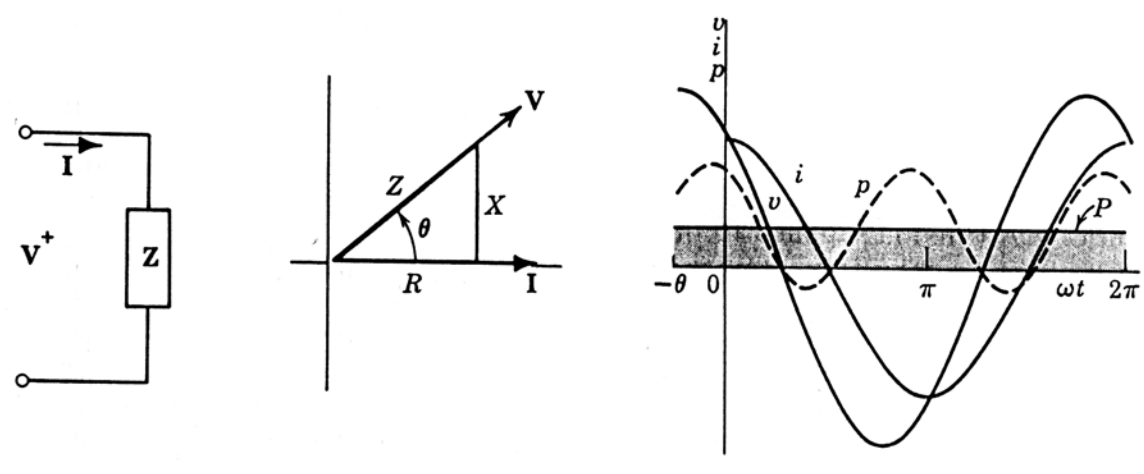
$$2cos(A)cos(B) = cos(A - B)\ cos(A + B)$$

- Since the average over time

$$\int_0^T V_{rms}I_{rms}cos(\omega t + \theta)dt = 0$$

- Thus the time averaged power is

$$P_{avg} = V_{rms}I_{rms}cos(\theta)$$



(a) Circuit diagram (b) Phasor diagram (c) Time variation

AC Power Factor

- The apparent power is in Volt-Amperes

$$P_{apparent} = V_{rms} I_{rms}$$

- VI related to the real power by the Power Factor pf

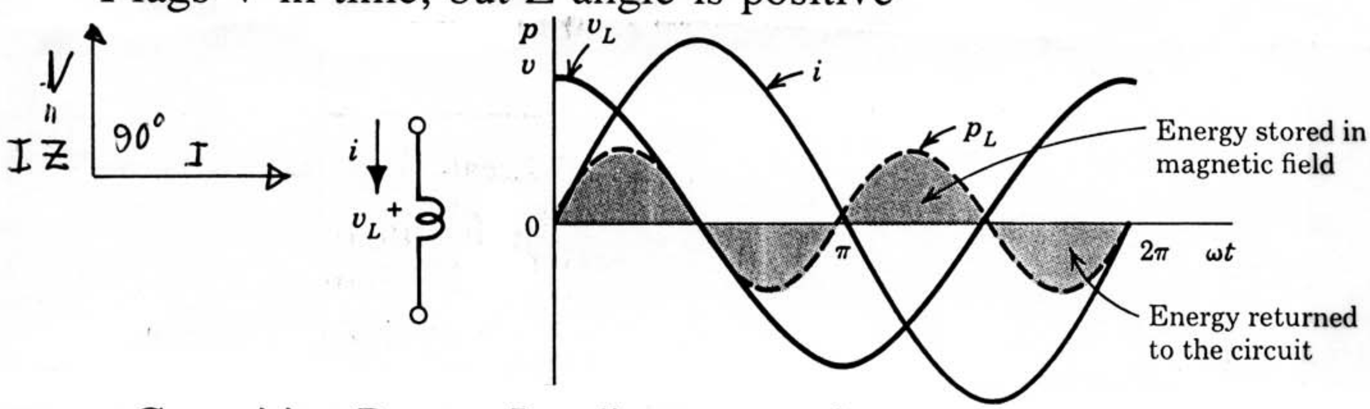
$$pf = cos(\theta) = \frac{P_{avg}}{V_{rms} I_{rms}}$$

- NOTE: the power angle is that of the impedance

$$\theta_{Pavg} = \theta_Z$$

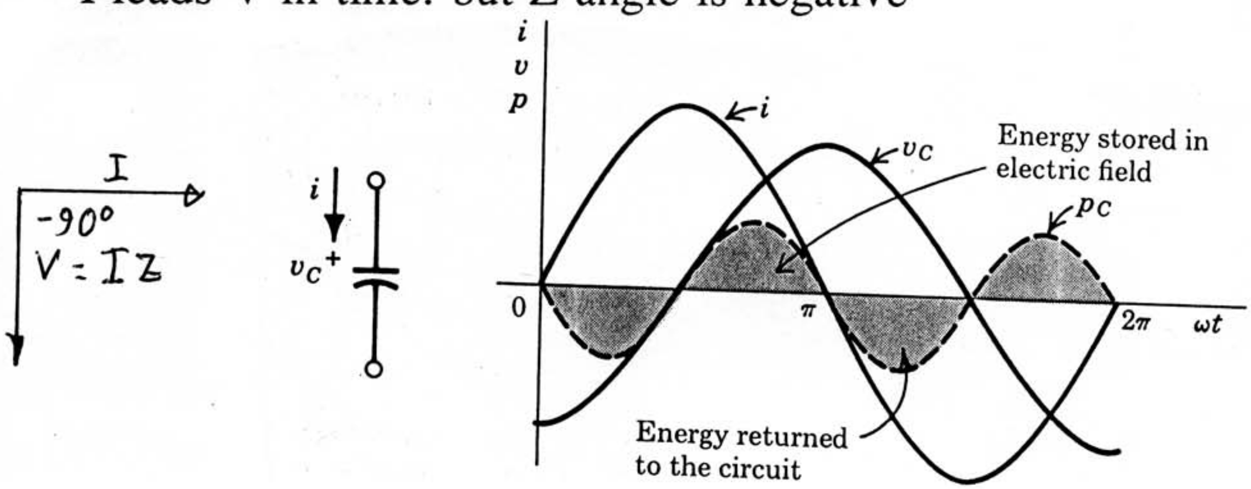
- Inductive power (most common): Lagging power factor

I lags V in time, but Z angle is positive



- Capacitive Power: Leading power factor

I leads V in time: but Z angle is negative



AC Reactive Power

- Reactive Power: Power must be supplied but will be returned in part of the cycle

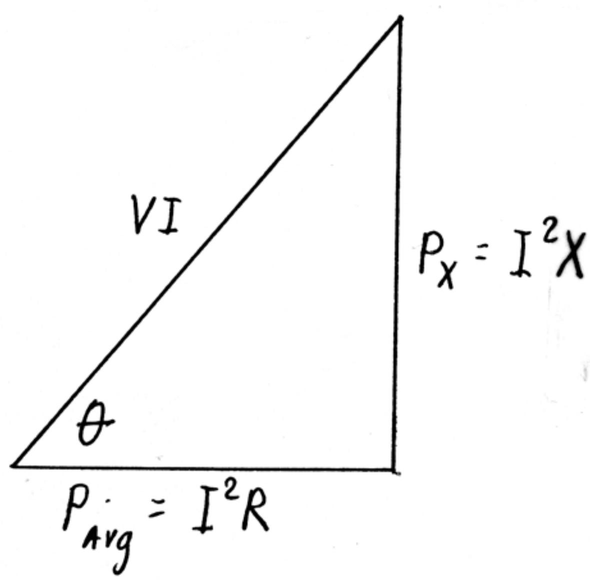
$$P_X = Q = I_{rms}^2 X = I_{rms}^2 Z \sin(\theta) = V_{rms} I_{rms} sin(\theta)$$

- units: Volts-Amperes Reactive (VAR)

- Power just like other complex unites

$$\vec{P}_{app} = S = P_R + jQ = |V_{rms} I_{rms}| \underline{/\theta}$$

- Often draw the Power Triangle:
I^2X plotted against I^2 R



Example AC Power

- Example: AC electric motor has R = 4 ohms coils Inductance of 10 mH, what is power factor at 60 Hz, 120 V

- Impedance is

$$Z = R + j\omega L = 4 + j377x0.01 = 4+j3.77 = 5.50\underline{/43.3^o}$$

$$\theta = arctan\left[\frac{3.77}{4}\right] = 43.3^o$$

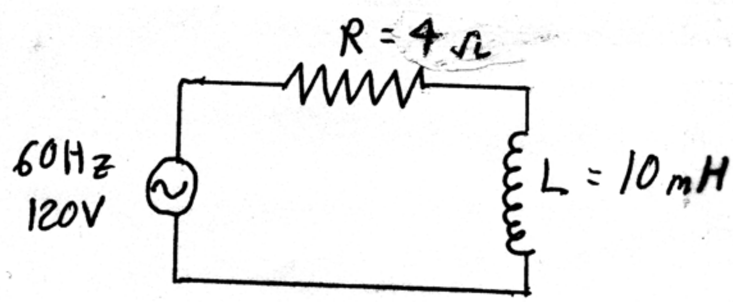
- power factor is

$$pf = \cos(43.3^o) = 0.728$$

- Current required is

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{120}{5.50\underline{/43.3^o}} = 21.8\underline{/-43.3^o}\ Amps$$

•: NOTE: this is the I that a current meter would read



Example AC Power continued

- Thus for AC power: Volt-Amps is:

$$VA = V_{rms} I_{rms} = 120x21.8 = 2618\ VA$$

- Average Power consumed is

$$P_{avg} = V_{rms} I_{rms} pf = 2618x0.729 = 1.905\ KW$$

KW = KiloWatts

$$\vec{P}_{avg} = 1.905/\underline{43.3^o}$$

- Reactive power is

$$Q = V_{rms} I_{rms} \sin(\theta)$$

$$= 2618x\sin(43.3^o) = 2618x0.689 = 1795\ VAR$$

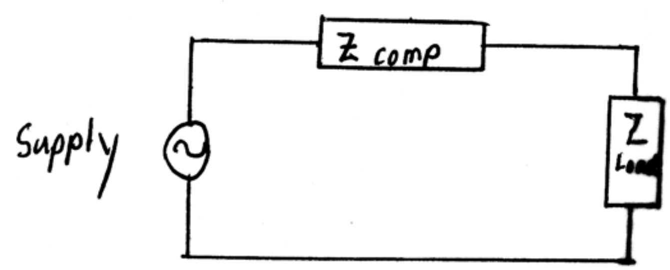
Power Factor compensation

- Most heavy devices (motors, heater coils) inductive
- thus tendency is to get lagging power factor
- But large users charged extra for low power factor

- reason:
- generators must supply the I(t): thus larger generators
- also power lines mostly resistive:

Thus VARS create higher real losses in the delivery lines
That loss is the Utilities cost, not the users!

- thus users must often build capacitor banks to minimize their power factor charges



Power Factor Compensation cont'd

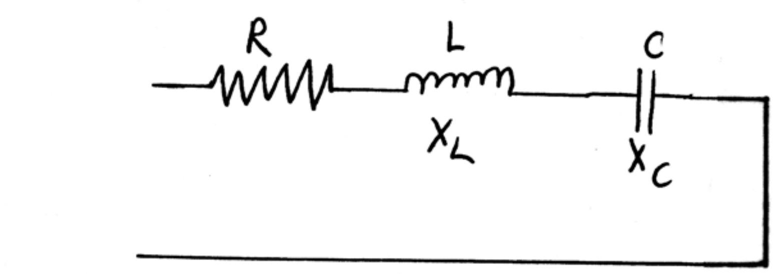
- To compensate want only real power

$$0 = X_L + X_C = j\omega L - j\frac{1}{\omega C}$$

- Thus

$$C = \frac{1}{\omega^2 L}$$

- actually often rate the capacitor in its reactive VAR's
- rated for specific frequency
- thus compensation is generally not perfect
- average power is the only thing that does real work
- thus compensation increases work that can be done



Example compensating for power factor

- What C compensation must be supplied for the motor
- of the previous example: R = 4 ohms coils

Inductance of 10 mH, what is power factor at 60 Hz, 120 V

- Impedance is

$$Z = R + j\omega L = 4 + j3.77 = 5.50\underline{/43.3^o}$$

- power factor is

$$pf = \cos(43.3^o) = 0.728$$

$$P_{avg} = V_{rms} I_{rms} pf = 2618 x 0.729 = 1.905\ KW$$

- To compensate

$$C = \frac{1}{\omega^2 L} = \frac{1}{377^2 x 0.01} = \frac{1}{1421} = 704\ \mu F$$

- note that adding C reduces the Z of device
- this makes more current available at given V
- would increase the real power
- thus increases the real work that can be done

Example compensating for power factor

- In practice would reduce the applied V
- and keep work done constant.
- eg. What C compensation must be supplied for the motor
- In terms of VAR's need

$$Q = V_{rms} I_{rms} \sin(\theta) = 2618 x \sin(43.3^o)$$

$$= 2618 x 0.689 = -1.795\ KVAR$$

- Typical commercial unit 1.5 Kvar thus compensation

$$Q_{comp} = 1795 - 1500 = 295\ var$$

- if keep work done by motor constant then real power remains constant. Thus as before

$$P_{avg:comp} = P_{avg:uncomp} = V_{rms} I_{rms} pf_{uncomp}$$

$$= 2618 x 0.729 = 1.905\ KW$$

$$\theta_{comp} = arctan\left[\frac{Q_{comp}}{P_{avg}}\right] = arctan\left[\frac{295}{1905}\right] = arctan(0.154) = 8.77^o$$

- thus the new power factor is

$$pf_{comp} = cos(\theta_{comp}) = cos(8.77^o) = 0.988$$

Problem with power meters

- Getting real power meters is difficult

- older moving coil meters average VI
two coils let IV interact
Mechanical effects of needle average signal
Problem with frequency effects

- new electronic meters:
- "True RMS" measure IV instantaneously
convert to digital, do numerical integral
and time average