KVL Example Resistor Voltage Divider

- Consider a series of resistors and a voltage source
- Then using KVL

$$V - V_1 - V_2 = 0$$

- Since by Ohm's law

$$V_1 = I_1 R_1 \quad V_2 = I_1 R_2$$

- Then

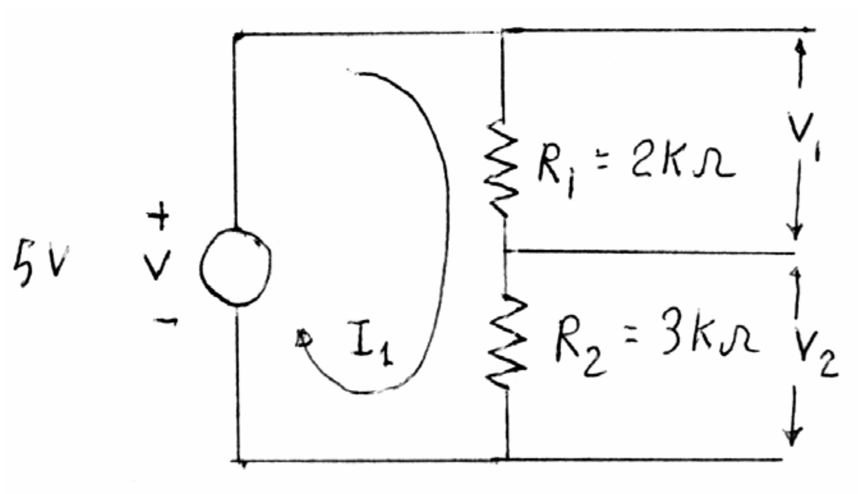
$$V - I_1 R_1 - I_1 R_2 = V - I_1 \left(R_1 + R_2 \right) = 0$$

- Thus

$$I_1 = \frac{V}{R_1 + R_2} = \frac{5}{2000 + 3000} = 1\, mA$$

- i.e. get the resistors in series formula

$$R_{total} = R_1 + R_2 = 5\ K\Omega$$



KVL Example Resistor Voltage Divider Continued

- What is the voltage across each resistor
- Now we can relate V_1 and V_2 to the applied V
- With the substitution

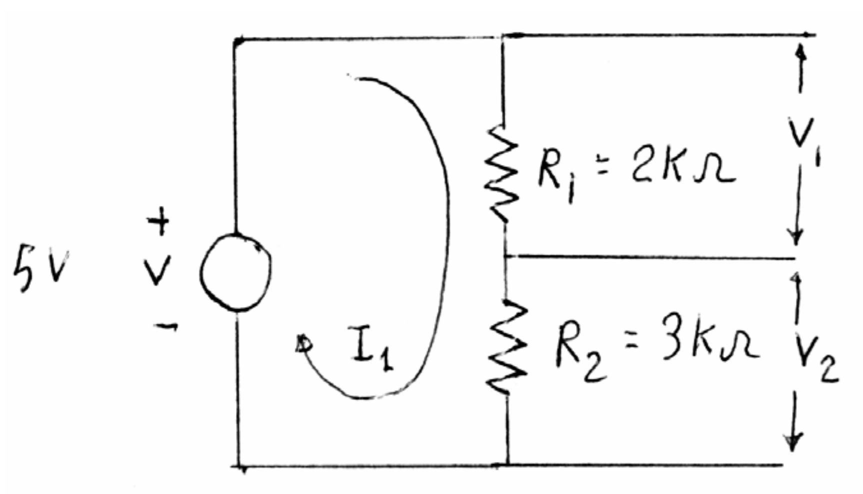
$$I_1 = \frac{V}{R_1 + R_2}$$

- Thus V_1

$$V_1 = I_1 R_1 = \frac{V R_1}{R_1 + R_2} = \frac{5(2000)}{2000 + 3000} = 2\,V$$

- Similarly for the V_2

$$V_1 = I_1 R_2 = \frac{V R_2}{R_1 + R_2} = \frac{5(3000)}{2000 + 3000} = 3\,V$$



General Resistor Voltage Divider

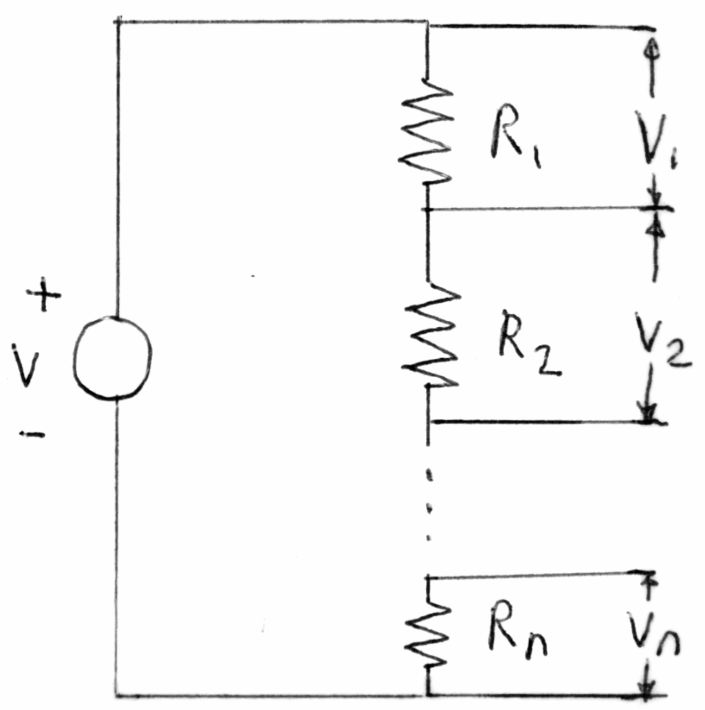
- Consider a long series of resistors and a voltage source
- Then using KVL or series resistance to get

$$V = I_1 \sum_{j=1}^{N} R_j \ldots or \ldots I_1 = \frac{V}{\sum_{j=1}^{N} R_j}$$

- The general voltage V_k across resistor R_k is

$$V_k = I_1 R_k = \frac{V R_k}{\sum_{j=1}^{N} R_j}$$

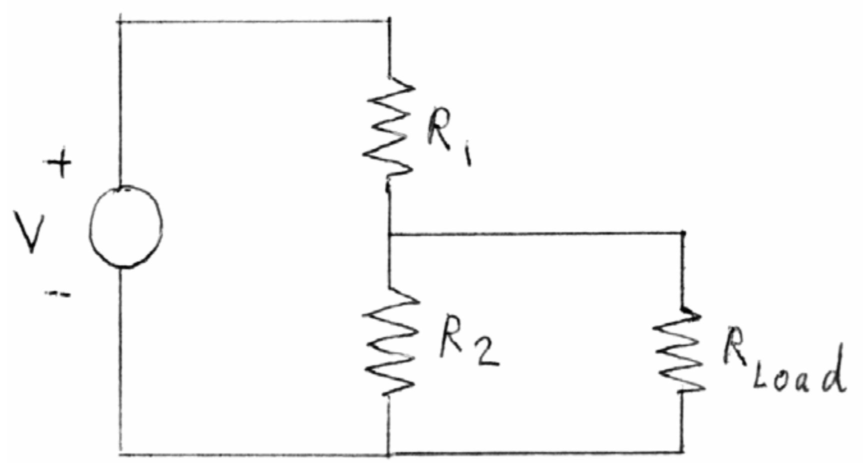
- Note important assumption: current is the same in all R_j



Usefulness of Resistor Voltage Divider

- A voltage divider can generate several voltages from a fixed source
- Common circuits (eg. IC's) have one supply voltage
- Use voltage dividers to create other values at low cost/complexity
- Eg. Need different supply voltages for many transistors

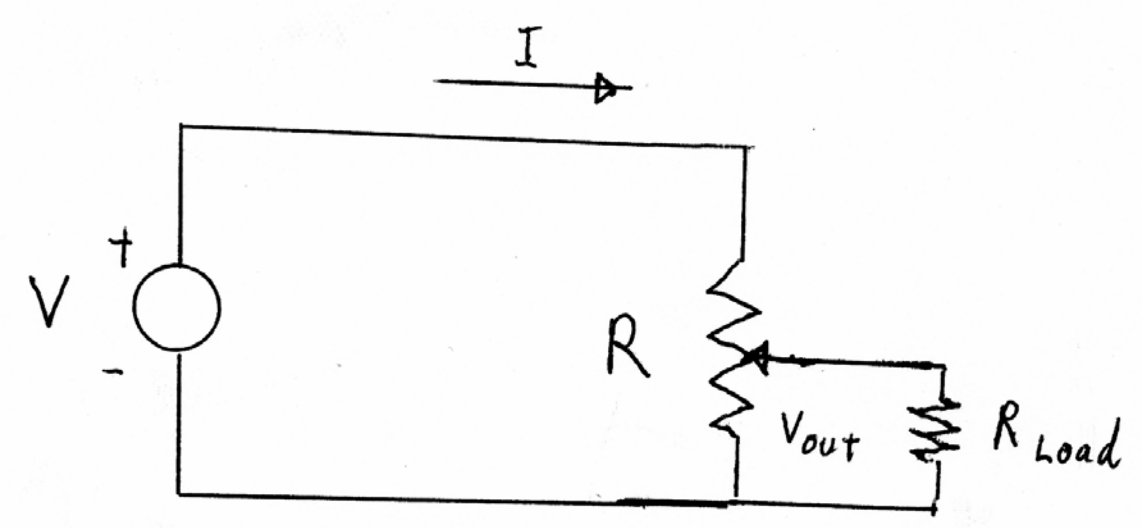
- Eg. Common computer outputs 5V (called TTL)
- But modern chips (CMOS) are lower voltage (eg. 2.5 or 1.8V)
- Quick interface – use a voltage divider on computer output
- Gives desired input to the chip



Variable Voltage and Resistor Voltage Divider

- If we have one fixed and one variable resistor (rheostat)
- Changing variable resistor controls output V_{out} across rheostat
- Simple power supplies use this

- Warning: ideally no additional loads can be applied
- Loads are current drawing devices
- In practice the load resistance >> the divider output resistor
- Best if R_{load} >100R



Current Divider: Example of KCL

- KCL equivalent of voltage divider is a current divider
- Consider a current source with resistors in parallel
- At node 1 the KCL laws state:

$$I - I_1 - I_2 = 0$$

- Define V_1 as the voltage between node 1 & node 0
- Then

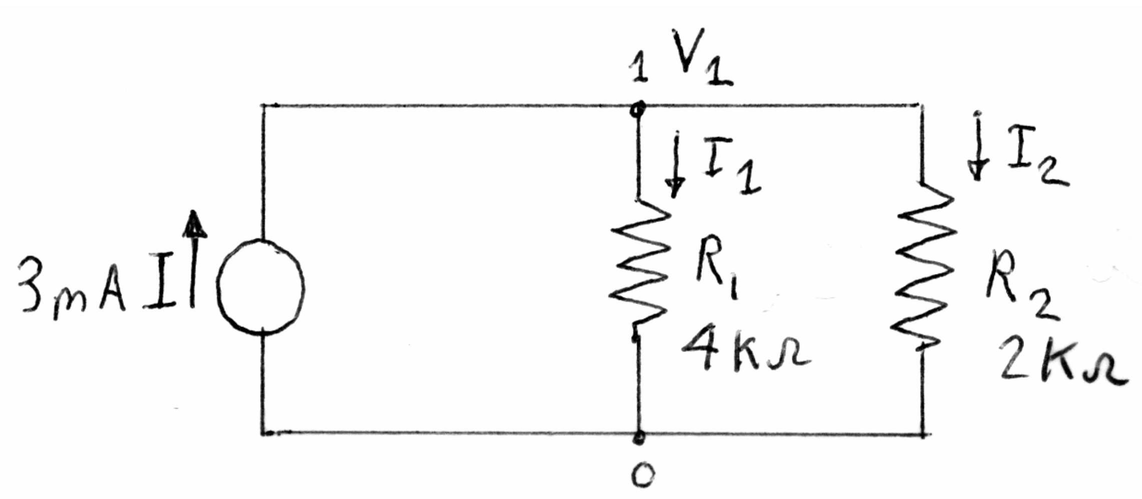
$$I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_1}{R_2}$$

- Thus from KCL

$$I = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_1}{R_2} = V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

- This produces the parallel resistors formula

$$I = V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{V_1}{R_{total}}$$



Current Divider Continued

- To get the currents through R_1 and R_2

$$I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_1}{R_2}$$

- First get the voltage from the KCL equation

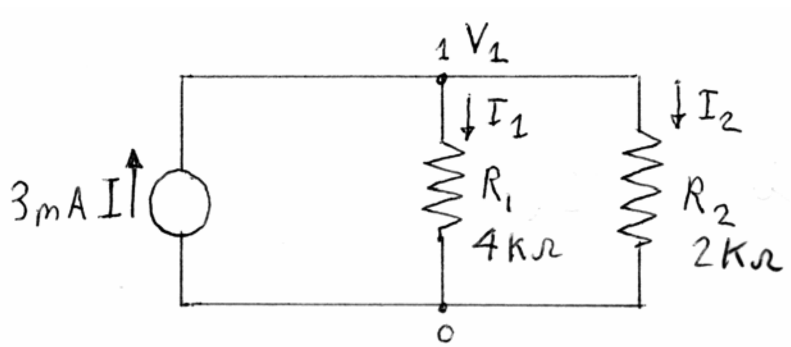
$$V_1 = I\left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1} = I\left[\frac{1}{R_{total}}\right]^{-1}$$

- Solving for I_1

$$I_1 = \frac{V_1}{R_1} = I\frac{\left[\frac{1}{R_1}\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

- Similarly solving for I_2

$$I_2 = \frac{V_1}{R_2} = I\frac{\left[\frac{1}{R_2}\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$



Example of Current Divider

- Consider 4KΩ and 2KΩ in parallel with a 3 mA current source
- The the current divider to obtain I_1

$$I_1 = \frac{V_1}{R_1} = I\frac{\left[\frac{1}{R_1}\right]}{\left[\frac{1}{R_1}+\frac{1}{R_2}\right]} = 0.003\frac{\left[\frac{1}{4000}\right]}{\left[\frac{1}{4000}+\frac{1}{2000}\right]} = 1\,mA$$

- Similarly for I_2

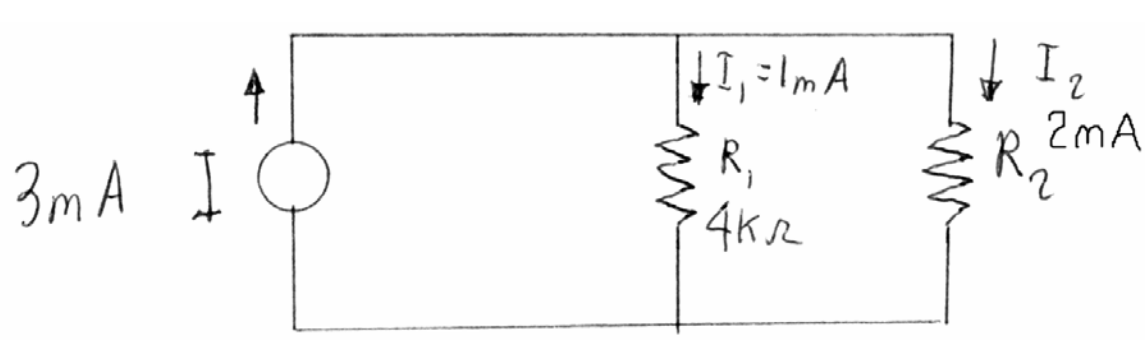
$$I_2 = \frac{V_1}{R_2} = I\frac{\left[\frac{1}{R_2}\right]}{\left[\frac{1}{R_1}+\frac{1}{R_2}\right]} = 0.003\frac{\left[\frac{1}{2000}\right]}{\left[\frac{1}{4000}+\frac{1}{2000}\right]} = 2\,mA$$

- Note the smaller resistor = larger current

- Checking: the voltage across the resistors

$$V_1 = I_1 R_1 = 0.001\times 4000 = 4\,V$$

$$V_1 = I_2 R_2 = 0.002\times 2000 = 4\,V$$



General Current Divider

- Using KCL to get the currents into the node

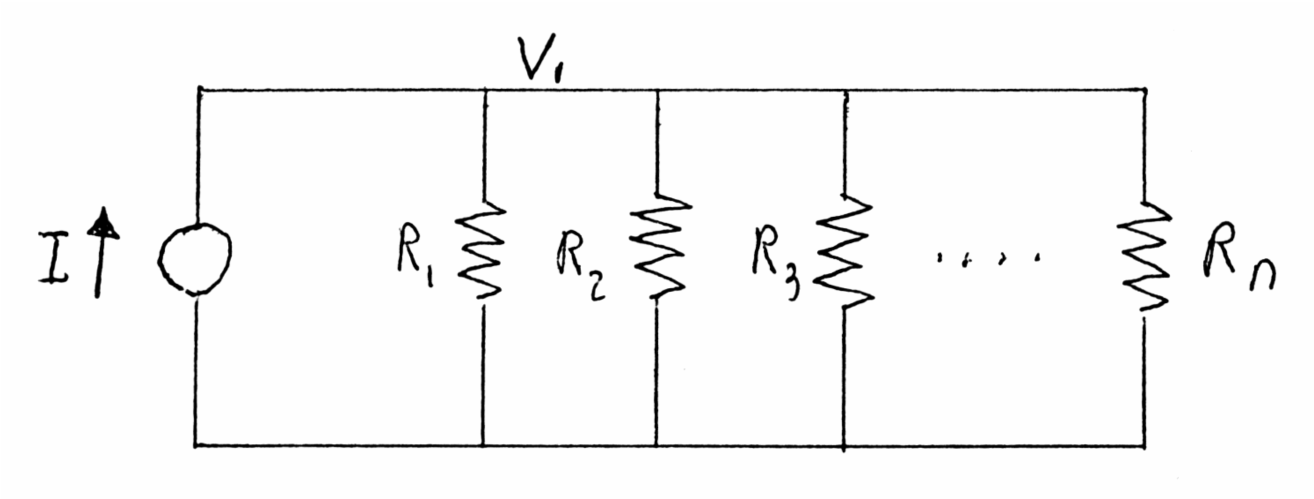
$$I = \sum_{j=1}^{n} I_j$$

- Getting the voltage from the KCL equation

$$V_1 = I\left[\sum_{j=1}^{N}\frac{1}{R_j}\right]^{-1} = I\left[\frac{1}{R_{total}}\right]^{-1}$$

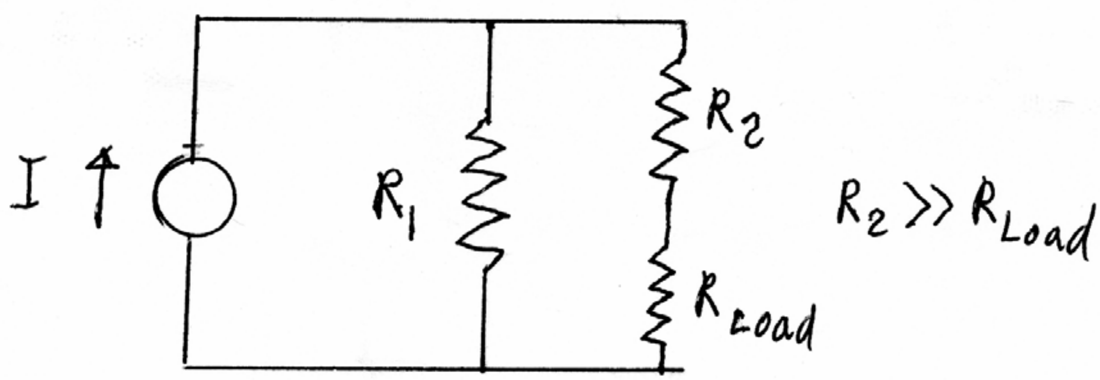
- Solving for current I_k with using the R_k

$$I_k = I\frac{\frac{1}{R_k}}{\left[\sum_{j=1}^{N}\frac{1}{R_j}\right]}$$



Practical Current Divider

- Create current dividers for use with current sources
- Less common than Voltage dividers as a circuit application
- Again any load used must not be significant
- Load in this case in series with the output resistor
- Load must be very small compared to R_2
- Best if load is << 0.01 of R_2



General Current Divider using Conductance

- Often better with parallel circuits to use conductance
- Again the KCL says at the node

$$I = \sum_{j=1}^{N} I_j$$

- Total conductance is resistors in parallel is

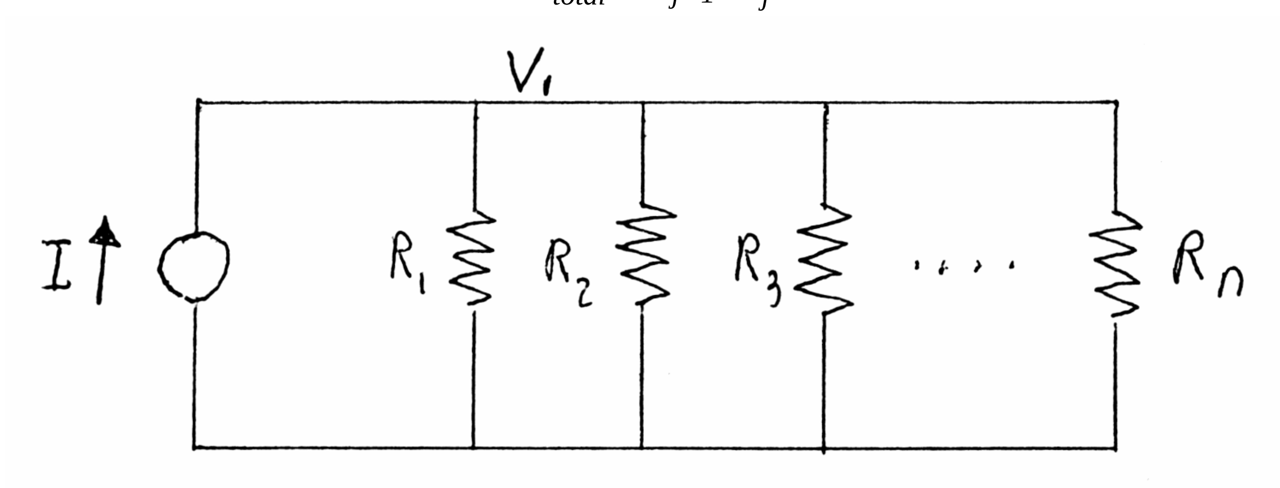
$$G_{total} = \sum_{j=1}^{N} G_j = \sum_{j=1}^{N} \frac{1}{R_j}$$

- The general current divider equation for I_k through resistor R_k

$$I_k = \frac{I G_k}{\sum_{j=1}^{N} G_j}$$

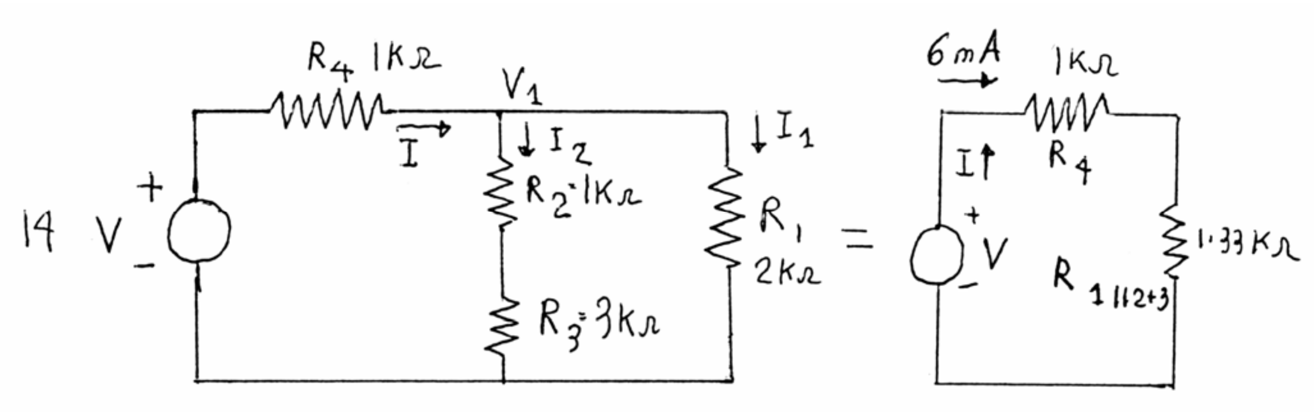
- Conductance calculations useful for parallel resistors
- Conductance equation for I is equivalent of voltage divider eqn
- Note for resistors in series the conductance is

$$\frac{1}{G_{total}} = \sum_{j=1}^{N} \frac{1}{G_j}$$



Solving Circuits with Equivalent Resistors

- Series and parallel resistor equivalents can solve some circuits
- Method, make equivalent resistance to simplify
- Go between series and parallel as needed
- Produce one final equivalent resistance
- Use voltage and current divider equations
- Get I & V for each element



Example Solving Circuits with Equivalent Resistors

- Consider circuit with R_2, R_3 in parallel R_1
- All in series with R_4
- For the R_2, R_3 side

$$R_{2+3} = R_2 + R_3 = 1000 + 3000 = 4000$$

- Now get the parallel equivalent

$$\frac{1}{R_{1||2+3}} = \frac{1}{R_1} + \frac{1}{R_{2+3}} = \frac{1}{2000} + \frac{1}{4000} = \frac{3}{4000}$$

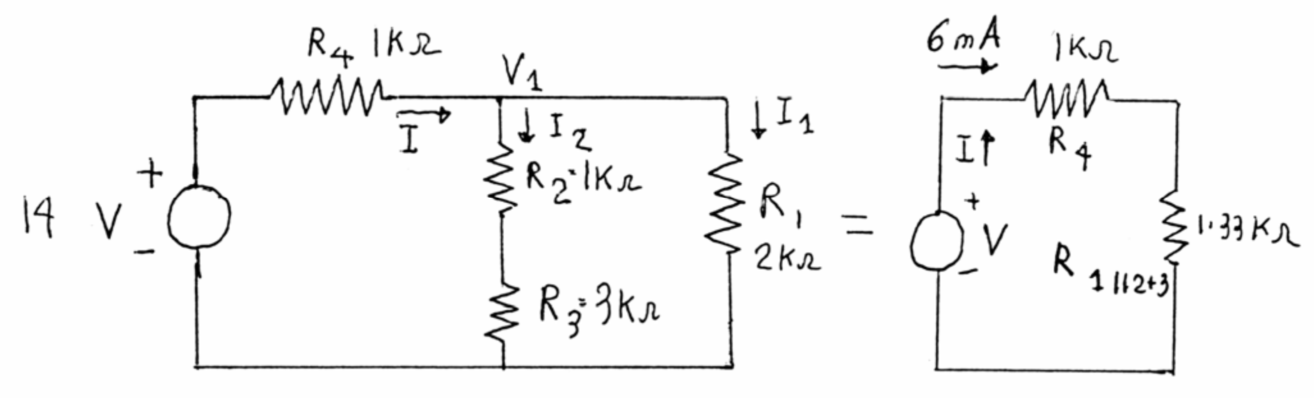
$$R_{1||2+3} = \frac{4000}{3} = 1333.3\,\Omega$$

- Adding the series resistance

$$R_{total} = R_4 + R_{1||2+3} = 1000 + 1333.3 = 2333.3\,\Omega$$

- Thus current from the source is

$$I_{total} = \frac{V}{R_{total}} = \frac{14}{2333.3} = 6\ mA$$



Example Circuits with Equivalent Resistors Continued

- Voltage across R_4 and parallel section is

$$V_{R4} = I_4 R_4 = 1000 \times 0.006 = 6\ V$$

$$V_1 = V - I_4 R_4 = 14 - 1000 \times 0.006 = 8\ V$$

- And the current in the parallel resistors

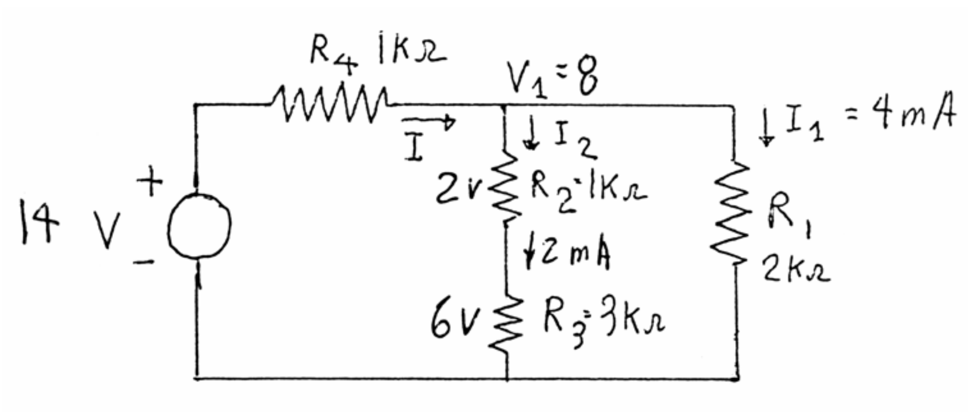
$$I_1 = \frac{V_1}{R_1} = \frac{8}{2000} = 4\ mA$$

$$I_2 = \frac{V_1}{R_{2+3}} = \frac{8}{4000} = 2\ mA$$

- Solving for the voltages

$$V_{R2} = I_2 R_2 = 0.002 \times 1000 = 2\ V$$

$$V_{R3} = I_2 R_3 = 0.002 \times 3000 = 6\ V$$



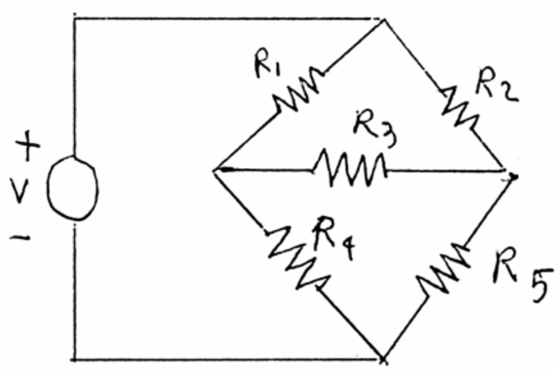
Advantages & Disadvantages: Equivalent Resistors Method

Advantages

- Simply guided by pattern of circuit
- Easy to understand

Disadvantages

- Can be quite time consuming
- Some circuits cannot be solved this way



Measuring Small Values: the Wheatstone Bridge

- Resistor dividers are set by ratios of resistance
- Thus can compare unknown R to a known set of R
- Called a Wheatstone Bridge
- Left side known resistance R_1 and variable resistor R_3
- Right side known R_2 and unknown R_s
- Place a very sensitive meter between the middle nodes
- Best is a galvonometer
- Voltages balance and no current i_g flows if

$$\frac{R_3}{R_1} = \frac{R_s}{R_2}$$

- If we know the R_1 R_2 R_3 very accurately can measure R_s accurately

$$R_s = \frac{R_3}{R_1} R_2$$

- Must use very accurate variable resistance

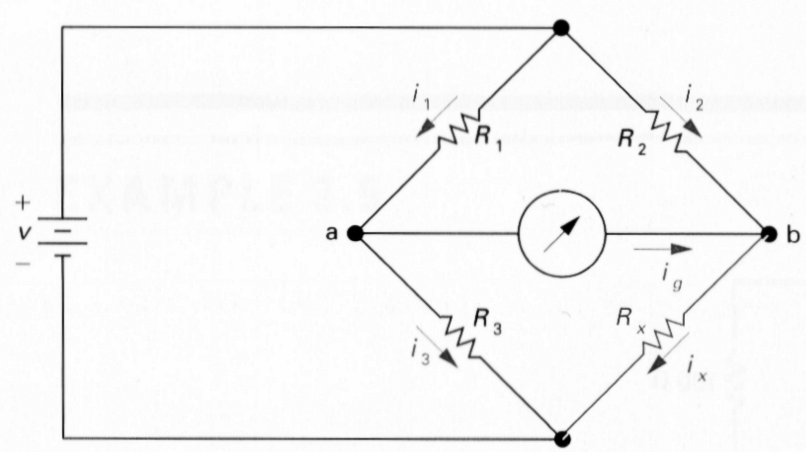


Figure 3.24 A balanced Wheatstone bridge ($i_g = 0$).

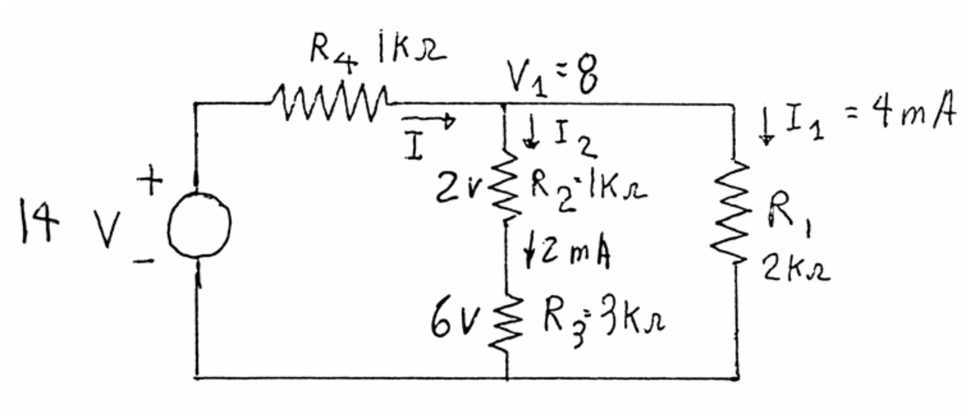
Circuit Analysis with Kirchhoff's Laws Circuits (EC 4)

- Task of Circuit analysis:
- Find the current through and the voltage across every element

Four methods used:

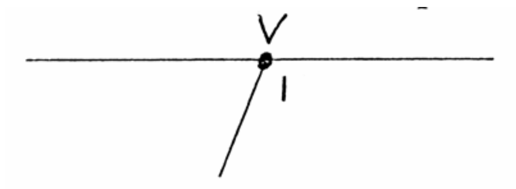
- Resistor substitution
- Mesh analysis (KVL)
- Node analysis (KCL)
- Superposition (simple circuits)

- Computer methods use aspects of these

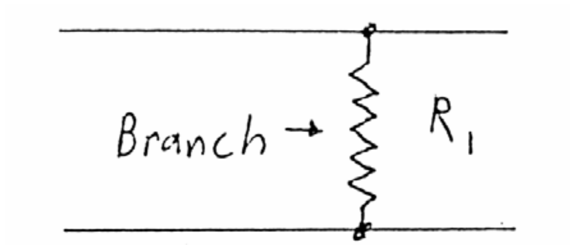


Circuit Definitions

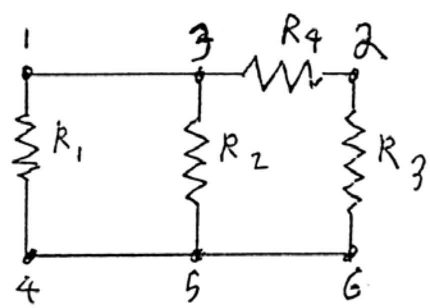
- Node: point where several current paths meet:

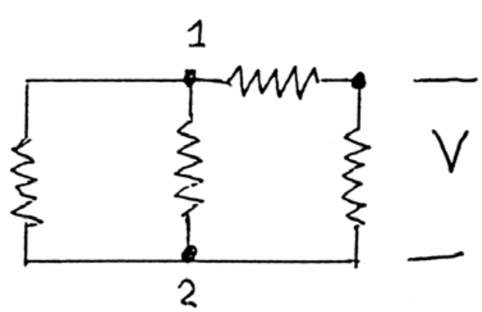


- Branch: a current path connecting only two nodes
- Branch contains 1 or more devices eg. resistors
- Note: a node may have many branch connections



- If 2 nodes are connected by a wire
- Then combine them into a single node

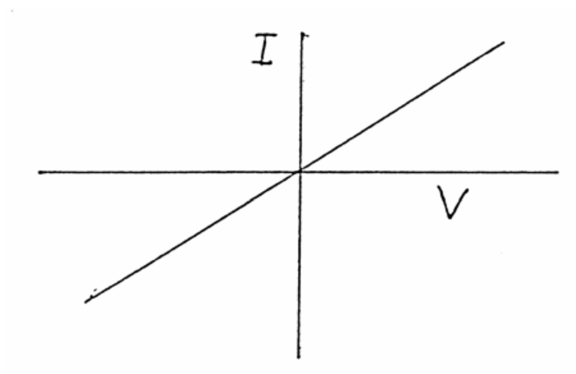




Linear & Nonlinear Circuit Elements

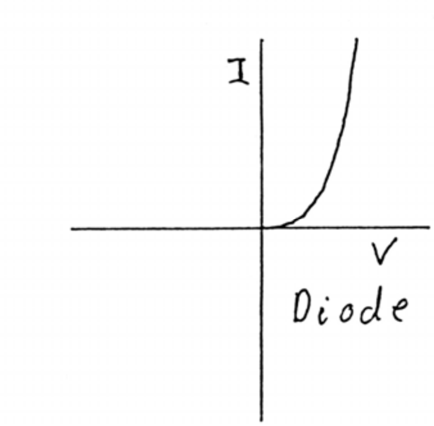
Linear devices

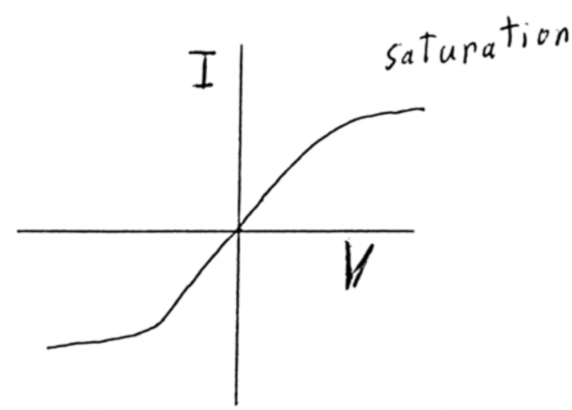
- Response is linear for the applied Voltage or Current
- eg Double voltage get twice the current
- eg devices: resistors, capacitors, inductors (coils)



Non-Linear devices

- Response is non-linear for applied Voltage or Current
- Eg. may have different response for different polarity of V
- Eg devices Semiconductor Diodes, iron core inductors





Kirchhoff's Laws and Complex Circuits

- Kirchoff's laws provide all the equations for a circuit
- But if know the currents then can calculate the voltages
- If know the voltages then can calculate the currents
- Thus only need to solve for one or the other.
- Use the other laws to obtain the missing quantity