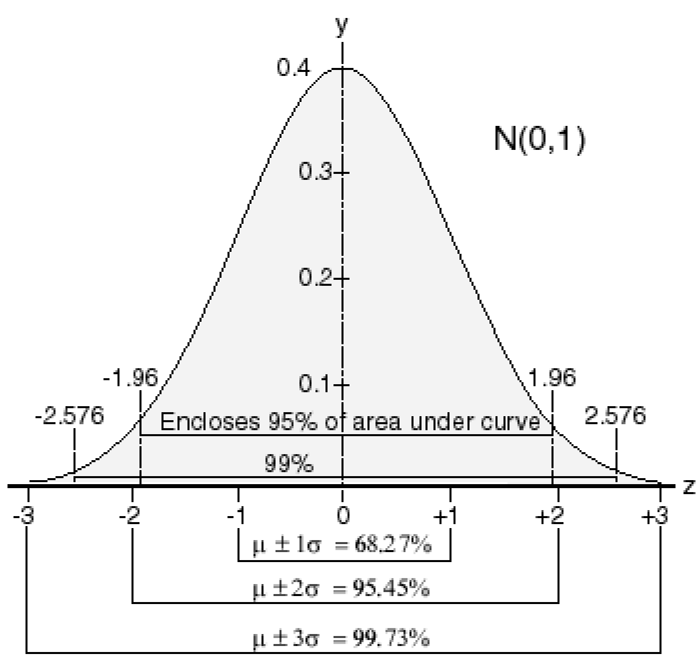
Measurement Errors and the Lab

- In every measurement must determine the accuracy or error range
- Typically give as $Y \pm \Delta y$
- Where Y is the measurement

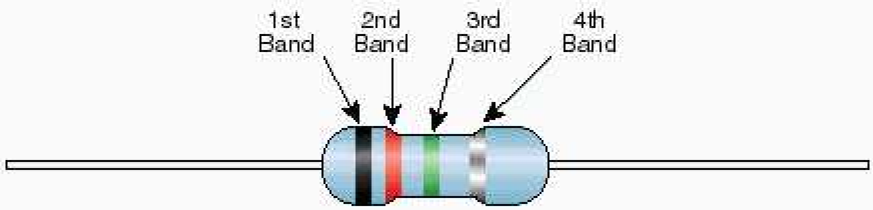
  Δy is the error on the measurement
- Typically follows a Gaussian distribution so
- $\pm\Delta y$ 68% of time
- $\pm 2\Delta y$ 95% of time
- $\pm 3\Delta y$ 99.7% of time
- Error is derived from the instruments or the measurement system
- In ENSC 220 lab 1:
- Error on resistors
- Measurement error from meters or instruments
- Use meter not power supply to measure V and I
- Must always check for the error for the instrument in its manual
- Try the web for the instrument specs in general



Resistors and Errors

- Resistors are marked for their error limits
- Precision determined by the 4th band
- Typically 5%, 10%, 20%
- 5% most common in lab
- Precision resistors 1% or 2%
- Resistors of all precision are all manufactured at same time
- Then just selected for precision
- Thus not really Gaussian distribution

Standard EIA Color Code Table **4 Band:** ±2%, ±5%, and ±10%



Color	1st Band (1st figure)	2nd Band (2nd figure)	3rd Band (multiplier)	4th Band (tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	±2%
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Gray	8	8	10^8	
White	9	9	10^9	
Gold			10^{-1}	±5%
Silver			10^{-2}	±10%

Worst Case Errors

- In engineering use Worst Case Error analysis
- Thus errors always add to do most damage
- Consider resistors of R_1=2.2K, and R_2=1.0K with 5% precision
- Then expected error on each is

$$\Delta R_1 = 2200 \times 0.05 = 110\ \Omega$$
$$R_1 = 2200 \pm 110\ \Omega$$
$$R_2 = 1000 \pm 50\ \Omega$$

- What if the resistors are in series?

$$R_{total} = R_1 + R_2 = 2200 + 1000 = 3200\ \Omega$$
$$\Delta R_{total} = \Delta R_1 + \Delta R_2 = 110 + 50 = 160\ \Omega$$

- Thus expect total resistance to be from 3040 to 3360Ω
- Could improve if you measure the values
- For division and multiplication add percentage errors

$$\frac{R_1}{R_2} = \frac{2200}{1000} = 2.2$$

$$\Delta \frac{R_1}{R_2} = \Delta\% R_1 + \Delta\% R_2 = 5\% + 5\% = 10\%$$

Digital MultiMeters and Accuracy

- Digital MultiMeter DMM accuracy is very dependent on the meter
- In lab we use a two meters: Fluke 45 and Fluke 8010A

Fluke 45 Autoranging meter

- Must look at the spec sheet for the accuracy in each range
- We have put the spec sheet in pdf form on the lab page
- Accuracy is in Appendix A

- Example of Accuracy from Appendix A

Range	Resolution			Accuracy	
	Slow	Medium	Fast	(6 Months)	(1 Year)
300 mV	—	10 µV	100 µV	0.002 % + 2	0.025 % + 2
3 V	—	100 µV	1 mV	0.02 % + 2	0.025 % + 2
30 V	—	1 mV	10 mV	0.02 % + 2	0.025 % + 2
300 V	—	10 mV	100 mV	0.02 % + 2	0.025 % + 2
1000 V	—	100 mV	1 V	0.02 % + 2	0.025 % + 2
100 mV	1 µV	—	—	0.02 % + 6	0.025 % + 6
1000 mV	10 µV	—	—	0.02 % + 6	0.025 % + 6
10 V	100 µV	—	—	0.02 % + 6	0.025 % + 6
100 V	1 mV	—	—	0.02 % + 6	0.025 % + 6
1000 V	10 mV	—	—	0.02 % + 6	0.025 % + 6

How to Get Reading Accuracy in a DMM

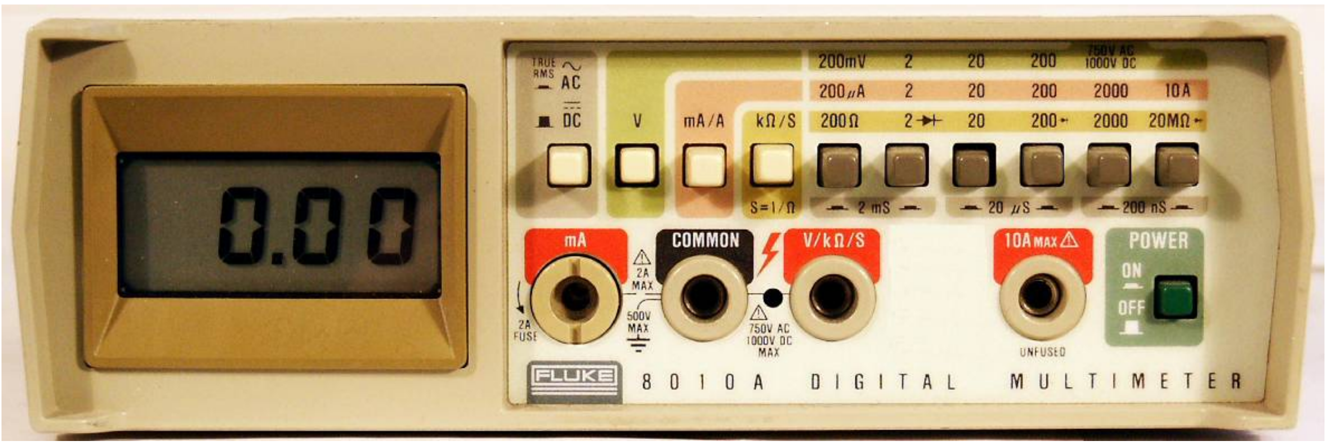
- Meter has certain number of displayed digits
- Accuracy depends on the range of the units measured
- Also number of digits displayed for your reading
- Typically has two parts
- A percentage of the actual reading eg 1%
- Percentage is overall accuracy of the internal resistors etc
- A count for the last digit
- Measures accuracy of the Digital to Analog converter
- The count is how many tenths of final digit will cause it to flip
- Eg. if meter accuracy was 1% + 2 in spec sheet
- Meter reading was reading 100 V
- Then accuracy due to last digit
- Digit is 0 but will stay 0 for 0.2 below to 0.2 above
- Thus from % expect a range of 99 to 101 V
- From last digit it is 98.8 to 101.2
- Some DMM are 3 ½ digits
- There error is % of reading ±0.5 of last digit value

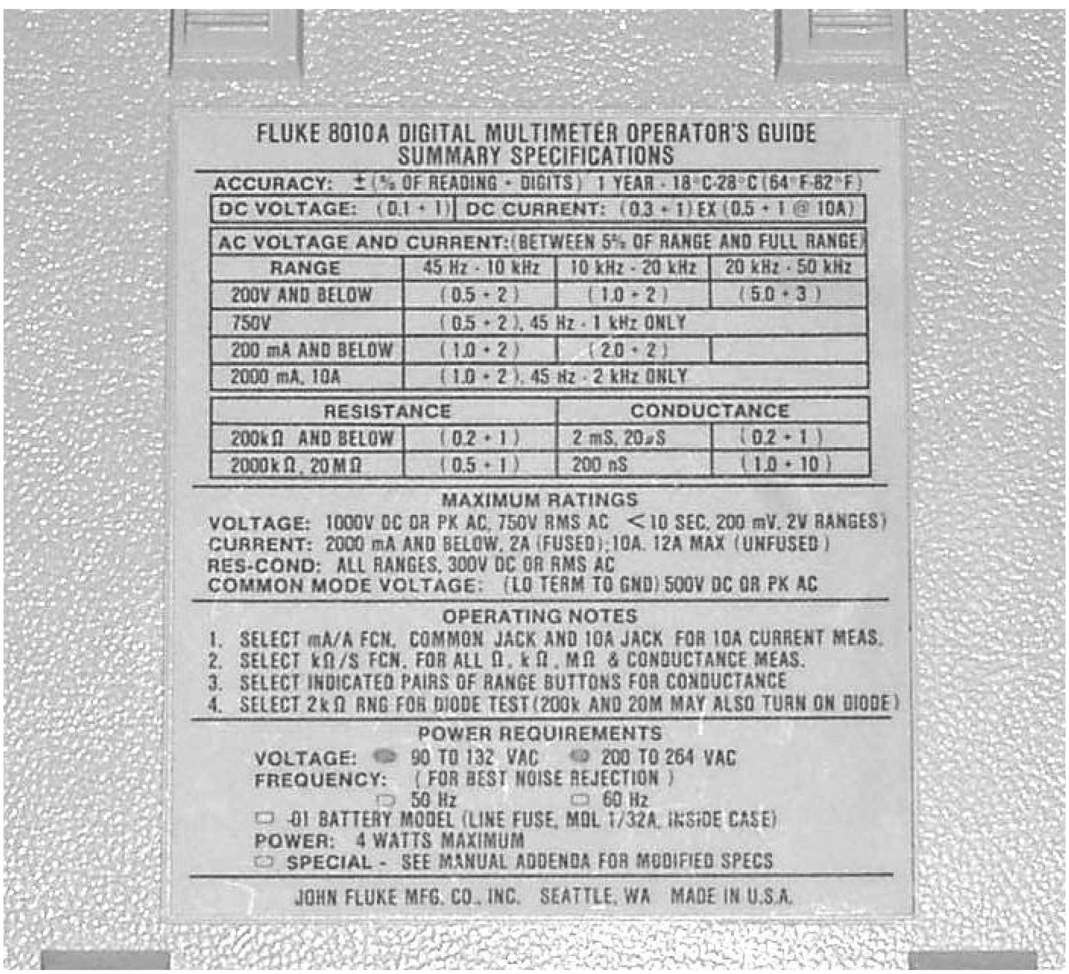
- Good reference

http://www.electroline.com.au/elc/feature_article/item_082005a.asp

Fluke 8010A DMM

- Fluke 8010 is 3 1/2 digit meter autoranging
- Error specifications to it
- Error is % plus ½ of last digit
- ie. ±0.5 of last digit value





Example of Errors in Experiments

- Consider a circuit with 2 resistors in series
- R_1=2.2K, and R_2=1.0K with 5% precision

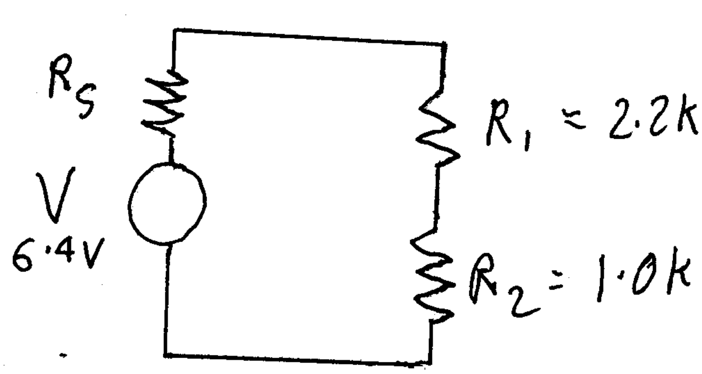
Apply a voltage source based on its meter of 3.2 V to it
Expect a current of

$$I = \frac{V}{R_1 + R_2} = \frac{3.2}{2200 + 1000} = 1\ mA$$

Voltages you get not those you expect
Assume the meters have 1% error & source ½ digit

	Vexpected	Vmeter	Vmax error	Vmin error
V	6.40	6.40	6.405	6.395
Rsource				
R_1	4.40	4.47	4.49	4.12
R_2	2.00	1.82	2.04	1.87
Vtotal	6.40	6.32	6.53	5.99
V_{R1}/V_{R2}	2.20	2.46	2.42	1.98

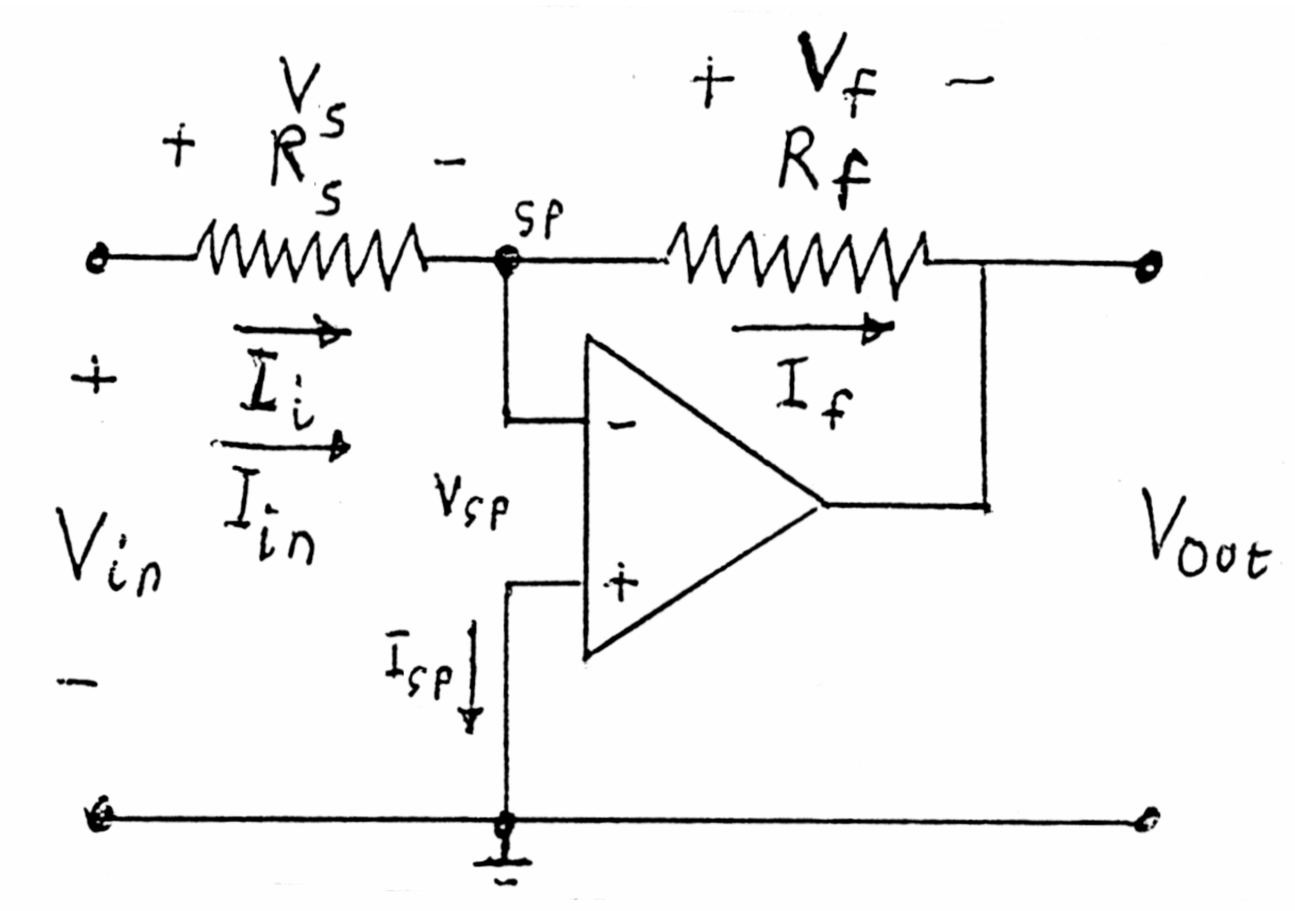
- Sources of error
- Resistance of source ~75Ω
- Errors in resistors 5%
- Errors in meters 1%



Inverting Op Amplifier

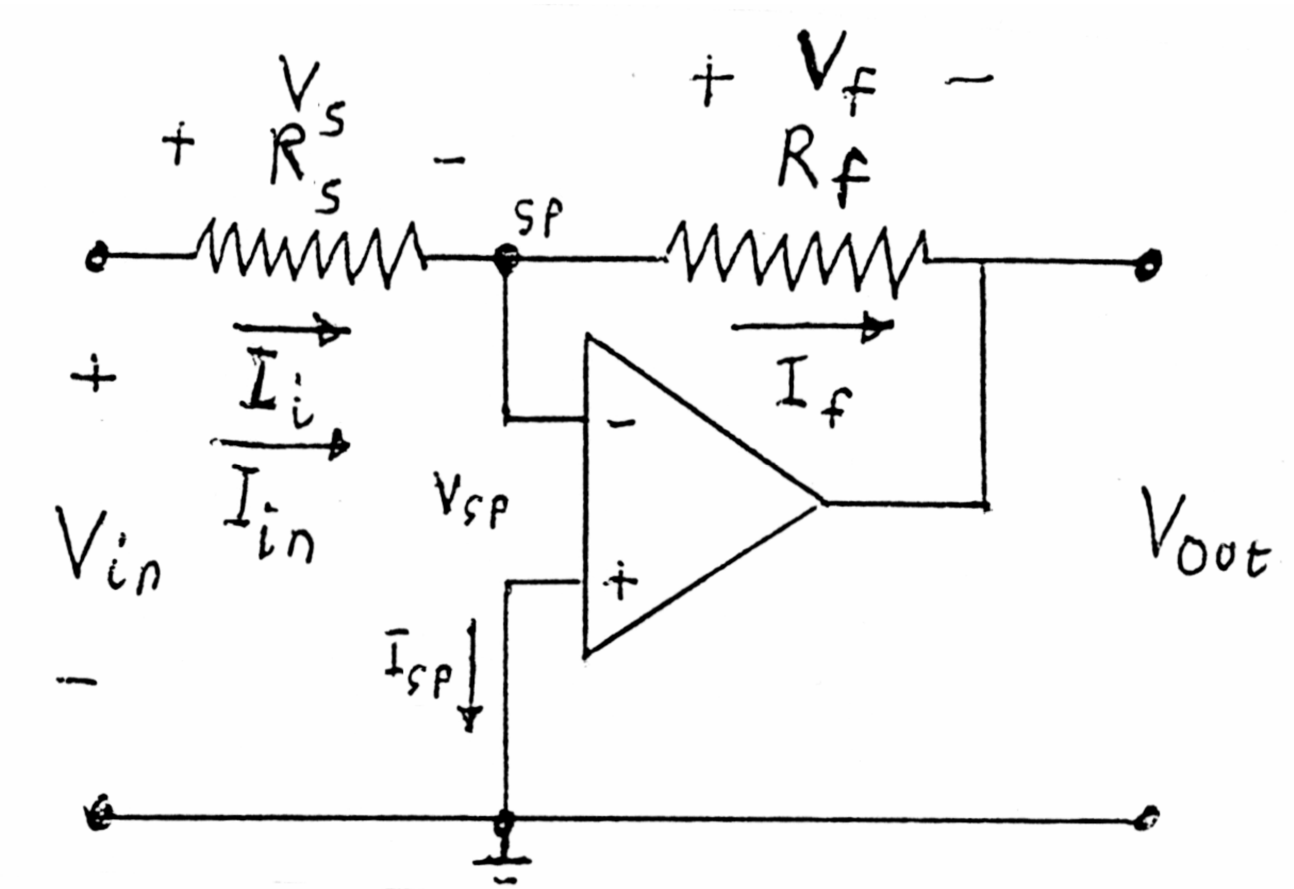
- Adding resistors to op amp can control the gain
- Gain controlled by "Feedback":
- Feeding input back into output
- This circuit gives negative gain:
- Called Inverting op amp

- SP = Summing Point, where output and input signals sum



Inverting Op Amplifier

- Place a feedback resistor R_f from op amp output to positive input
- The R_s between the summing point SP and input $V_s = V_{in}$
- Sometimes see SP called set point
- Allows current to flow from input and output to SP
- This circuit gives negative gain:
- Increases signal but makes it negative
- Called Inverting op amp



Inverting Op Amplifier

- Putting the input into v_n gives negative gain
- Output becomes the opposite polarity to input

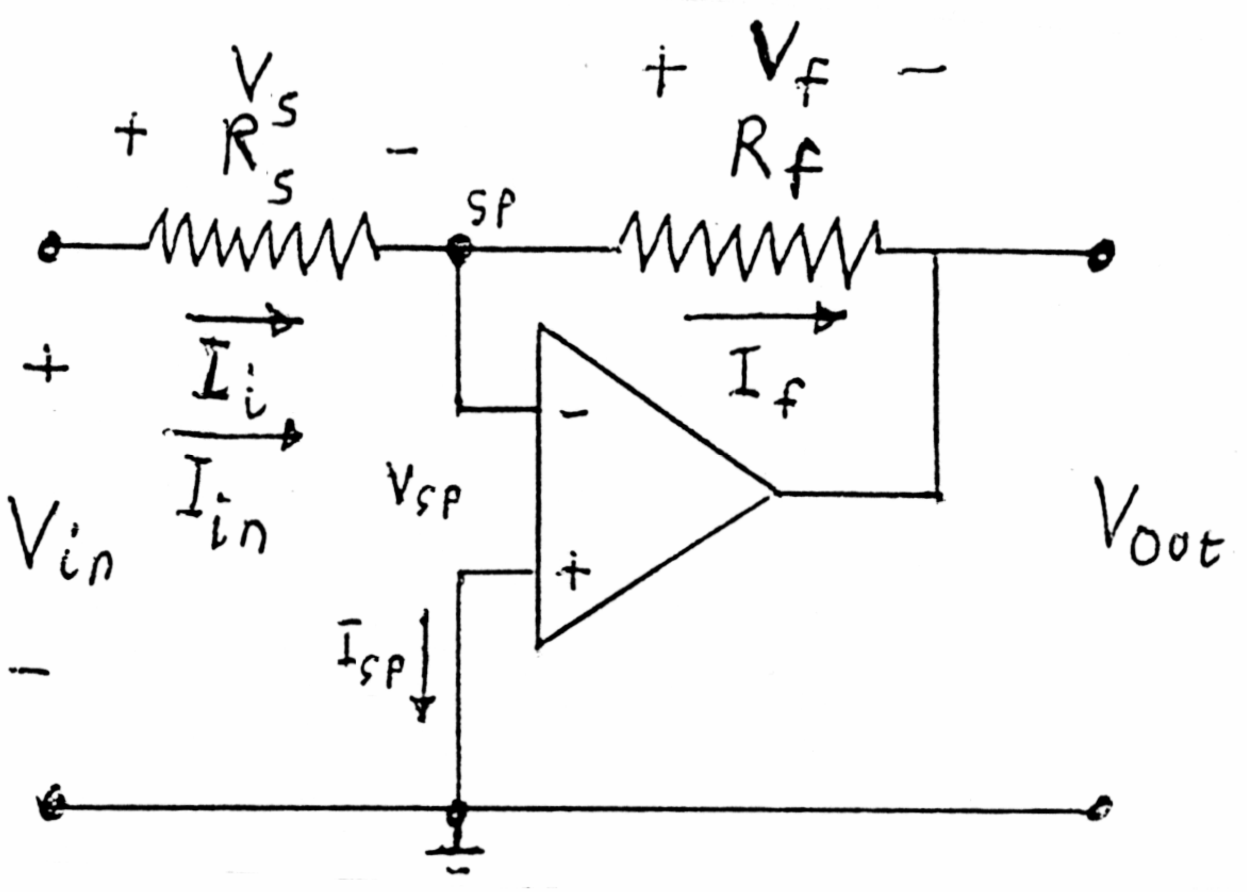
$$V_{sp} = 0 \qquad I_{sp} = 0$$

- Thus SP = a virtual ground
- In practice has small voltage offset

$$V_s = V_i = V_{in}$$

$$I_s = I_i = \frac{V_{in}}{R_s} = I_{in}$$

- Note I_f is not supplied by input current I_{sp}
- I_f is supplied by power supplies V_{cc} and V_{ee} of amp
- If load (eg. a resistor) is added to Vout
- The power supplies create that current also up to amp's I limit



Inverting Op Amplifier Gain Equations

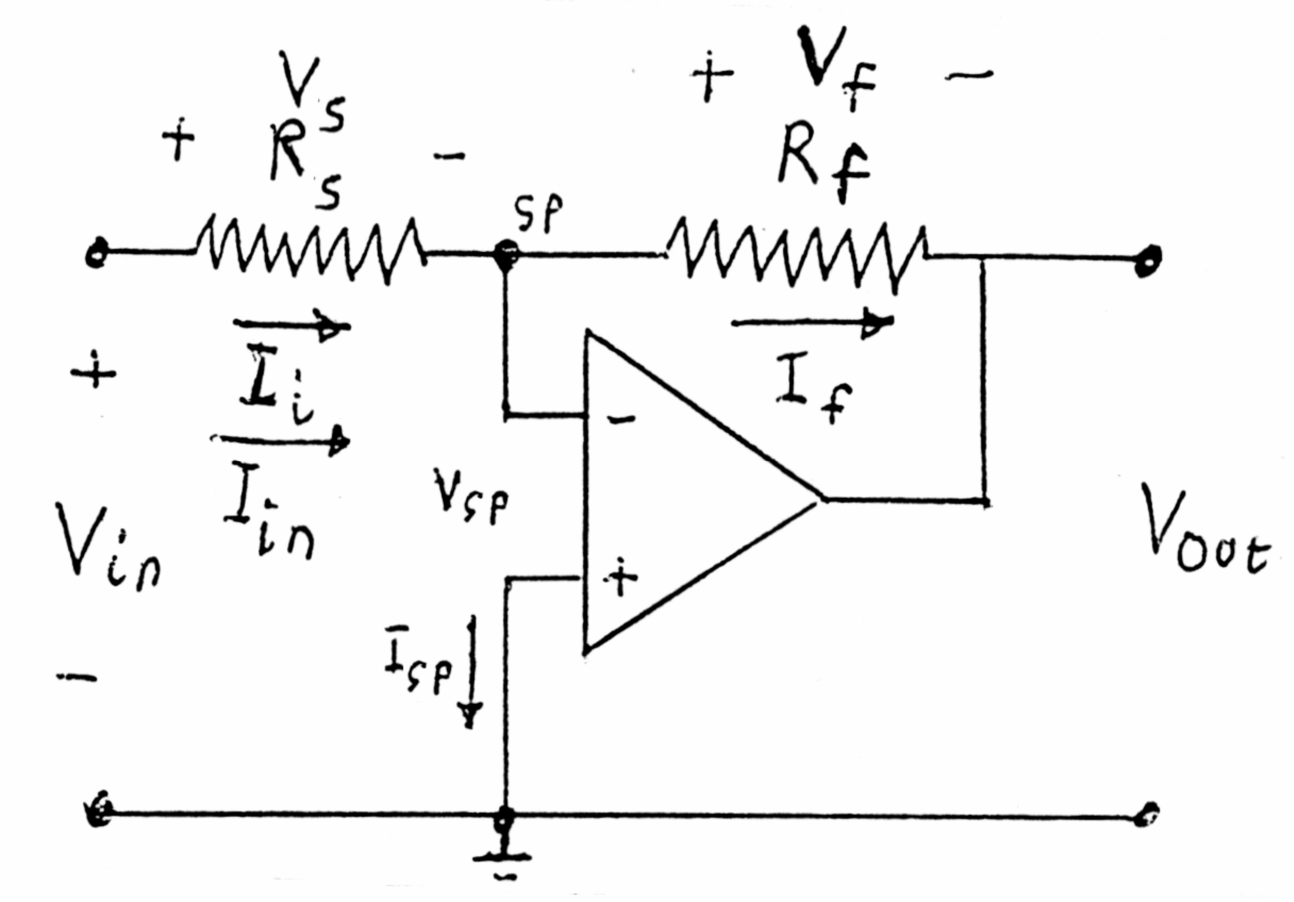
- Virtual ground requires V_f to reverse relative to $V_{in} = V_s$

$$I_s = I_f = \frac{V_s}{R_s} = \frac{V_{out}}{R_f}$$

$$V_{out} = V_f = I_f R_f = -V_s \frac{R_f}{R_s}$$

- The amplification (or gain) is:

$$A_v = \frac{V_o}{V_s} = -\frac{R_f}{R_s}$$



Example Inverting Op Amplifier

- For an inverting op am with

$$R_s = 1\ K\Omega \qquad R_f = 10\ K\Omega$$

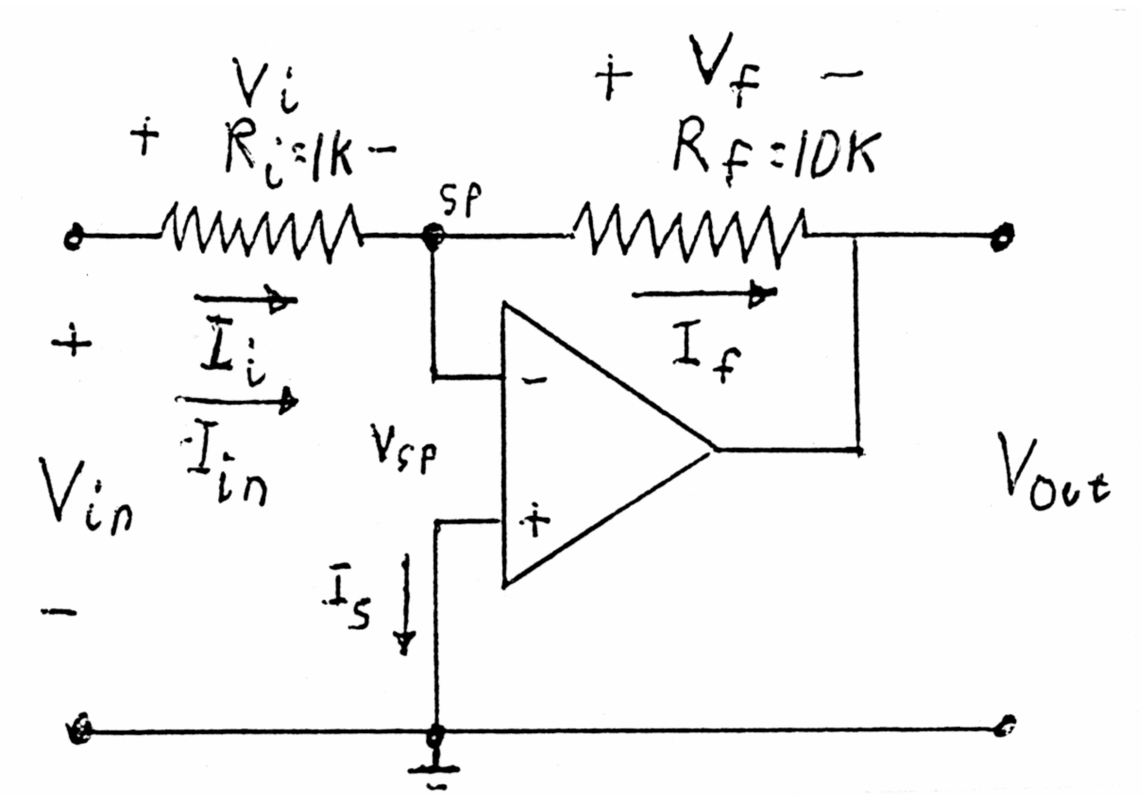
$$V_{CC} = +15\ V \qquad V_{EE} = -15\ V$$

- What is the output for a 0.5 V input?
- The voltage gain is:

$$A_v = \frac{V_o}{V_s} = -\frac{R_f}{R_s} = -\frac{10000}{1000} = -10$$

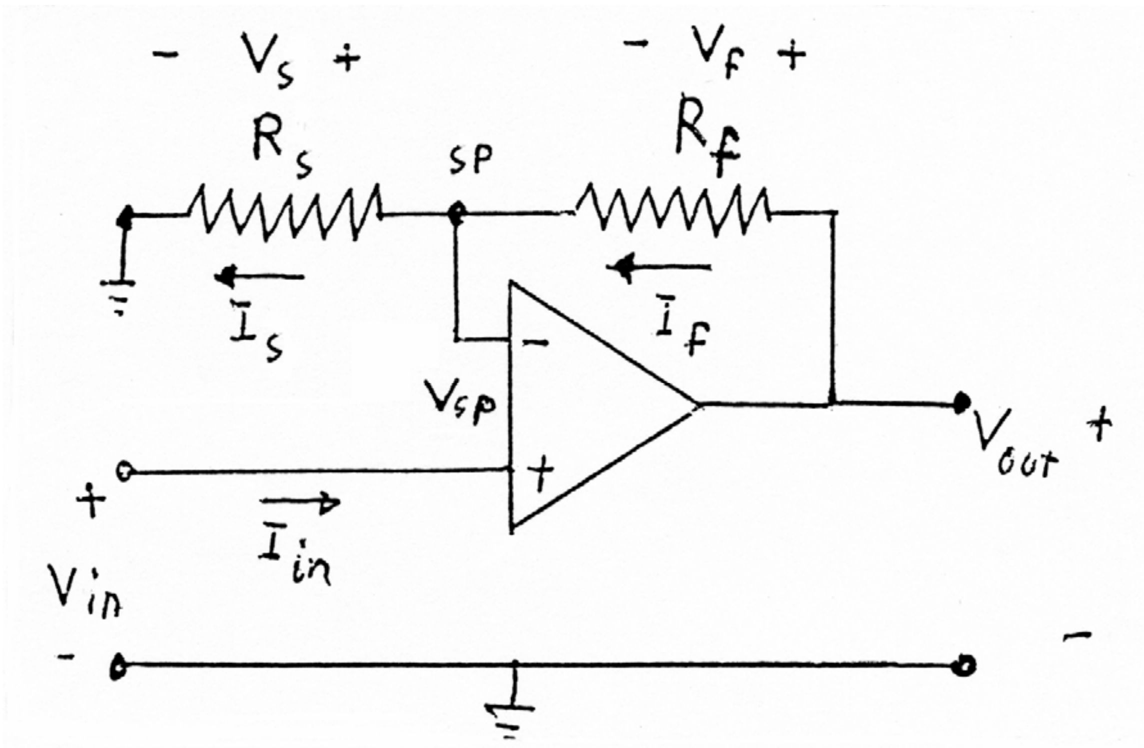
- Thus the output is:

$$V_o = A_v V_s = -10 \times 0.5 = -5\,V$$



Non-inverting Op Amplifier

- Place a feedback resistor R_f from op amp output to neg input
- The R_i between summing point and ground
- Allows current to flow from output to input
- Voltage divider R_f /R_s sets voltage at input
- This circuit gives positive gain
- Called Non-inverting op amp
- Note: text uses V_g for V_{in}
- Also shows a R_g on input which is not needed



Non-inverting Op Amplifier Gain

- Key point: Infinite op amp input resistance means no input current
- Thus voltage across the op amp input V_{sp} must be zero

$$I_{in} = 0 \qquad thus \qquad V_{sp} = 0$$

- Hence voltage at summing point sp must equal input voltage

$$V_s = V_{in}$$

- Since no input current to the op amp neg input

$$I_s = \frac{V_{in}}{R_s} \qquad I_s = I_f$$

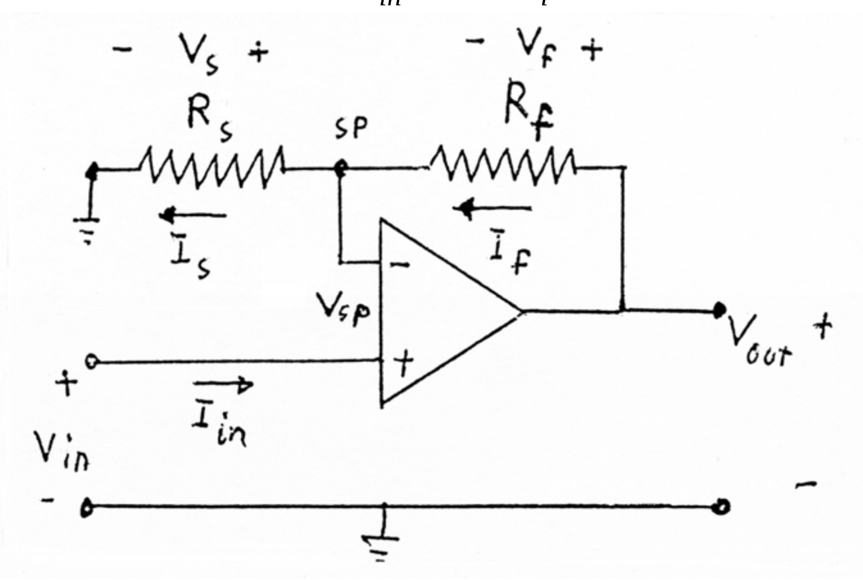
- Thus voltage across the feedback resistor becomes

$$V_f = I_f R_f = I_s R_f = V_{in} \frac{R_f}{R_s}$$

$$V_o = V_s + V_f = V_{in} \frac{R_i + R_f}{R_i}$$

- The voltage amplification (or gain) is:

$$A_v = \frac{V_o}{V_{in}} = \frac{R_i + R_f}{R_i}$$



Example Non-inverting Op Amplifier

- For an non-inverting op amp with

$$R_s = 1\ K\Omega \quad and \quad R_f = 9\ K\Omega$$
$$V_{CC} = +15\ V \qquad V_{EE} = -15$$

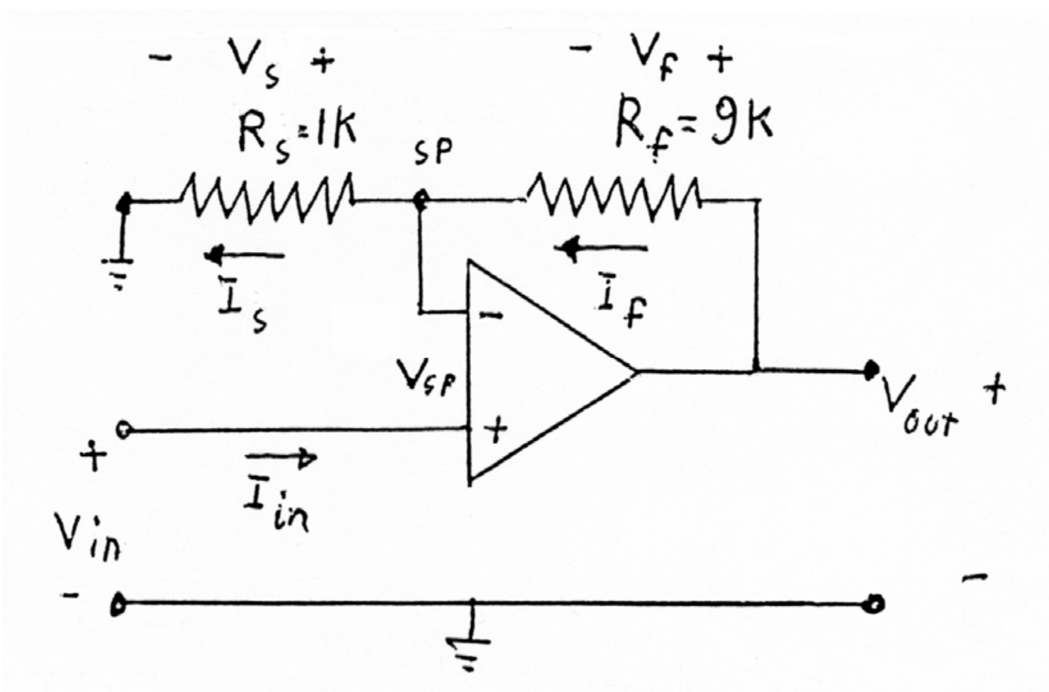
- What is the output for a 0.5 V input?
- The voltage gain is:

$$A_v = \frac{V_o}{V_{in}} = \frac{R_i + R_f}{R_i} = \frac{1000 + 9000}{1000} = 10$$

- Thus the output is:

$$V_o = V_{in}\frac{R_i + R_f}{R_i} = V_{in}A_v = 0.5 \times 10 = 5\,V$$

- NOTE: must keep output <power supply voltages - input limited
- In real circuits use resistors in Kilo-ohm range
- Thus reduce effects of smaller resistance eg. contacts



Summing Inverting Op Amplifier (EC 6.4)

- Using inverting op amps to combine many signals
- Have each input resistance connected to SP
- But only one feedback resistor
- Each signal can have different amplification
- Again the SP is a virtual ground

$$V_{sp} = 0 \qquad I_{sp} = 0$$

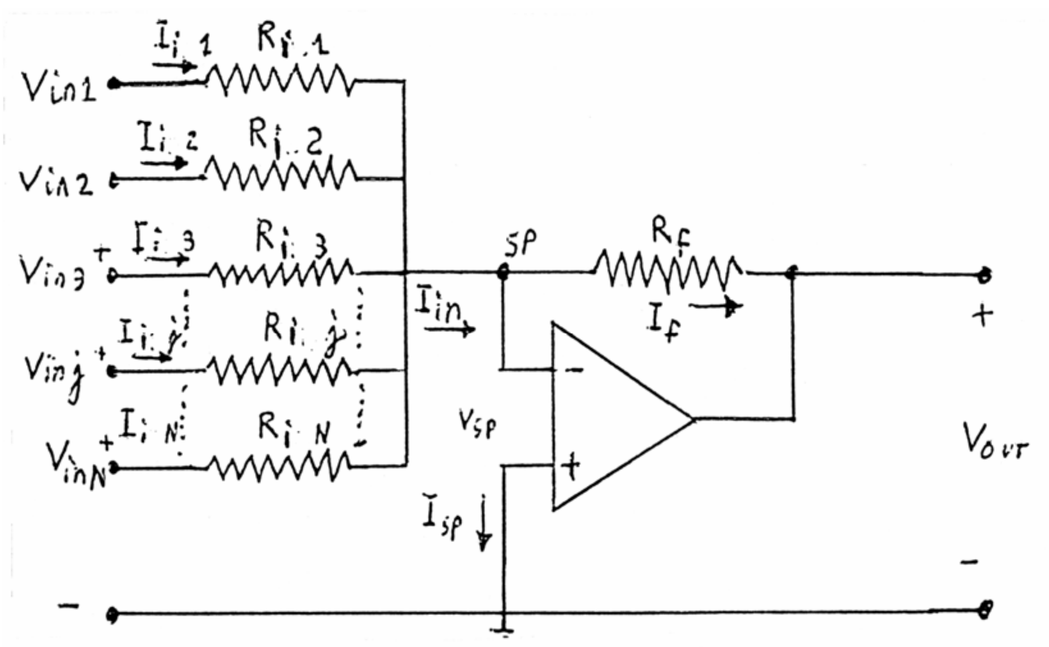
- Current from each input V_{sj} is

$$I_{sj} = \frac{V_{sj}}{R_{sj}}$$

- By KCL the total input current is

$$I_{s-total} = \sum_{j=1}^{N} I_{sj} = \sum_{j=1}^{N} \frac{V_{sj}}{R_{sj}}$$

- True because summing point a virtual ground
- Note each input is not affected by the other inputs



Summing Inverting Op Amp Gain

- Then by KCL the feedback current = input current

$$I_f = -\frac{V_f}{R_f} = I_{s-total} = \sum_{j=1}^{N} I_{sj} = \sum_{j=1}^{N} \frac{V_{sj}}{R_j}$$

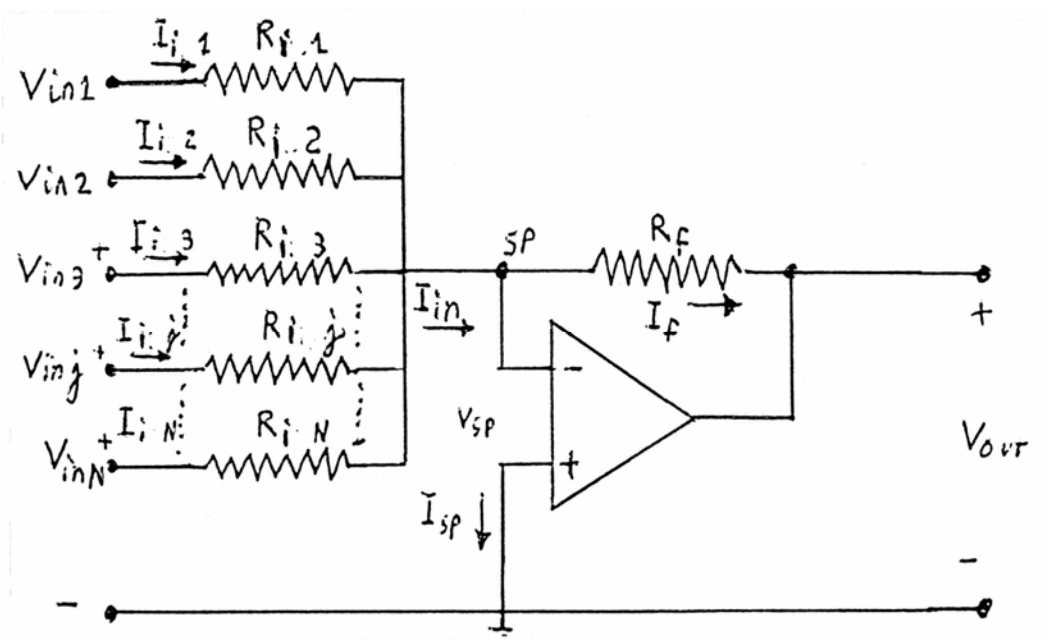
- Then in terms of output voltage

$$V_o = V_f = \sum_{j=1}^{N} -V_{sj} \frac{R_f}{R_j} = \sum_{j=1}^{N} -V_{sj} A_{vj}$$

- Note: V_o must not exceed V_{EE} or V_{CC}
- The amplification (or gain) per channel is:

$$A_{vj} = -\frac{R_f}{R_{sj}}$$

- Simple control systems use this summing input



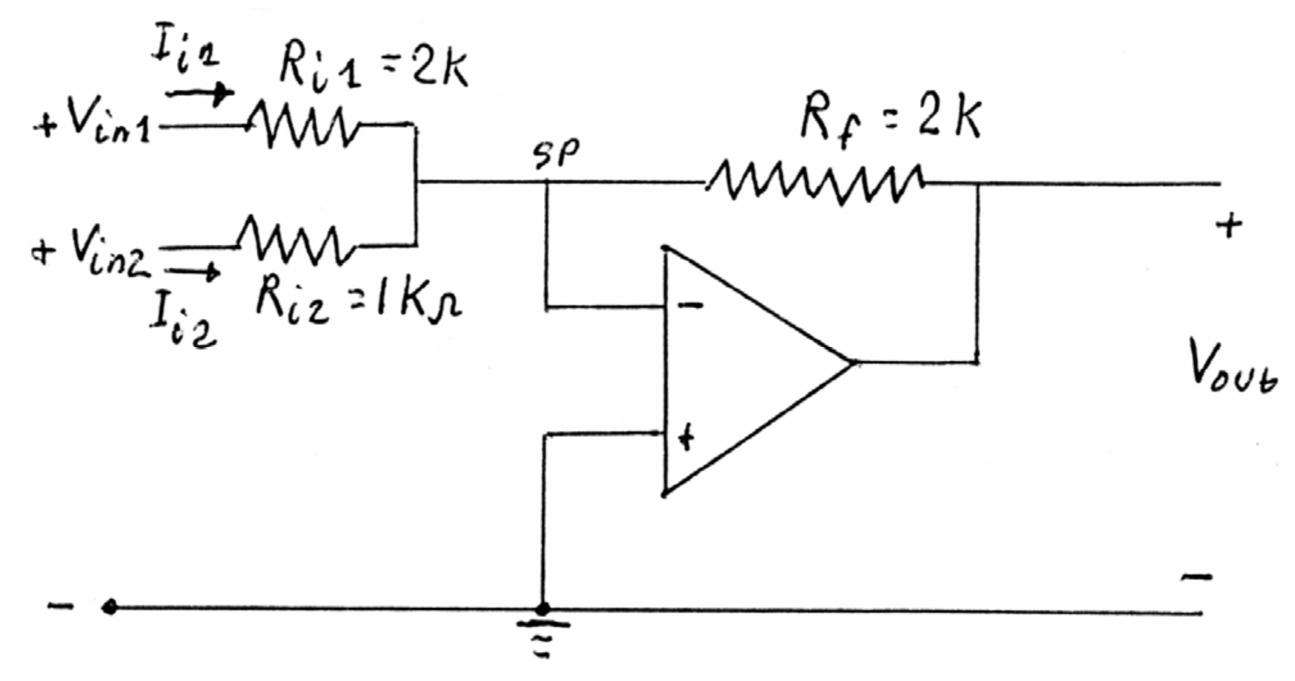
Example of Summing Op Amp

- Create an op amp circuit that will generate the sum equation

$$V_o = -V_1 - 2V_2$$

- To begin set R_f to the largest multiplication factor
- Ie.: times some common R for the design
- Here: largest gain A_{max} is for V_2 which is 2
- Now choose the minimum resistance: ie common R = 1 KΩ

$$R_f = A_{max} R_{min} = 2 \times 1000 = 2K\Omega$$



Example of Summing Op Amp Cont'd

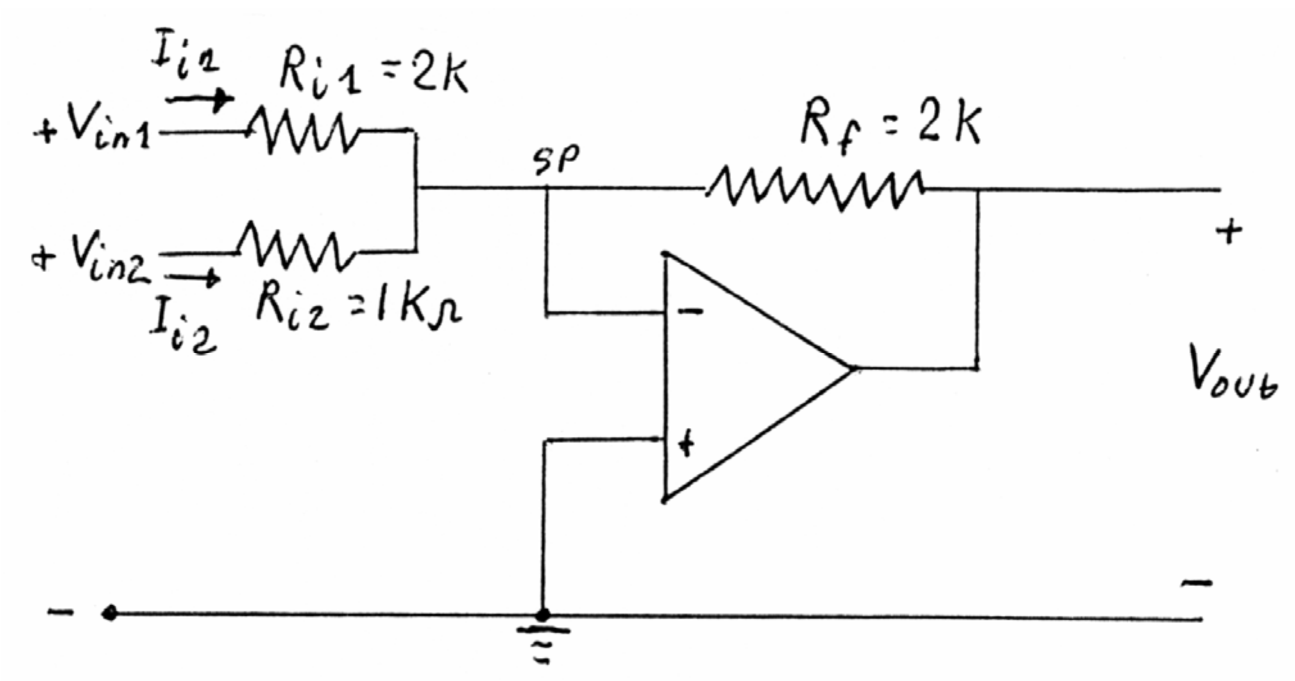
- For the input resistances
- Set the R_{sj} equal to common R_f times the multiplier for that input

$$R_j = \frac{R_f}{A_j}$$

- Thus for the example

$$R_{i1} = \frac{R_f}{A_1} = \frac{2000}{1} = 2\ K\Omega$$

$$R_{i2} = \frac{R_f}{A_2} = \frac{2000}{2} = 1\ K\Omega$$



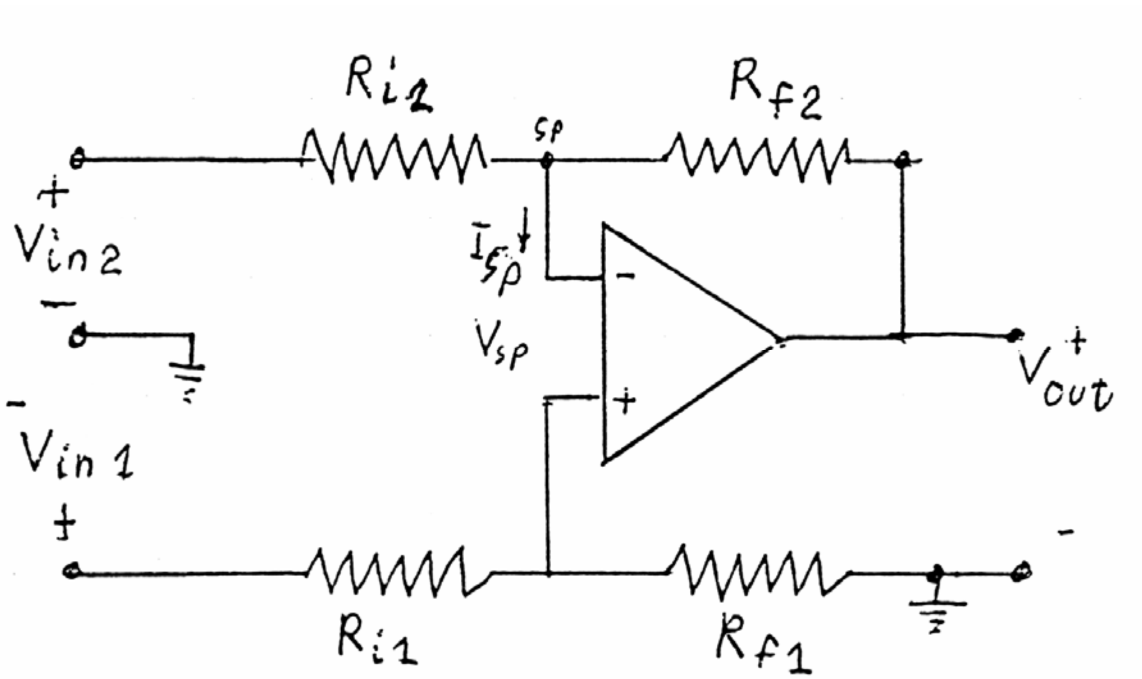
Differential (Difference) Op Amplifier

- Want to determine the difference between two voltages
- Often used as comparators (ie. is the output near zero)
- Note Ground is no longer part of the op amp input
- Put input resistors R_{i1} and R_{i2} on both inputs
- Arrangement like combining inverting & non-inverting op amp

$$R_{i1} = R_{i2} \qquad R_{f1} = R_{f2}$$
$$V_{sp} = 0 \qquad I_s = 0$$

- Consider input 2 side
- To measure V_{in2} only: ground V_{in1}
- This reduces to an inverting amplifier

$$V_{out2} = -V_{in2}\frac{R_{f2}}{R_{i2}}$$



Differential Op Amplifier Cont'd

- Consider input 1 side only
- For V_{in1} only; ground V_{in2}
- This reduces to non-inverting like amplifier but with changed input
- The effective non-inverting V_{in} is:

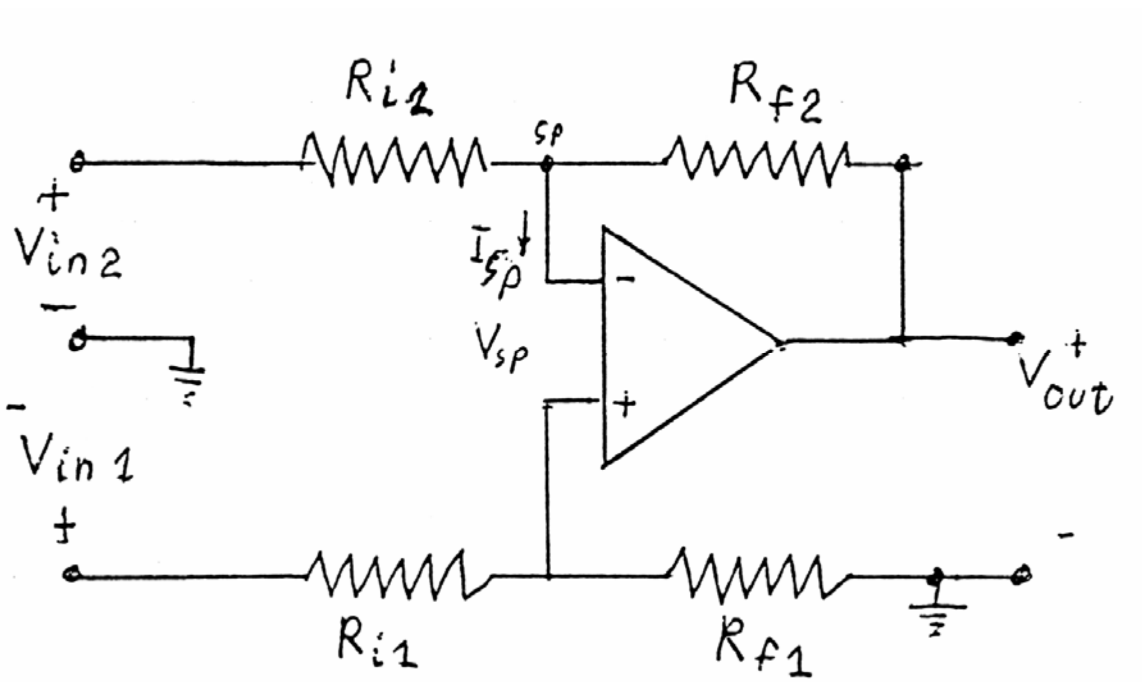
$$V_{in} = V_{in1} \frac{R_{f1}}{R_{i1} + R_{f2}}$$

- Thus

$$V_{out1} = V_{in}\left[\frac{R_{i2} + R_{f1}}{R_{i2}}\right] = V_{in1}\left[\frac{R_{f1}}{R_{i1} + R_{f2}}\right]\left[\frac{R_{i2} + R_{f1}}{R_{i2}}\right]$$

$$V_{out1} = V_{in1} \frac{R_{f1}}{R_{i1}}$$

- This reduces to a non-inverting amplifier
- But with input voltage divider on input



Differential Op Amplifier Cont'd

- The op amps are linear as long as output/input outside of saturation
- Now using superposition the total output is:

$$V_{out} = V_{out1} + V_{out2}$$
$$= V_{in1}\frac{R_{f1}}{R_{i1}} - V_{in2}\frac{R_{f2}}{R_{i2}}$$

- Since

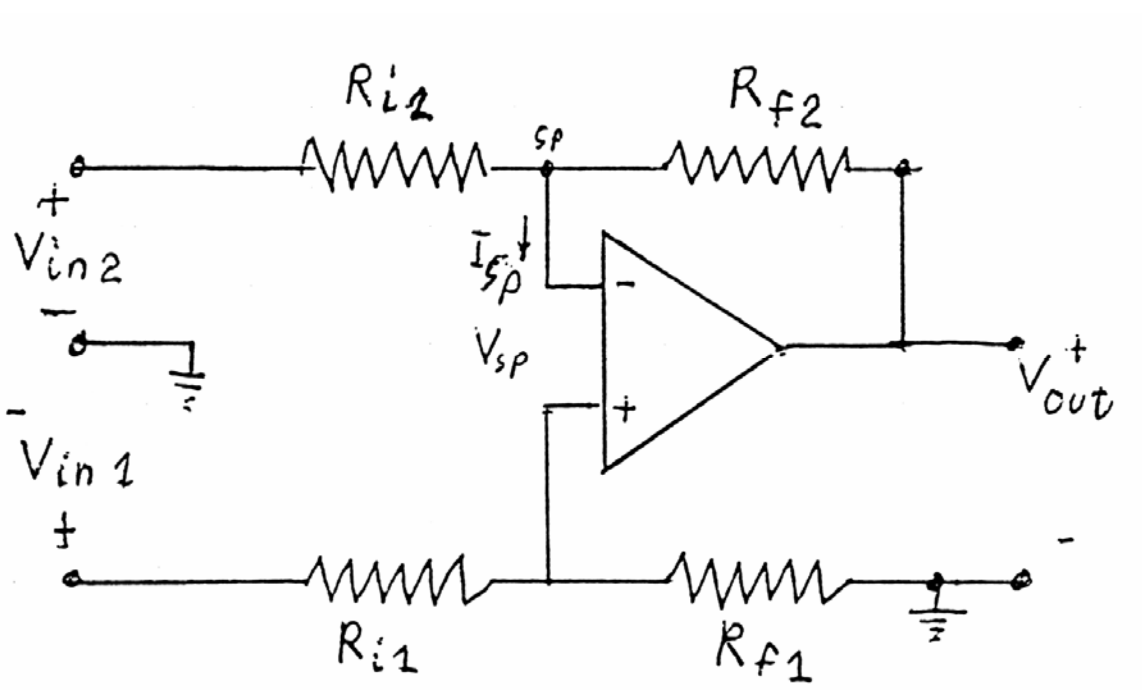
$$R_{i1} = R_{i2} \qquad R_{f1} = R_{f2}$$

- Thus

$$V_{out} = (V_{in1} - V_{in2})\left[\frac{R_{f1}}{R_{i1}}\right]$$

- Thus the amplification (gain) is:

$$A_v = \frac{V_{out}}{V_{in1} - V_{in2}} = \frac{R_{f1}}{R_{i1}}$$



Non ideal Op Amplifier

- The real op amps, like the 741, have
- Finite input resistance R_i ~ 2 MΩ
- Finite amplification A ~ 200,000
- Real out resistance R_o ~ 75 Ω
- Commonly will have an offset current as well

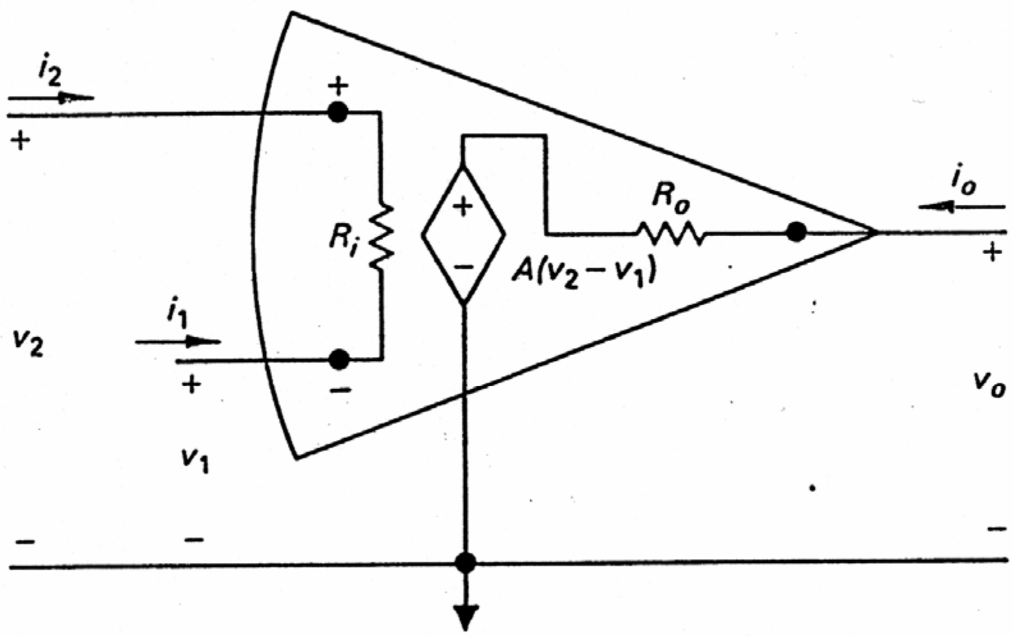


FIGURE 6.15 An equivalent circuit for an operational amplifier.