ENSC 220 Assignment 4 (Oct. 28, 2005, due Nov. 4, 2005)

1.

[a] $v_o = -\frac{220}{44}v_a - \frac{220}{27.5}v_b - \frac{220}{80}v_c = -5v_a - 8v_b - 2.75v_c$

$v_o = -5(1) - 8(1.5) - 2.75(-4)$

$v_o = -5 - 12 + 11 = -6\text{ V}$

[b] $v_o = 6 - 8v_b = \pm 10$

$\therefore\ v_b = -0.5\text{ V}$ when $v_o = 10\text{ V}$; $\quad v_b = 2.0\text{ V}$ when $v_o = -10\text{ V}$

$\therefore\ -0.5\text{ V} \le v_b \le 2.0\text{ V}$

2.

[a] Assume the op-amp is operating within its linear range, then

$$i_L = \frac{9}{5} = 1.8\text{ mA}$$

For $R_L = 4\text{ k}\Omega \qquad v_o = (4+5)(1.8) = 16.2\text{ V}$

Now since $v_o < 18\text{ V}$ our assumption of linear operation is correct, therefore

$$i_L = 1.8\text{ mA}$$

[b] $i_L = 1.8(5 + R_L)$

$5 + R_L = 10\text{ k}\Omega, \qquad R_L = 5\text{ k}\Omega$

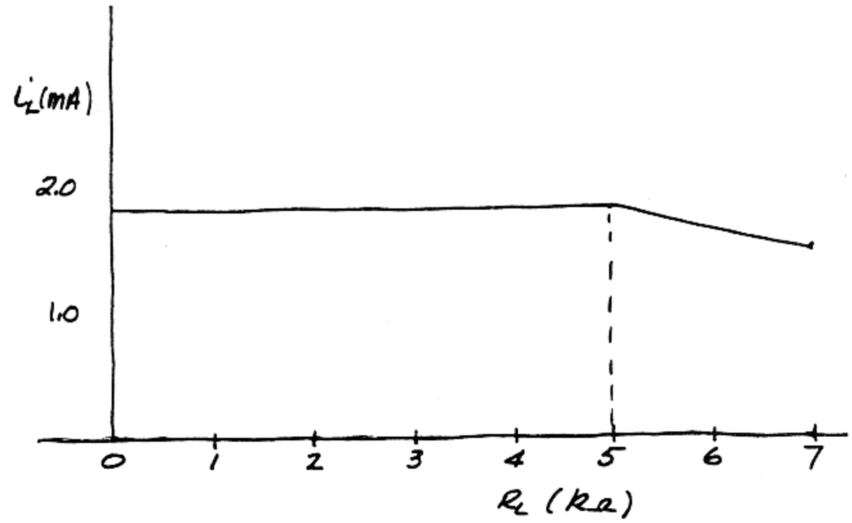
[c] Op-amp saturates for $R_L \ge 5\text{ k}\Omega$, $\qquad \therefore\ i_L = \frac{18}{5+7} = 1.5\text{ mA}$

If we assume $i_1 = i_2 \approx 0$, the voltage across the input terminals of the op-amp is

$$v_{21} \approx 9 - 5(1.5) = 1.5\text{ V}$$

If we assume $R_{in} = 500\text{ k}\Omega$, then $i_1 = -i_2 = -(1.5/500) = -3\,\mu\text{A}$, which is small compared to i_L.

[d]



3.

[a] $\dfrac{v_1}{R_g} + \dfrac{v_1 - v_o}{180} = 0$

$\therefore\; v_o = \left(1 + \dfrac{180}{R_g}\right) v_1 = Gv_1 = Gv_2$

$\dfrac{v_2 - v_a}{8} + \dfrac{v_2 - v_b}{2} + \dfrac{v_2 - v_c}{1} + \dfrac{v_2}{9} = 0$

$\therefore\; 125v_2 = 9v_a + 36v_b + 72v_c$

$\therefore\; v_o = \dfrac{9Gv_a}{125} + \dfrac{36Gv_b}{125} + \dfrac{72Gv_c}{125}$

It follows that $\dfrac{9G}{125} = 1.8, \qquad G = 0.2(125) = 25$

$\therefore\; 1 + \dfrac{180}{R_g} = 25, \qquad R_g = \dfrac{180}{24} = 7.5\,\text{k}\Omega$

[b] $v_o = 1.8(0.5) + 7.2(0.25) + 14.40(0.15) = 4.86\text{ V} = 194.40\,\text{mV}$

4.86 V

$\text{V2} = \text{V1} = \dfrac{4.86(7.5)}{187.5} = 194.4\,\text{mV}$

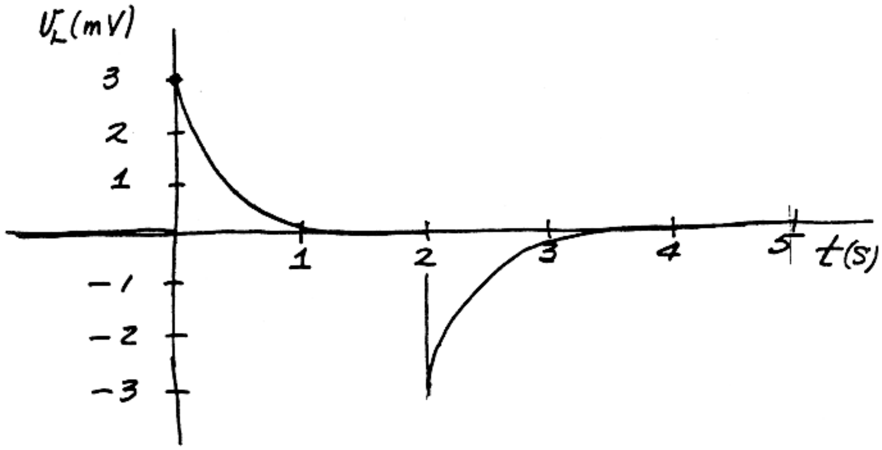
$i_a = \dfrac{500 - 194.4}{8} = 38.20\,\mu\text{A}$

$i_b = \dfrac{250 - 194.4}{2} = 27.80\,\mu\text{A}$

$i_c = \dfrac{150 - 194.4}{1} = -44.40\,\mu\text{A}$

$$i_g = \frac{194.40}{7.5} = 25.92\,\mu\text{A}$$
$$i_h = \frac{194.40}{9} = 21.60\,\mu\text{A}$$
$$i_o = -\frac{4860}{3.6} - 25.92$$
$$i_o = -1375.92\,\mu\text{A}$$

4.



$0 \le t \le 2\,\text{s}$

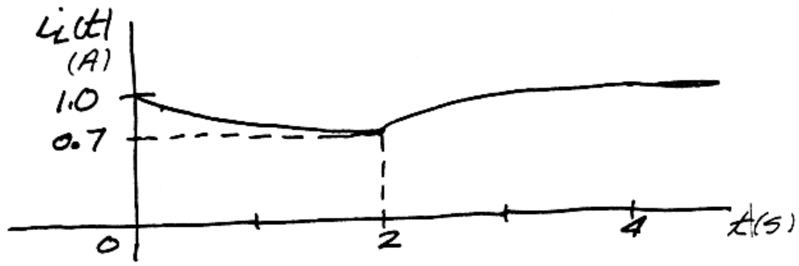
$$i_L = \frac{-10^3}{2.5}\int_0^t 3\times 10^{-3}e^{-4x}\,dx + 1 = -1.2\frac{e^{-4x}}{-4}\Bigg|_0^t + 1$$
$$= 0.3e^{-4t} + 0.7\,\text{A}, \qquad 0 \le t \le 2\,\text{s}$$

$2\,\text{s} \le t \le \infty$

$$i_L = \frac{-10^3}{2.5}\int_2^t -3\times 10^{-3}e^{-4(x-2)}\,dx + 0.7$$

Note: $i_L(2) = 0.3e^{-8} + 0.7 \approx 0.7\,\text{A}$

$$i_L = 1.2\left[\frac{e^{-4(x-2)}}{-4}\Bigg|_2^t + 0.7 = -0.3\left[e^{-4(t-2)} - 1\right] + 0.7\right.$$
$$= -0.3e^{-4(t-2)} + 1.0\,\text{A}, \qquad 2\,\text{s} \le t \le \infty$$



5.

[a] $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.2) \times 10^{-6}(150)^2 = 2250 \times 10^{-6} = 2.25\,\text{mJ}$

[b] $v = (A_1 t + A_2)e^{-5000t}$

$v(0) = A_2 = 150\,\text{V}$

$$\frac{dv}{dt} = -5000e^{-5000t}(A_1 t + A_2) + e^{-5000t}(A_1)$$

$$= (-5000A_1 t - 5000A_2 + A_1)e^{-5000t}$$

$$\frac{dv}{dt}(0) = A_1 - 5000A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{250 \times 10^{-3}}{0.2 \times 10^{-6}} = 1250 \times 10^3$$

$$\therefore 1.25 \times 10^6 = A_1 - 5000(150)$$

$$A_1 = 1.25 \times 10^6 + 75 \times 10^4 = 2.0 \times 10^6\,\frac{\text{V}}{\text{s}}$$

[c] $v = (2 \times 10^6 t + 150)e^{-5000t}$

$$i = C\frac{dv}{dt} = 0.2 \times 10^{-6}\frac{d}{dt}(2 \times 10^6 t + 150)e^{-5000t}$$

$$i = \frac{d}{dt}\left[(0.4t + 30 \times 10^{-6})e^{-5000t}\right]$$

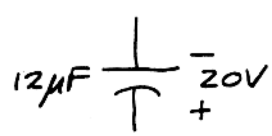
$$= (0.4t + 30 \times 10^{-6})(-5000)e^{-5000t} + e^{-5000t}(0.4)$$

$$= (-2000t - 150 \times 10^{-3} + 0.4)e^{-5000t}$$

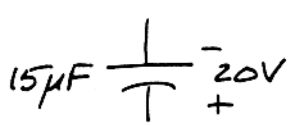
$$= (0.25 - 2000t)e^{-5000t}\,\text{A}, \qquad t \geq 0$$

6.

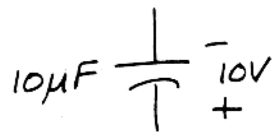
48 μF in series with 16 μF is 12 μF with an initial voltage of 20 V, i.e.,



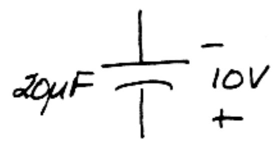
12 μF in parallel with 3 μF is 15 μF charged to 20 V, i.e.,



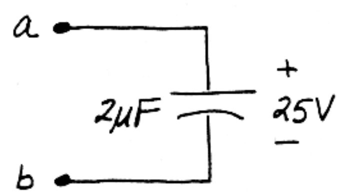
$15\,\mu F$ in series with $30\,\mu F$ is $10\,\mu F$ with a net charge of $10\,V$, i.e.,



$10\,\mu F$ in parallel with $10\,\mu F$ is $20\,\mu F$ with a charge of $10\,V$, i.e.,

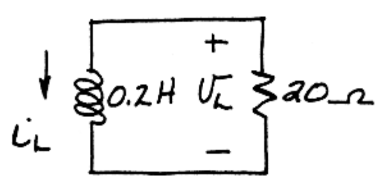


$5\,\mu F$ in series with $20\,\mu F$ in series with $4\,\mu F$ is $2\,\mu F$ with a net charge of $25\,V$.



7.

$t > 0$: $\quad i_L(0) = \left(\frac{80}{20}\right)\left(\frac{50}{80}\right) = 2.5\,A$



$$\tau = \frac{0.2}{20} = 0.01\,s$$

$$\frac{1}{\tau} = 100$$

$$i_L = 2.5e^{-100t}\,A, \qquad t \geq 0^+$$

$$v_L = -20i_L = -50e^{-100t}\,V, \qquad t \geq 0^+$$

$$v_o = -15i_L = -37.5e^{-100t}\,V, \qquad t \geq 0^+$$