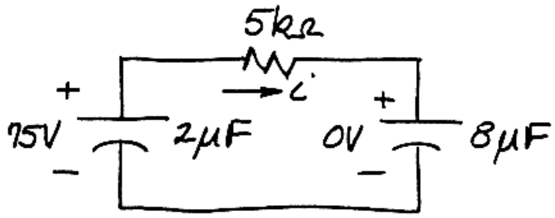
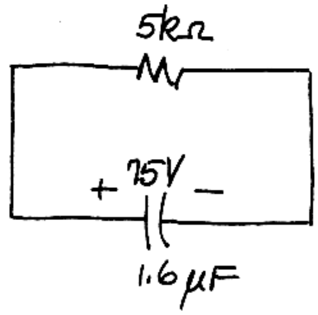
ENSC 220 Assignment 5 solutions (2005)

1.

[a] $t > 0$:





$\tau = 8 \times 10^{-3} = 8\,\text{ms}$

$1/\tau = 125$

$i(0^+) = \frac{75}{5} \times 10^{-3} = 15\,\text{mA}$

$i(t) = 15e^{-125t}\,\text{mA}, \quad t \geq 0^+$

$$v_1 = -\frac{10^6}{2}\int_0^t 15 \times 10^{-3} e^{-125x}\,dx + 75$$
$$= 60(e^{-125t} - 1) + 75 = 60e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$
$$v_2 = \frac{10^6}{8}\int_0^t 15 \times 10^{-3} e^{-125x}\,dx + 0$$
$$= -15(e^{-125t} - 1) = -15e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(2 \times 10^{-6})(75)^2 = 5625\,\mu\text{J}$

[c] $W_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(15)^2 + \frac{1}{2}(8 \times 10^{-6})(15)^2 = 225[10^{-6} + 4 \times 10^{-6}]$

$W_{\text{trapped}} = 1125\,\mu\text{J}$

$W_{\text{diss}} = w(0) - W_{\text{trapped}} = 4500\,\mu\text{J}$

Check: $W_{\text{diss}} = \int_0^\infty 225 \times 10^{-6} e^{-250t}(5000)\,dt$

$$= 1125 \times 10^{-3} \frac{e^{-250t}}{-250}\Big|_0^\infty = 4500\,\mu\text{J}$$

2.

[a] $v_o(0^-) = v_o(0^+) = 120\,\text{V}; \qquad v_o(\infty) = -4(37.5) = -150\,\text{V}$

$\tau = (37.5 + 12.5)10^3(0.04 \times 10^{-6}) = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$

$\therefore\ v_o(t) = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0^+$

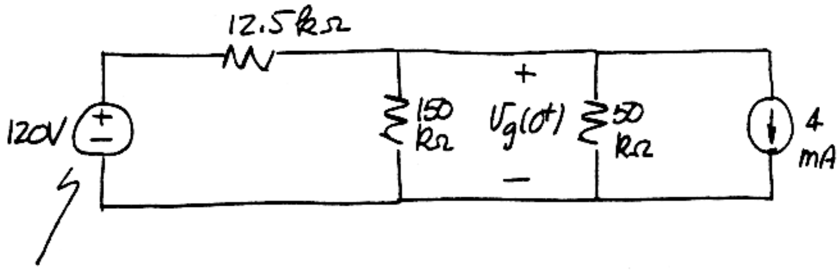
[b] $i_o(t) = -C\frac{dv_o}{dt} = -0.04 \times 10^{-6}(-135{,}000e^{-500t}) = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g(t) = v_o - 12.5i_o(t) = -150 + 270e^{-500t} - 67.5e^{-500t}$

$= -150 + 202.5e^{-500t}\,\text{V}, \qquad t \geq 0^+$

[d] $v_g(0^+) = 52.50\,\text{V}$

We can check $v_g(0^+)$ as follows: At $t = 0^+$ we have:



Capacitor acts like a voltage source at $t = 0^+$.

$$\frac{v_g(0^+) - 120}{12.5} + \frac{v_g(0^+)}{150} + \frac{v_g(0^+)}{50} + 4 = 0$$

$$12v_g(0^+) - 1440 + v_g(0^+) + 3v_g(0^+) + 600 = 0$$

$$16v_g(0^+) = 840$$

$$v_g(0^+) = 52.50\,\text{V} \quad \text{(ok)}$$

3.

[a] $RC = 25(0.4) \times 10^{-3} = 10\,\text{ms}; \qquad \frac{1}{RC} = 100; \qquad v_o = 0, \quad t < 0$

[b] $0 \leq t \leq 250\,\text{ms}$:

$$v_o = -100\int_0^t -0.20\,dx = 20t\,\text{V}$$

[c] $250\,\text{ms} \leq t \leq 500\,\text{ms}$:

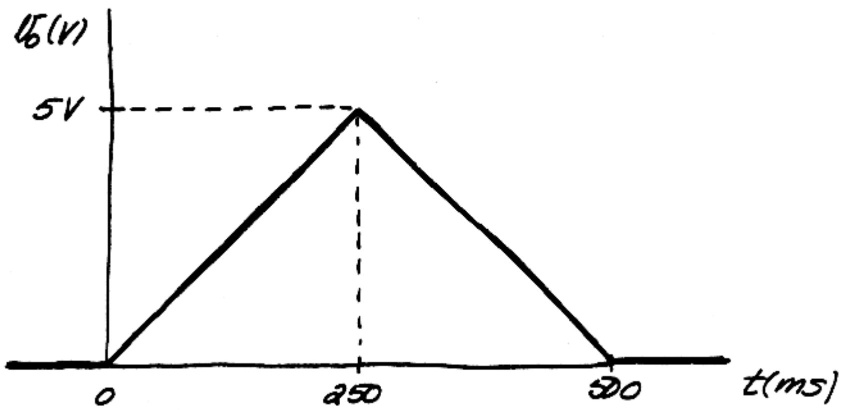
$$v_o(0.25) = 20(0.25) = 5\,\text{V}$$

$$v_o(t) = -100\int_{0.25}^t 0.20\,dx + 5 = -20(t - 0.25) + 5 = -20t + 10\,\text{V}$$

[d] $500\,\text{ms} \leq t \leq \infty$:

$$v_o(0.5) = -10 + 10 = 0\,\text{V}$$

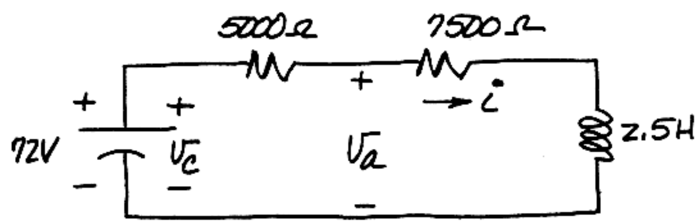
$$v_o(t) = 0\,\text{V}$$



4.

[a] $v_a(0) = v_c(0) = 72\,\text{V}$

[b] $t > 0$:



$$v_a = v_c - 5000i$$

$$\frac{dv_a}{dt} = \frac{dv_c}{dt} - 5000\frac{di}{dt}$$

$$\frac{dv_c(0)}{dt} = 0 \quad \text{since} \quad i(0) = 0; \qquad \therefore\ \frac{dv_a(0)}{dt} = -5000\frac{di(0)}{dt}$$

$$\frac{di(0)}{dt} = \frac{72}{2.5} = 28.80\,\text{A/s}; \qquad \therefore\ \frac{dv_a(0)}{dt} = -144{,}000\,\text{V/s}$$

[c] $\alpha = \frac{R}{2L} = \frac{12{,}500}{5} = 2500\,\text{rad/s}; \qquad \alpha^2 = 625 \times 10^4$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(0.1)(2.5)} = 4 \times 10^6 = 400 \times 10^4$$

$$\therefore\ \alpha^2 > \omega_o^2 \quad \text{overdamped}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2500 + \sqrt{225 \times 10^4} = -2500 + 1500 = -1000\,\text{rad/s}$$

$$s_2 = -2500 - 1500 = -4000\,\text{rad/s}$$

$$v_a = A_1e^{-1000t} + A_2e^{-4000t}; \qquad v_a(0) = A_1 + A_2 = 72$$

$$\frac{dv_a(0)}{dt} = -1000A_1 - 4000A_2 = -144{,}000$$

$$\therefore\ A_1 + 4A_2 = 144; \qquad \therefore\ 3A_2 = 72, \quad A_2 = 24, \quad A_1 = 48$$

$$\therefore\ v_a = 48e^{-1000t} + 24e^{-4000t}\,\text{V}, \qquad t \ge 0$$

5.

[a] $i_o(0) = \frac{80}{800} = 0.10\,\text{A}; \qquad v_o(0) = 500(0.1) = 50\,\text{V}$

$L\frac{di_o(0)}{dt} = 0; \qquad \therefore \frac{di_o}{dt}(0) = 0$

$\alpha = \frac{R}{2L} = \frac{500}{5} \times 10^3 = 10^5; \qquad \alpha^2 = 10^{10}$

$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^9)}{(2.5)(40)} = 10^{10}$

$\therefore \omega_o^2 = \alpha^2$ critically damped

$i_o(t) = D_1te^{-10^5t} + D_2e^{-10^5t}$

$i_o(0) = D_2 = 0.10\,\text{A}$

$\frac{di_o}{dt}(0) = -\alpha D_2 + D_1 = 0; \qquad D_1 = \alpha D_2 = 10^5(0.1) = 10^4$

$i_o(t) = 10^4te^{-10^5t} + 0.1e^{-10^5t}A, \qquad t \geq 0$

[b] $v_o(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}$

$v_o(0) = 50 = D_2$

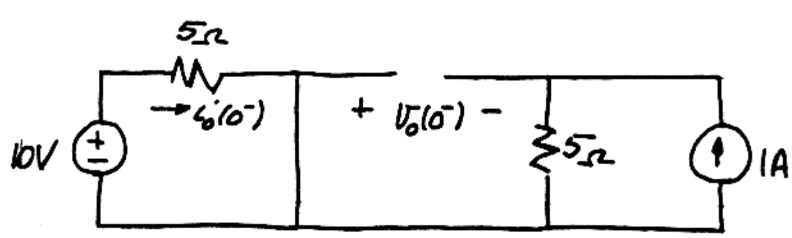
$C\frac{dv_o}{dt}(0) = -0.10; \qquad \frac{dv_o}{dt} = -25 \times 10^5$

$\frac{dv_o}{dt}(0) = -\alpha D_2 + D_1 = -25 \times 10^5; \qquad D_1 = -25 \times 10^5 + 10^5(50) = 25 \times 10^5$

$\therefore v_o(t) = 25 \times 10^5te^{-10^5t} + 50e^{-10^5t}\,\text{V}, \qquad t \geq 0$

6.

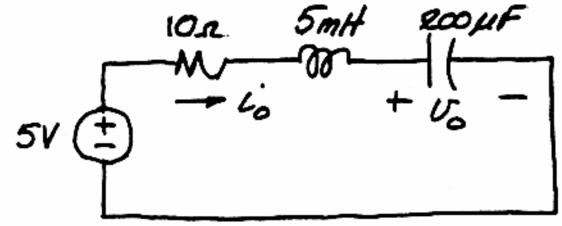
[a] $t < 0$:



$i_o(0^-) = 10/5 = 2\,\text{A} = i_o(0^+)$

$v_o(0^-) = -5(1) = -5\,\text{V} = v_o(0^+)$

$t > 0^+$:



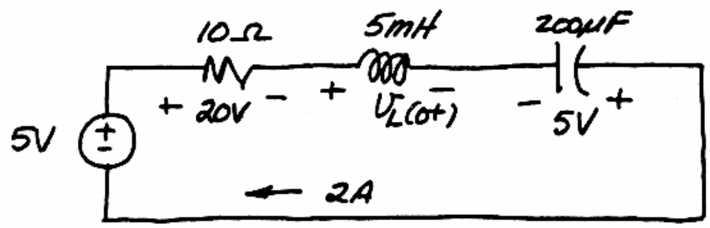
$$\alpha = \frac{R}{2L} = \frac{10}{10} \times 1000 = 1000\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^6)}{(5)(200)} = 10^6$$

$\alpha^2 = \omega^2 \quad \therefore$ critically damped

$\therefore\ i_o = i_F + D_1' te^{-1000t} + D_2' e^{-1000t}; \qquad i_F = 0$

$\therefore\ i_o = D_1' te^{-1000t} + D_2' e^{-1000t}; \qquad i_o(0^+) = D_2' = 2$

At $t = 0^+$:



$-5 + 20 + v_L(0^+) - 5 = 0, \qquad v_L(0^+) = -10\,\text{V}$

$$\therefore\ \frac{di}{dt}(0^+) = \frac{-10}{5} \times 10^3 = -2000\,\text{A/s}$$

$$\frac{di_o}{dt} = -1000e^{-1000t}(D_1' t + D_2') + e^{-1000t} D_1'$$

$$\frac{di_o}{dt}(0^+) = -1000D_2' + D_1' = -2000$$

$-2000 + D_1' = -2000; \qquad D_1' = 0$

$\therefore\ i_o(t) = 2e^{-1000t}\,\text{A}, \qquad t \geq 0$

[b] $v_o = v_F + D_3' te^{-1000t} + D_4' e^{-1000t}$

$v_o(0^+) = v_F + D_4' = -5$

$v_F = 5\,\text{V}; \qquad \therefore\ D_4' = -10\,\text{V}$

$v_o = 5 + D_3' te^{-1000t} - 10e^{-1000t}$

$$\frac{dv_o}{dt} = -1000e^{-1000t}(D_3' t - 10) + D_3' e^{-1000t}$$

$$\frac{dv_o}{dt}(0^+) = 10{,}000 + D_3'$$

$$\frac{dv_o}{dt}(0^+) = \frac{2}{200} \times 10^6 = 10^4 = 10{,}000$$

$\therefore\ D_3' = 0$

$\therefore\ v_o = 5 - 10e^{-1000t}\,\text{V}, \qquad t \geq 0$