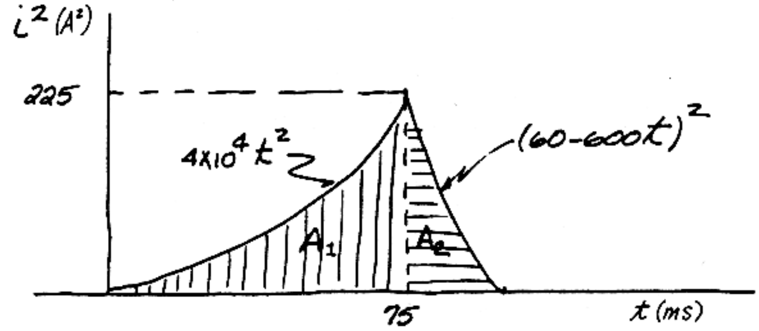
ENSC 220 – Assignment #7 Solutions (Fall 2005)

1.

P 11.5



$$A_1 = \int_0^{0.075} 4 \times 10^4 t^2\, dt = 4 \times 10^4 \frac{t^3}{3}\Big|_0^{0.075} = 5.625$$

$$A_2 = \int_{0.075}^{0.1} (3600 - 72{,}000t + 36 \times 10^4 t^2)\, dt$$

$$= 3600t \Big|_{0.075}^{0.10} - 36{,}000t^2 \Big|_{0.075}^{0.10} + 12 \times 10^4 t^3 \Big|_{0.075}^{0.10} = 90 - 157.50 + 69.375 = 1.875$$

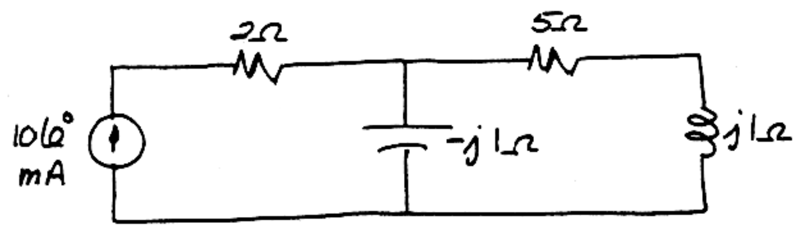
$$A_1 + A_2 = 5.625 + 1.875 = 7.5$$

$$\frac{1}{T} = \frac{1}{0.10} = 10$$

$$\therefore\ I_{\text{rms}} = \sqrt{10(7.5)} = \sqrt{75} = 8.66\ \text{A} \quad \text{(rms)}$$

2.

P 11.12 $\frac{1}{j\omega C} = -j1\,\Omega; \qquad j\omega L = j1\,\Omega$



$$Z_1 = -j1 // (5 + j1) = 0.2 - j1.0\,\Omega; \qquad Z_{\text{eq}} = 2 + 0.2 - j1 = 2.2 - j1\,\Omega$$

$$P_{\text{del}} = P_{\text{diss}} = \left(\frac{10 \times 10^{-3}}{\sqrt{2}}\right)^2 (2.2) = 110\,\mu\text{W}$$

3.

P 11.17 [a] $Z^* = \dfrac{|\mathbf{V}_{\text{eff}}|^2}{S}; \qquad \mathbf{V}_{\text{eff}} = 480\text{ V}$

$$S = [25 + j25 + 15(0.8 - j0.6) + 11(1 + j0)]\text{ kVA}$$
$$= [25 + j25 + 12 - j9 + 11]\text{ kVA} = (48 + j16)\text{ kVA}$$
$$Z^* = \frac{(480)(480)}{48{,}000 + j16{,}000} = 4.32 - j1.44$$
$$Z = 4.32 + j1.44 = 4.55\underline{/18.43^\circ}\ \Omega$$

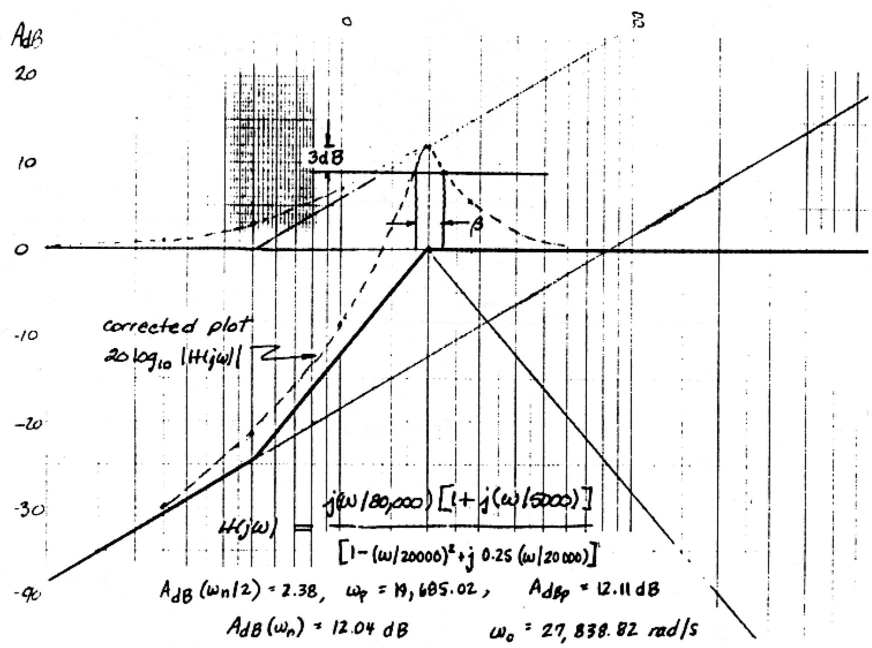
[b] $\text{pf} = \cos(18.43^\circ) = 0.9487 \quad \text{(lagging)}$

4.

P 17.29 [a] $\dfrac{1}{sC} = \dfrac{20\times 10^6}{s}; \qquad sL = 0.05s$

$$I_o = \frac{I_g(250 + 0.05s)}{[(20\times 10^6)/s] + 250 + 0.05s}$$
$$\frac{I_o}{I_g} = \frac{s(250 + 0.05s)}{0.05s^2 + 250s + 20\times 10^6}$$
$$H(s) = \frac{s(s + 5000)}{s^2 + 5000s + 4\times 10^8}$$

[b]



[c] $\omega_p = 19{,}685.02\text{ rad/s}$

[d] $A_{\text{dB}}(\text{max}) \cong 12.11\text{ dB}$

[e] $\omega_1 \cong 18{,}000\text{ rad/s}; \qquad \omega_2 \cong 23{,}500\text{ rad/s}$

[f] $\beta \cong 5{,}500\text{ rad/s}$

[g] $H(j\omega) = \dfrac{j(\omega/80{,}000)[1 + j(\omega/5000)]}{1 - (\omega/20{,}000)^2 + j0.25(\omega/20{,}000)}$

$$20\log_{10}|H(j18{,}000)| = 9.11\text{ dB}$$
$$20\log_{10}|H(j23{,}500)| = 9.35\text{ dB}$$

5.

P 17.31 [a] $H(j\omega) = \dfrac{10^8}{-\omega^2 + j3000\omega + 10^8} = \dfrac{1}{1-(\omega/10^4)^2 + j(3000/10^4)(\omega/10^4)}$

$\therefore\ 2\zeta = 0.30, \qquad \zeta = 0.15$

$\omega_o = \sqrt{2}\omega_p = \sqrt{2}\left[\omega_n\sqrt{1-2\zeta^2}\right] = \sqrt{2}\left[10^4\sqrt{1-2(0.15)^2}\right] = 13{,}820.27\,\text{rad/s}$

$\therefore\ \omega = 0$ and $\omega = 13{,}820.27\,\text{rad/s}$

[b] $\omega_p = 10^4\sqrt{1-2(0.15)^2} = 9772.41\,\text{rad/s}$

[c] $A_{\text{dBp}} = -10\log_{10}\left[4\zeta^2(1-\zeta^2)\right] = 10.56\,\text{dB}$

$20\log_{10}|H(j\omega)| = 10.56; \qquad \log_{10}|H(j\omega)| = 0.53; \qquad H(j\omega) = 3.37$

$\therefore$ Maximum ratio is 3.37.

6.

P 17.34 $A_{\text{dB}} = 3\,\text{dB}$ @ 1 rad/s, therefore 1 rad/s looks like a corner frequency. Assume 1 rad/s is a corner, then a straight line plot would predict 20 dB @ 10 rad/sec. On the plot, A_{dB} @ 10 rad/s is 17 dB, i.e., down 3 dB. Therefore, we suspect a simple pole at 10 rad/s. At this point, we have $H(s)$ is of the form $K(s+1)/(s+10)$. A straight line plot of this function would approach a constant as s increases. However, we see from Fig. 18.34 that A_{dB} drops off and is down to 17 dB @ 100 rad/s. Therefore, the transfer function has another simple pole at 100 rad/s, thus $H(s)$ is of the form $K(s+1)/(s+10)(s+100)$. Because $H(j\omega) \to 0\,\text{dB}$ as $\omega \to 0$, K must equal 1000.