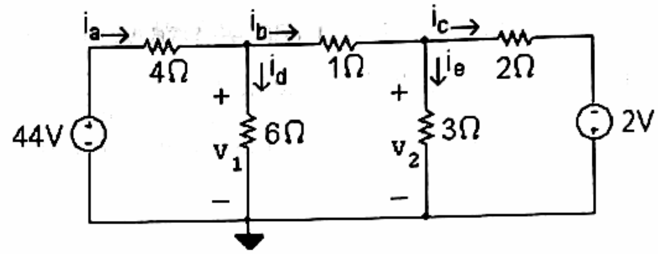
ENSC 220 – Hw#2 Solutions – Fall 2005

P 4.10 [a]



The two node voltage equations are:

$$\frac{v_1}{6} + \frac{v_1 - 44}{4} + \frac{v_1 - v_2}{1} = 0$$

$$\frac{v_2}{3} + \frac{v_2 - v_1}{1} + \frac{v_2 + 2}{2} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{6} + \frac{1}{4} + 1\right) + v_2(-1) = \frac{44}{4}$$

$$v_1(-1) + v_2\left(\frac{1}{3} + 1 + \frac{1}{2}\right) = -\frac{2}{2}$$

Solving, $v_1 = 12$ V; $v_2 = 6$ V

Now calculate the branch currents from the node voltage values:

$$i_a = \frac{44 - 12}{4} = 8 \text{ A}$$

$$i_b = \frac{12}{6} = 2 \text{ A}$$

$$i_c = \frac{12 - 6}{1} = 6 \text{ A}$$

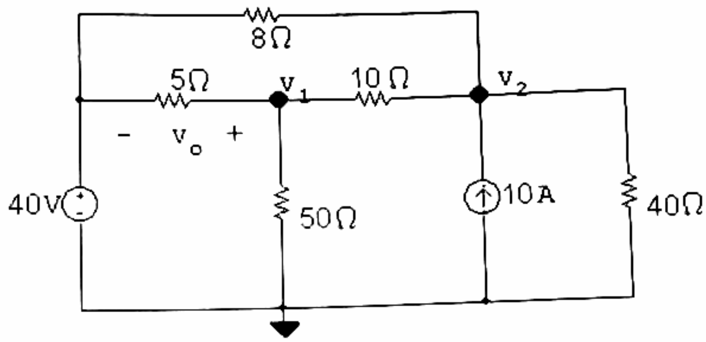
$$i_d = \frac{6}{3} = 2 \text{ A}$$

$$i_e = \frac{6 + 2}{2} = 4 \text{ A}$$

[b] $p_{\text{sources}} = p_{44\text{V}} + p_{2\text{V}} = -(44)i_a - (2)i_e = -(44)(8) - (2)(4) = -352 - 8 = -360$ **W**

Thus, the power developed in the circuit is 360 W. Note that the resistors cannot develop power!

P 4.21



The two node voltage equations are:

$$\frac{v_1 - 40}{5} + \frac{v_1}{50} + \frac{v_1 - v_2}{10} = 0$$

$$\frac{v_2 - v_1}{10} - 10 + \frac{v_2}{40} + \frac{v_2 - 40}{8} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{5} + \frac{1}{50} + \frac{1}{10}\right) + v_2\left(-\frac{1}{10}\right) = \frac{40}{5}$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(\frac{1}{10} + \frac{1}{40} + \frac{1}{8}\right) = 10 + \frac{40}{8}$$

Solving, $v_1 = 50$ V; $v_2 = 80$ V.

Thus, $v_o = v_1 - 40 = 50 - 40 = 10$ V.

POWER CHECK:

$$i_g = (50 - 40)/5 + (80 - 40)/8 = 7 \text{ A}$$

$$p_{40V} = (40)(7) = 280 \text{ W} \quad \text{(abs)}$$

$$p_{5\Omega} = (50 - 40)^2/5 = 20 \text{ W} \quad \text{(abs)}$$

$$p_{8\Omega} = (80 - 40)^2/8 = 200 \text{ W} \quad \text{(abs)}$$

$$p_{10\Omega} = (80 - 50)^2/10 = 90 \text{ W} \quad \text{(abs)}$$

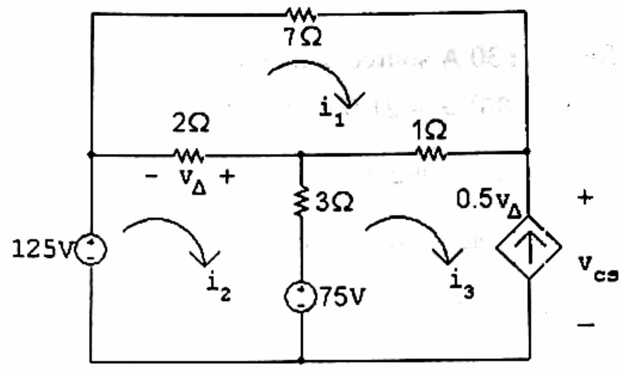
$$p_{50\Omega} = 50^2/50 = 50 \text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = 80^2/40 = 160 \text{ W} \quad \text{(abs)}$$

$$p_{10A} = -(80)(10) = -800 \text{ W} \quad \text{(del)}$$

$$\sum p_{\text{abs}} = 280 + 20 + 200 + 90 + 50 + 160 = 800 \text{ W} = \sum p_{\text{del}}$$

P 4.39



Mesh equations:

$$7i_1 + 1(i_1 - i_3) + 2(i_1 - i_2) = 0$$

$$-125 + 2(i_2 - i_1) + 3(i_2 - i_3) + 75 = 0$$

Constraint equations:

$i_3 = -0.5v_\Delta$; $\quad v_\Delta = 2(i_1 - i_2)$

Place these equations in standard form:

$$i_1(7+1+2) + i_2(-2) + i_3(-1) + v_\Delta(0) = 0$$

$$i_1(-2) + i_2(2+3) + i_3(-3) + v_\Delta(0) = 50$$

$$i_1(0) + i_2(0) + i_3(1) + v_\Delta(0.5) = 0$$

$$i_1(2) + i_2(-2) + i_3(0) + v_\Delta(-1) = 0$$

Solving, $i_1 = 6$ A; $\quad i_2 = 22$ A; $\quad i_3 = 16$ A; $\quad v_\Delta = -32$ V

Solve the outer loop KVL equation to find v_{cs}:

$-125 + 7i_1 + v_{cs} = 0$; $\quad\therefore\quad v_{cs} = 125 - 7(6) = 83$ V

Calculate the power:

$$p_{125\text{V}} = -(125)(22) = -2750 \text{ W}$$

$$p_{75\text{V}} = (75)(22-16) = 450 \text{ W}$$

$$p_{\text{dep source}} = -(83)[0.5(-32)] = 1328 \text{ W}$$

Thus, the total power developed is 2750 W.

CHECK:

$$p_{7\Omega} = (6)^2(7) = 252 \text{ W}$$

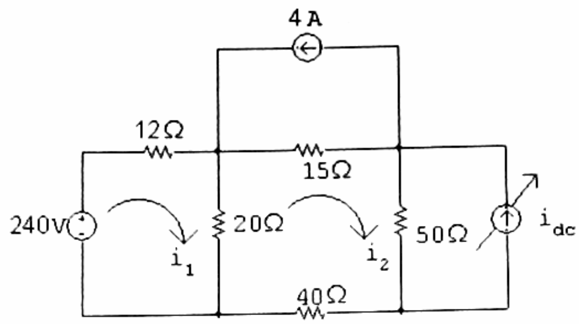
$$p_{2\Omega} = (22-6)^2(2) = 512 \text{ W}$$

$$p_{3\Omega} = (22-16)^2(3) = 108 \text{ W}$$

$$p_{1\Omega} = (16-6)^2(1) = 100 \text{ W}$$

$$\therefore \quad \sum p_{\text{abs}} = 450 + 1328 + 252 + 512 + 108 + 100 = 2750 \text{ W (checks!)}$$

P 4.50



The mesh current equations:

$$-240 + 12i_1 + 20(i_1 - i_2) = 0$$

$$20(i_2 - i_1) + 15(i_2 + 4) + 50(i_2 + i_{dc}) + 40i_2 = 0$$

Place these equations in standard form:

$$i_1(12 + 20) + i_2(-20) + i_{dc}(0) = 240$$

$$i_1(-20) + i_2(20 + 15 + 50 + 40) + i_{dc}(50) = -60$$

But if the power associated with the 4 A source is zero, the voltage drop across the source must be zero. This means that the voltage drop across the 15 Ω resistor is also zero, so the 15 Ω resistor is effectively removed from the circuit. Once this happens, $i_2 = -4$ A. Substitute this value into the first equation and solve for i_1:

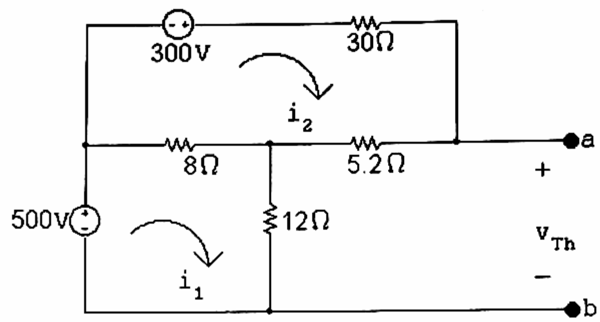
$32i_1 - 20(-4) = 240 \quad \therefore \quad 32i_1 = 160 \quad \text{so} \quad i_1 = 5$ A

Now substitute this value for i_1 into the second equation and solve for i_{dc}:

$-20(5) + 125(-4) + 50i_{dc} = -60 \quad \text{so} \quad 50i_{dc} = -60 + 100 + 500 = 540$

$\therefore \quad i_{dc} = 540/50 = 10.8$ A

P 4.61 After making a source transformation the circuit becomes



The mesh current equations are:

$$-500 + 8(i_1 - i_2) + 12i_1 = 0$$

$$-300 + 30i_2 + 5.2i_2 + 8(i_2 - i_1) = 0$$

Put the equations in standard form:

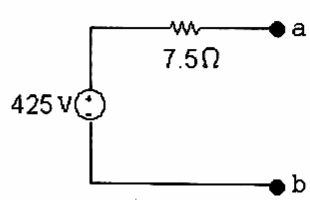
$$i_1(8 + 12) + i_2(-8) = 500$$

$$i_1(-8) + i_2(30 + 5.2 + 8) = 300$$

Solving, $i_1 = 30$ A; $i_2 = 12.5$ A

$V_{\mathrm{Th}} = 5.2i_2 + 12i_1 = 425$ V

$R_{\mathrm{Th}} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$



P 4.62 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to

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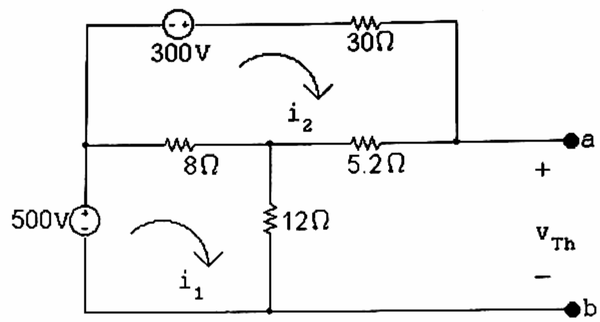

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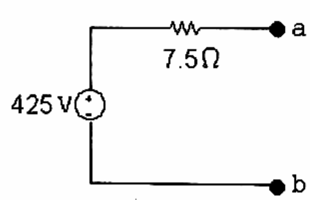

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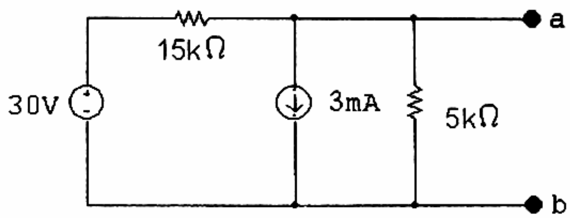
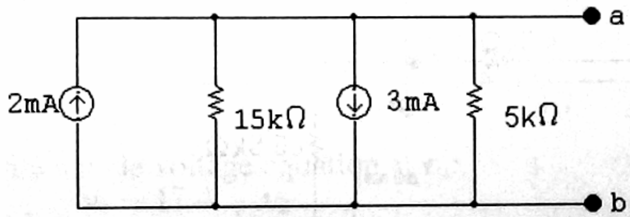


or



Therefore the Norton equivalent is determined by adding the current sources and combining the resistors in parallel: