MTF

- Generally measure MTF both horizontal and vertical

$$MTF = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

- Edge slope comes from the point spread function
- As frequency 1/N increases contrast decrease

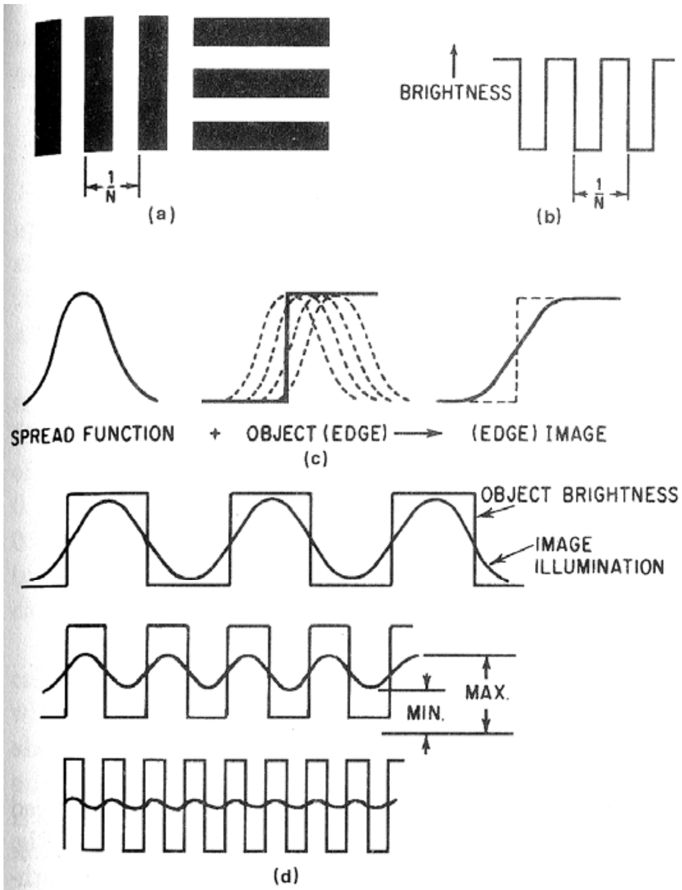


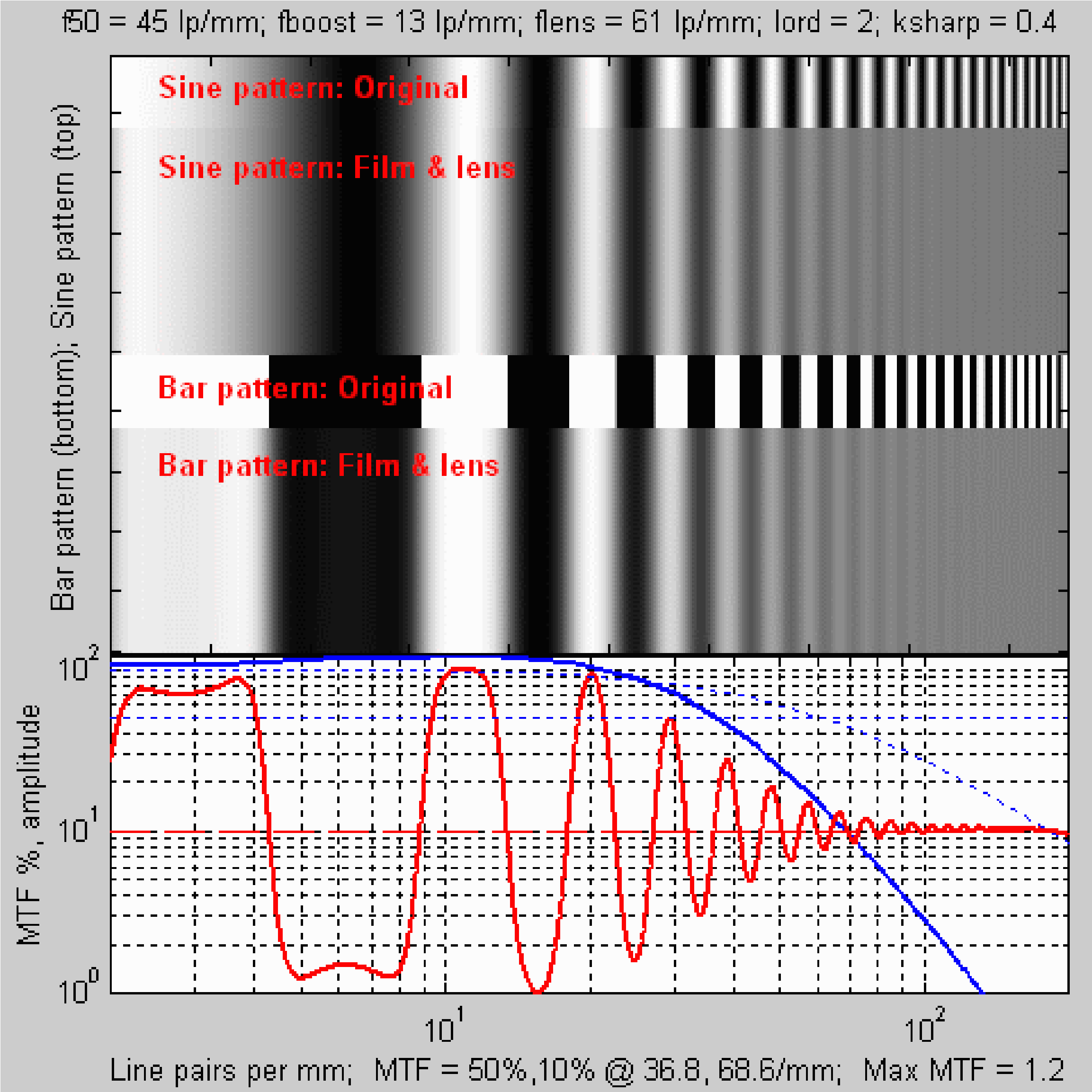
Figure 11.10 The imagery of a bar target. (a) A typical bar target used in testing optical systems consists of alternating light and dark bars. If the pattern has a frequency of *N* lines per millimeter, then it has a period of 1/*N* millimeters, as indicated. (b) A plot of the brightness of (a) is a square wave. (c) When an image is formed, each point is imaged as a blur, with an illumination distribution described by the spread function. The image then consists of the summation of all the spread functions. (d) As the test pattern is made finer, the contrast between the light and dark areas of the image is reduced.

Square Wave vs Sin wave

- Once MTF know for square wave can get sine wave response
- Use fourier components
- If S(v) at frequency v is for square waves
- Then can give response of sine wave

$$M(\nu)=\frac{\pi}{4}\left[S(\nu)+\frac{S(3\nu)}{3}-\frac{S(5\nu)}{5}+\frac{S(7\nu)}{7}-\cdots\right]$$

$$S(\nu)=\frac{4}{\pi}\left[M(\nu)-\frac{M(3\nu)}{3}+\frac{M(5\nu)}{5}-\frac{M(7\nu)}{7}+\cdots\right]$$



Diffraction Limited MTF

- For a perfect optical system

$$MTF = \frac{2}{\pi}(\phi - cos(\phi)sin(\phi))$$

Where

$$\phi = arccos\left(\frac{\lambda \nu}{2NA}\right)$$

Maximum or cutoff frequency v_0

$$\nu_0 = \frac{2NA}{\lambda} = \frac{1}{\lambda(f\#)}$$

In an afocal system or image at infinity then for lens dia D

$$\nu_0 = \frac{D}{\lambda}$$

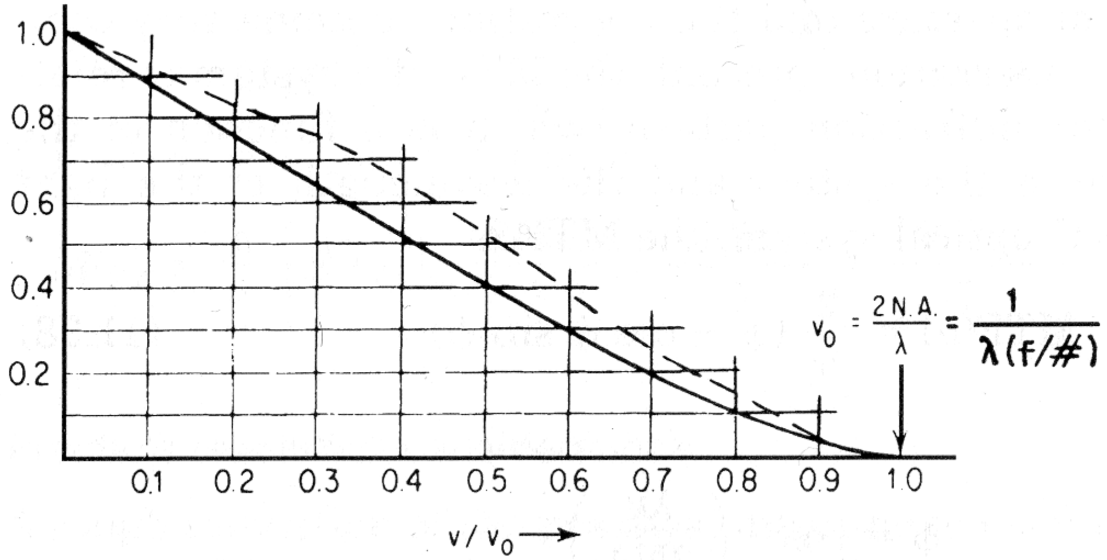


Figure 11.15 The modulation transfer function of an aberration-free system (solid line). Note that frequency is expressed as a fraction of the cutoff frequency. The dashed line is the modulation factor for a square wave (bar) target. Both curves are based on diffraction effects and assume a system with a uniformly transmitting circular aperture.

Defocus in MTF

- Adding defocus decreases MTF
- Defocus MTF

$$defocus\ MTF = \frac{2J_1(x)}{x}$$

Where x is

$$x = 2\pi\delta\ NA\,\frac{\nu(\nu_0-\nu)}{\nu_0}$$

- Max cutoff is 0.017 at $v=v_0/2$

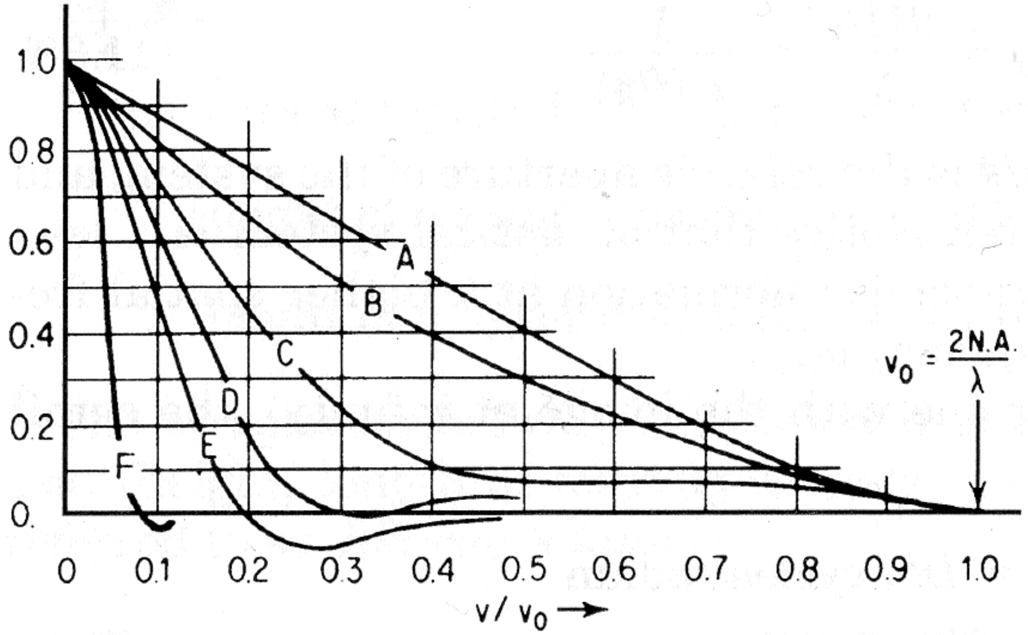


Figure 11.16 The effect of defocusing on the modulation transfer function of an aberration-free system.

(a) In focus OPD = 0.0
(b) Defocus $= \lambda/(2n \sin^2 U)$ OPD $= \lambda/4$
(c) Defocus $= \lambda/(n \sin^2 U)$ OPD $= \lambda/2$
(d) Defocus $= 3\lambda/(2n \sin^2 U)$ OPD $= 3\lambda/4$
(e) Defocus $= 2\lambda/(n \sin^2 U)$ OPD $= \lambda$
(f) Defocus $= 4\lambda/(n \sin^2 U)$ OPD $= 2\lambda$

(Curves are based on diffraction effects—not on a geometric calculation.)

MTF and Aberrations

- Aberrations degrade MTF
- Eg. 3rd order spherical aberrations
- Effect goes as wavelength defect

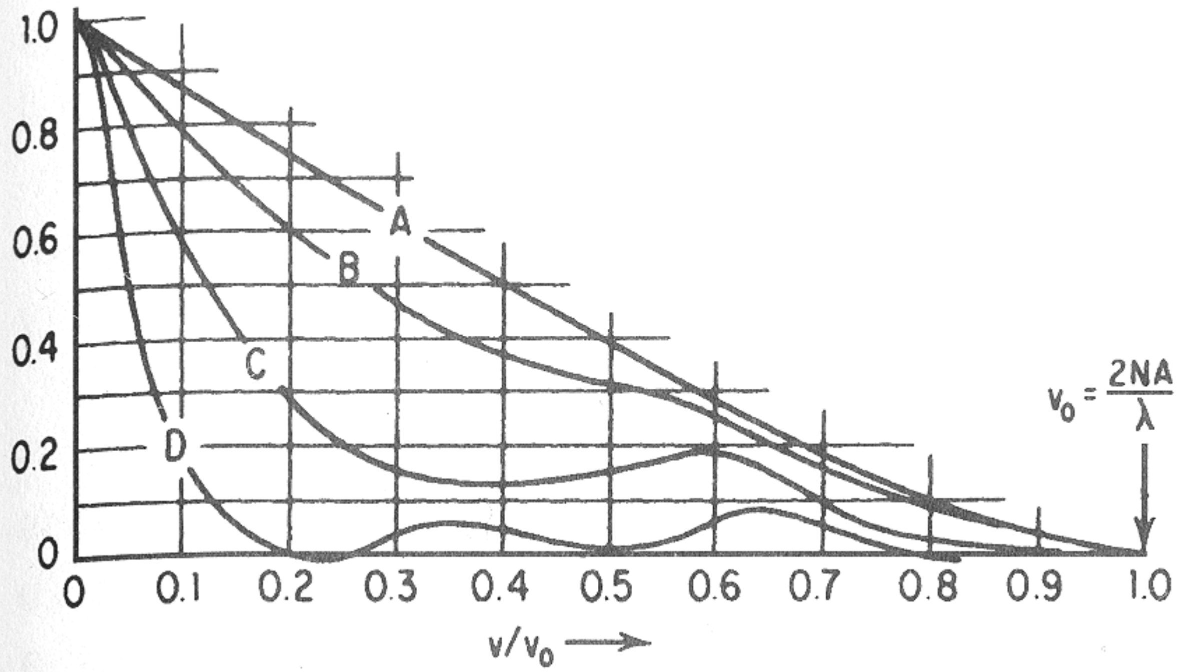


Figure 11.18 The effect of third-order spherical aberration on the modulation transfer function.

(a) $LA_m = 0.0$ OPD = 0
(b) $LA_m = 4\lambda/(n \sin^2 U)$ OPD = $\lambda/4$
(c) $LA_m = 8\lambda/(n \sin^2 U)$ OPD = $\lambda/2$
(d) $LA_m = 16\lambda/(n \sin^2 U)$ OPD = λ

These curves are based on diffraction wave-front computations for an image plane midway between the marginal and paraxial foci.

MTF and Filling Lens

- MTF decreases as lens is not filled
- Best result when image fills lens

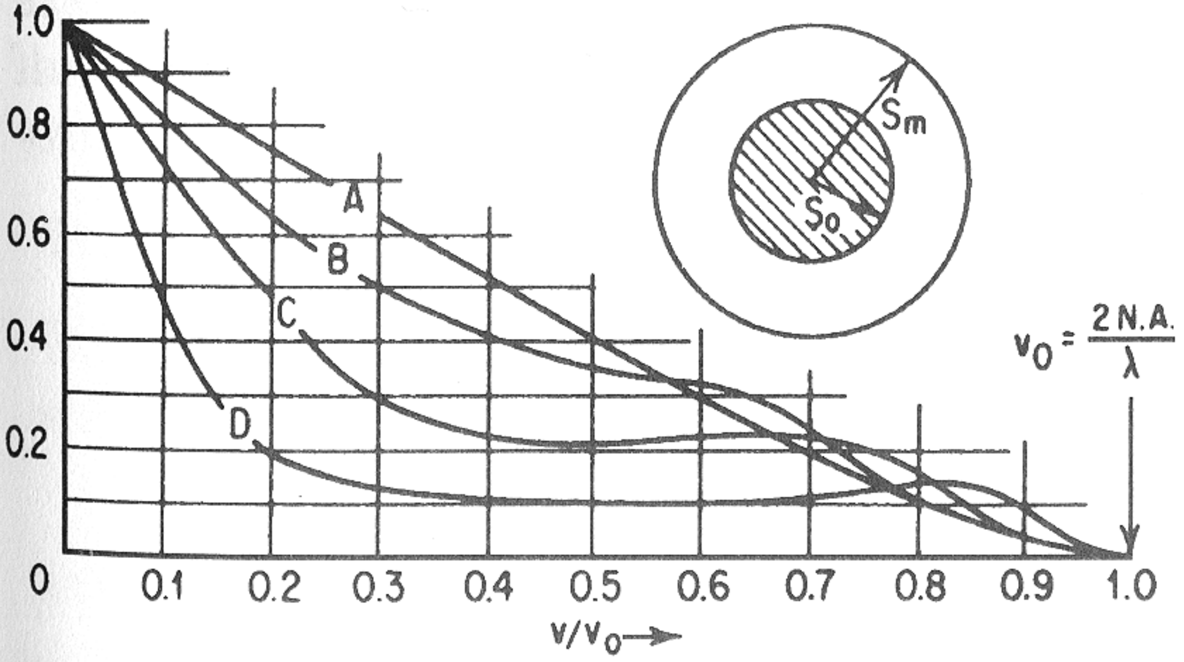
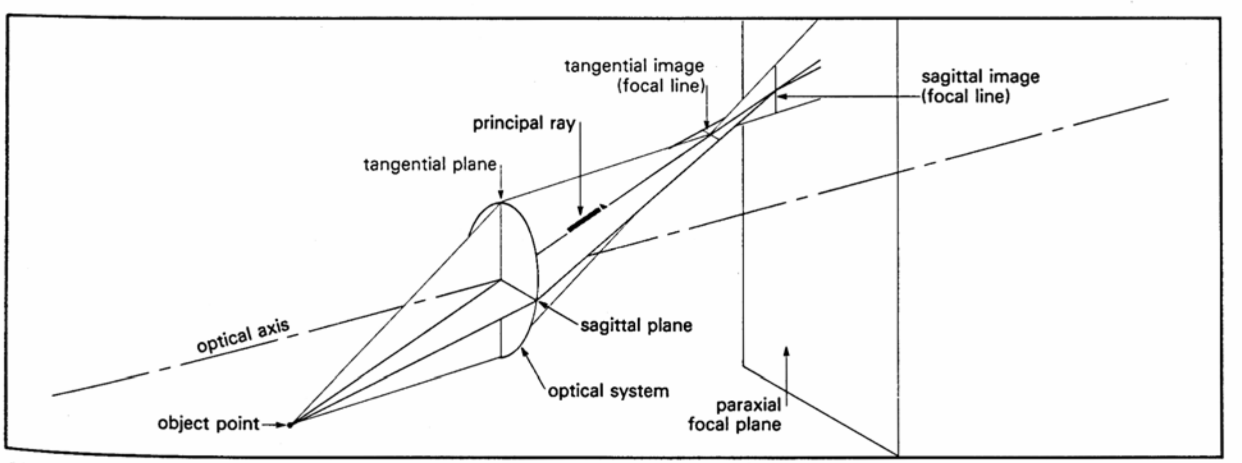


Figure 11.19 The effect of a central obscuration on the modulation transfer function of an aberration-free system.

(a) $s_0/s_m = 0.0$
(b) $s_0/s_m = 0.25$
(c) $s_0/s_m = 0.5$
(d) $s_0/s_m = 0.75$

MTF Specifications

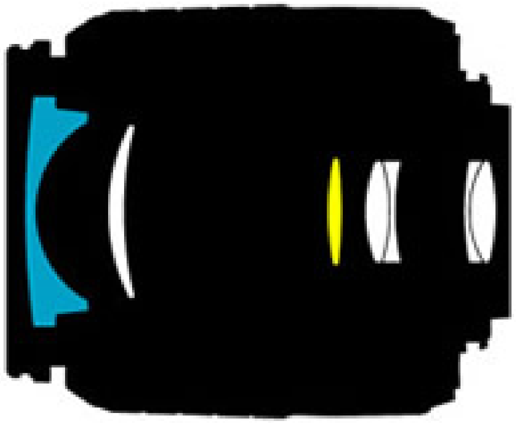
- MTF in lenses are specified in lines per millimetre
- Typically 10 and 30 lines
- Specified separately for Saggittal and tangential
- Saggittal – vertical aberrations on focus plane
- Tangential or Meridional: horizontal on focus plane

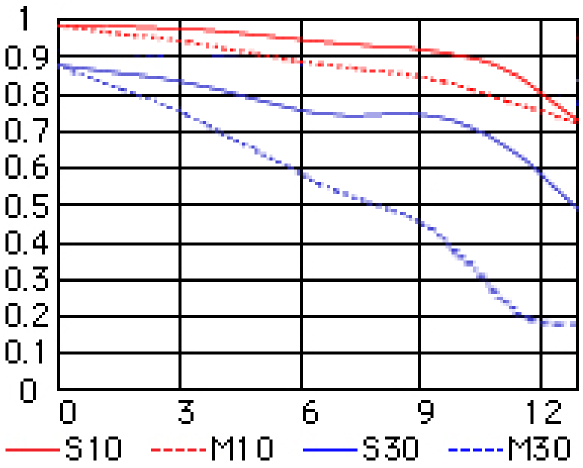


ASTIGMATISM can be represented by these sectional views.

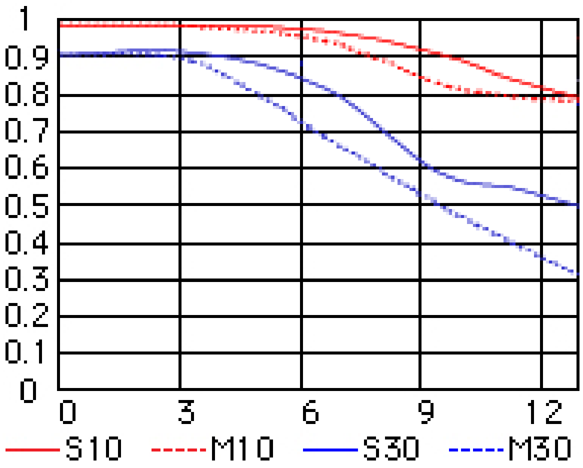
Reading MTF in Camera Lenses

- Camera lenses often publish MTF charts
- Below example for Nikon 18-55 mm zoom
- Plots show MTF at 10 lines/mm and 30/mm
- Shown with radius in mm from centre of image
- For a 24x15 mm image area
- Usually specified for single aperature (f/5.6 here)
- 10/mm measures lens contrast
- 30/mm lens resolution





Wide angle

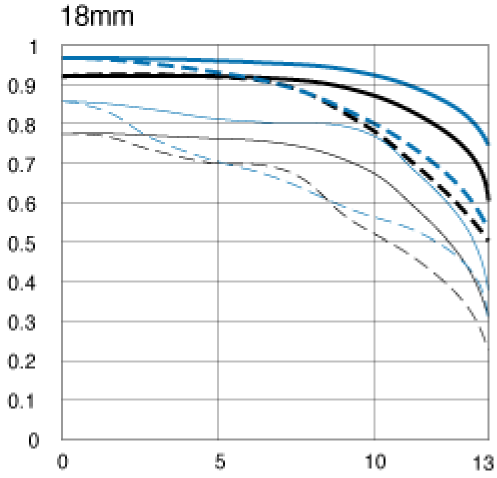


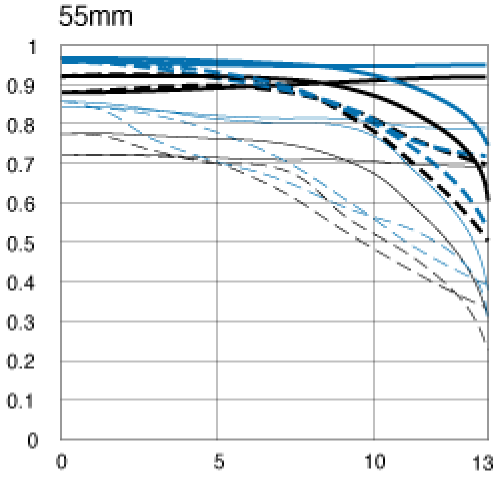
Telephoto

Spatial Frequencies	S: Sagittal	M: Meridional
10 lines/mm	——————	- - - - - - -
30 lines/mm	——————	- - - - - - -

Poor MTF Charts

- Some companies give charts but little info
- Entry level Cannon 18-55 mm lens
- Chart give MTF but does not say lines/mm
- Cannot compare without that





Aerial Image Modulation Curves

- Resolution set in Aerial Image Modulation (AIM)
- Combines the lens and the detector (eg film or digital sensor)
- Measures the smallest resolution detected by sensor
- Sensor can significantly change resolutions

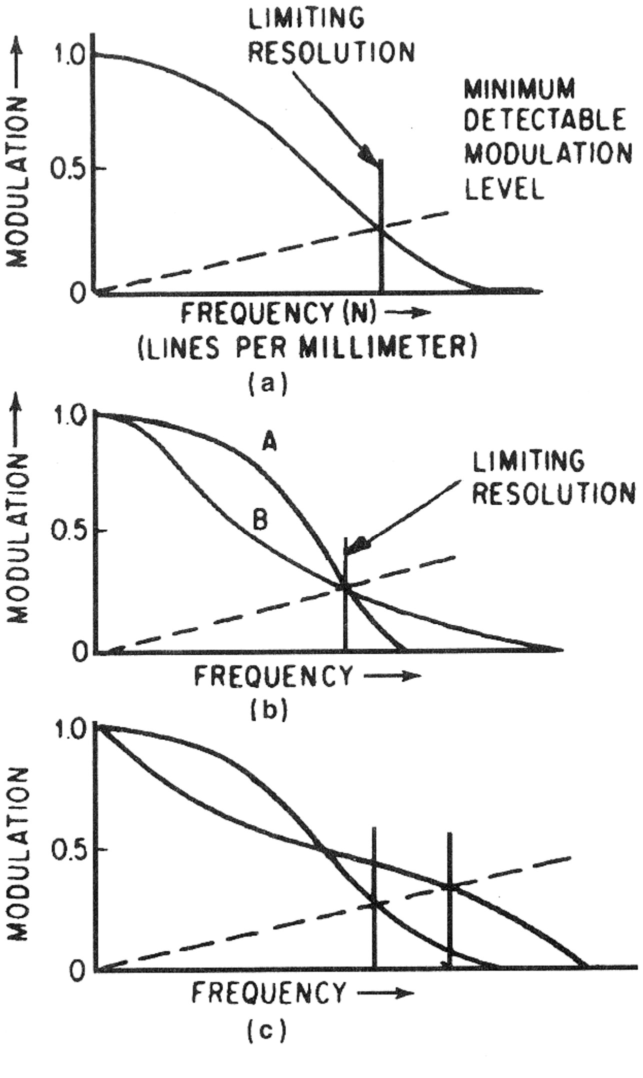
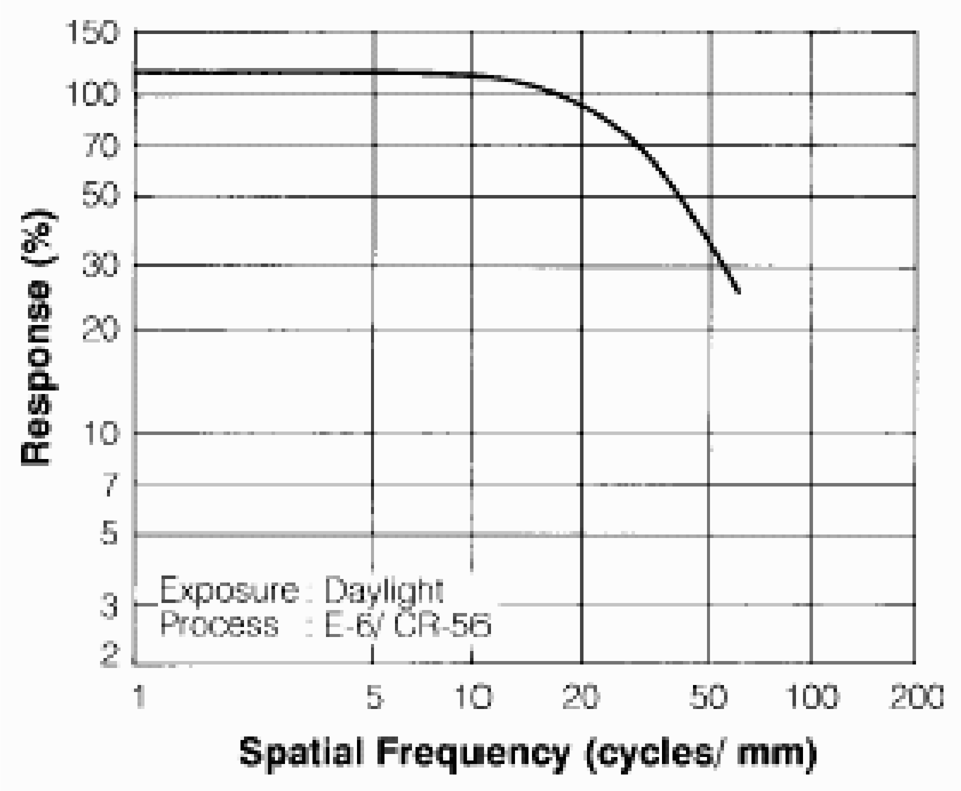
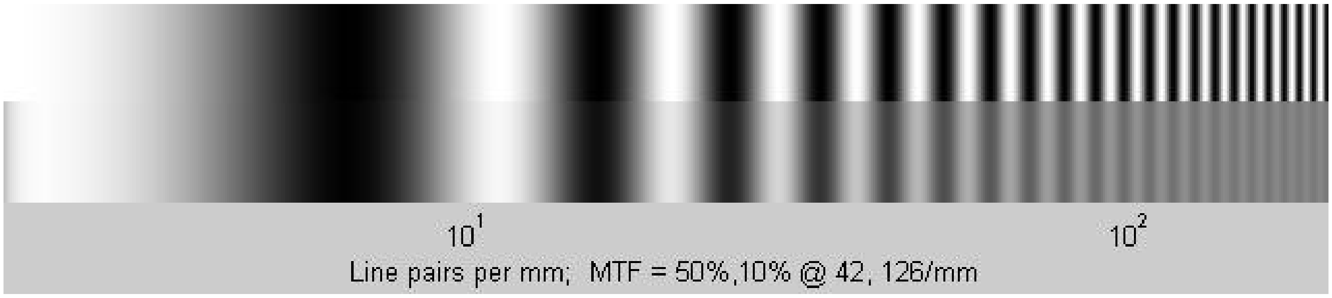


Figure 11.11 (a) The image modulation can be be plotted as a function of the frequency of the test pattern. When the modulation drops below the minimum that can be detected, the target is not resolved. (b) The system represented by (a) will produce a superior image, although both (a) and (b) have the same limiting resolution.

Film or Sensor MTF

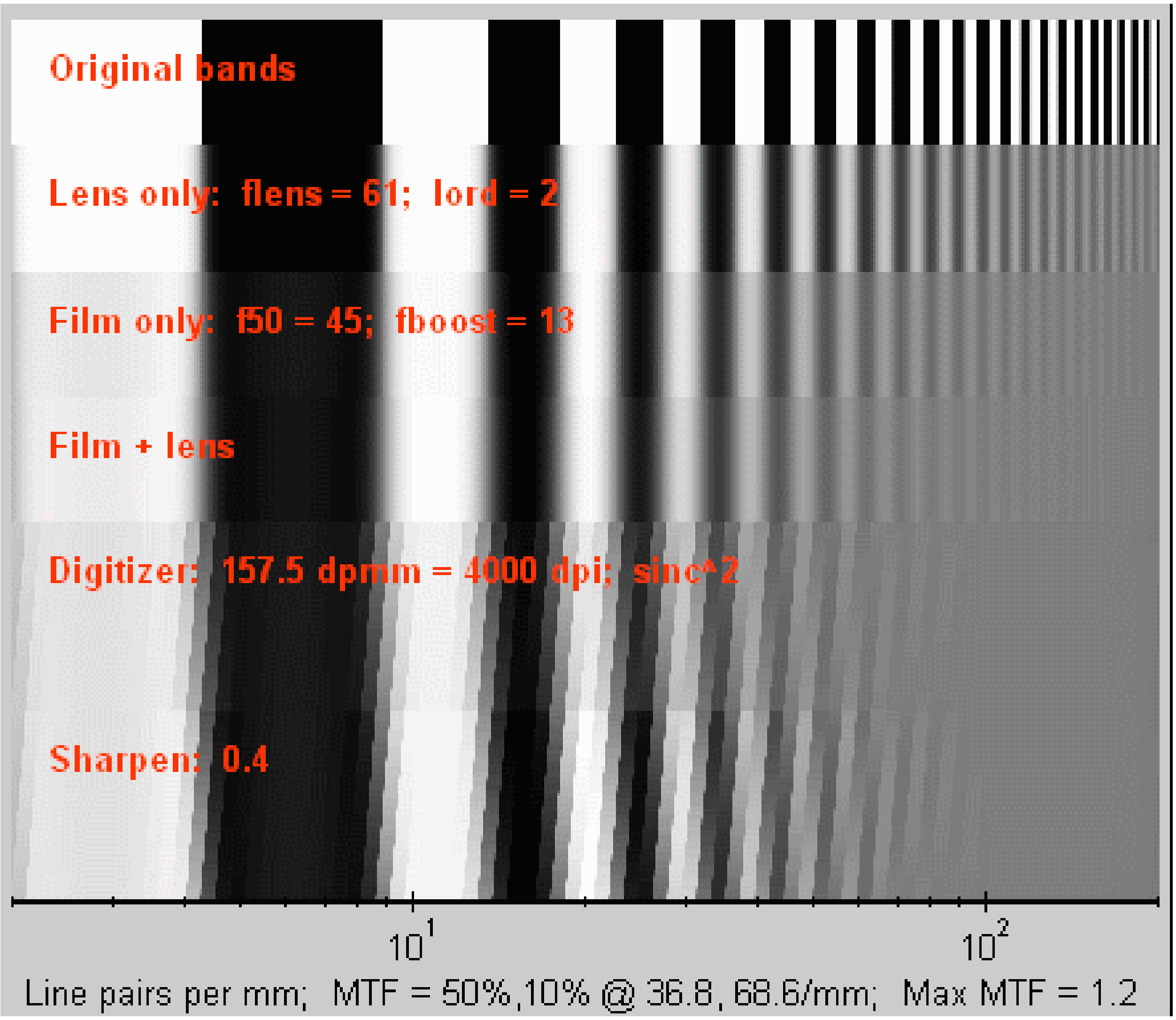
- Film or sensor has MTF measured
- Done with grating directly on sensor
- Eg Fuji fine grain Provia 100 slide film
- 50% MTF frequency (f_{50}) is 42 lp/mm





MTF/AIM and System

- Adding each item degrades system
- Also need to look at f/# for the lens
- Adding digitization degrades image
- This is 4000 dpi digitizing of negative



MTF and Coherent Light

- MTF is sharpest with coherent light
- Decreases as coherence decreases

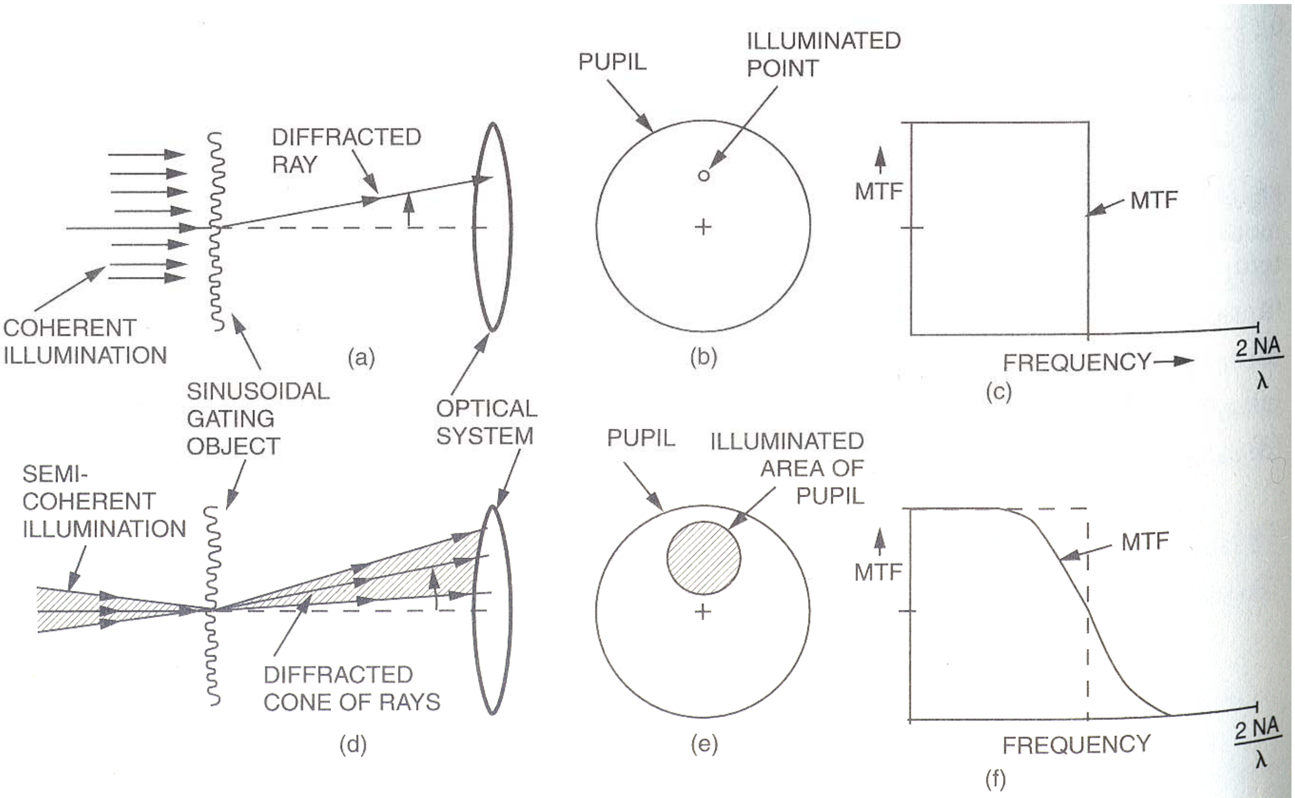


Figure 11.20 (a–c) The MTF with coherent illumination. (d–f) The MTF with semicoherent illumination (which partially fills the pupil).