Group Velocity and Phase Velocity

• Light speed is really the phase velocity v

 $\omega = vk$

Where $\omega = 2\pi f$ = angular frequency k = wave vector = $2\pi/\lambda$

- This is really the velocity of the waves
- However information travels in wave packets
- Wave packets have a frequency variation
- Frequency is modulated by slowly changing $\delta \omega$
- Have $\omega + \delta \omega$ and $\omega \delta \omega$ frequency rang
- This packet moves with wave vector δk
- Thus the group velocity is given by

$$v_g = \frac{d\omega}{dk}$$

• In a vacuum

$$v_g = \frac{d\omega}{dk} = c = phase \ velocity$$

- Each mode in a fiber travels with different group velocity
- Thus get spreading of information packet



FIGURE 1.6 Two slightly different wavelength waves traveling in the same direction result in a wave packet that has an amplitude variation which travels at the group velocity.

Multi Mode and Single Mode Fiber

- Multi mode fiber results in spreading of signal
- But number of modes is set by core diameter and index
- Can change number of modes by changing these
- Want a single mode fiber m=0
- Number of modes are set by

$$m \le \frac{(2V - \phi)}{\pi}$$

• Where V is the V number given by

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}$$

- Thus get a single mode when $V < \pi/2$
- To create single mode make diameter of core very small
- Result much reduced pulse spreading
- \bullet Multi mode fiber 380 μm diameter with 200 μm core
- Single mode $125 \,\mu m$ diameter with $10 \,\mu m$ core
- Also in indexes much smaller



Acceptance Angle and Fibers

- Problem with single mode is harder to get signal into fiber
- Called coupling with the fiber
- Acceptance angle $\theta_a = \alpha_{max}$ max angle light can enter fiber

$$\frac{\sin(\theta_a)}{\sin(90^o - \theta_c)} = \frac{n_1}{n_0} \qquad \sin(\theta_c) = \frac{n_2}{n_1}$$

• Thus

$$sin(\theta_a) = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} = \frac{NA}{n_o}$$

• The fiber's Numerical Aperture is

$$NA = \sqrt{n_1^2 - n_2^2}$$

• Total Acceptance Angle is $2\theta_a$



acceptance con e

Single & Multi-Mode Fiber Acceptance

- Multimode fiber example
- n_1 =1.480, n_2 = 1.460 with a core of 200 μ m

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.480^2 - 1.460^2} = 0.2425$$
$$\theta_a = \arcsin\left(\frac{NA}{n_o}\right) = \arcsin\left(\frac{0.2425}{1}\right) = 14^o$$

- And total acceptance angle is 28°
- Now consider a single mode fiber
- n_1 =1.460, n_2 = 1.464 (only 0.3% larger) with a core of 10 μ m

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.460^2 - 1.464^2} = 0.0.113$$
$$\theta_a = \arcsin\left(\frac{NA}{n_o}\right) = \arcsin\left(\frac{0.113}{1}\right) = 6.5^o$$

• Thus much harder to couple light



Graded Index (GRIN) Fiber

- Due to small acceptance angle want alternative to single mode
- Uuse a GRaded INdex in the core (GRIN)
- Made by doping fiber with material that varies with postion
- Light now bends rather than reflects
- Get acceptance angle close to multimode
- But bit rate much higher than multimode



Fiber Dispersion and Bit Rate

- Dispersion of pulse in fiber sets the limit of bit rate
- Look at the Full Width Half Power (FWHP) of signal $\Delta \tau_{1/2}$
- Two pulse must be at least $2\Delta \tau_{1/2}$ apart to be separated
- Bit rate is thus

$$B \cong \frac{1}{2\Delta \tau_{1/2}}$$

- $\Delta \tau_{1/2}$ increase with distance so measure
- Dispersion = Bandwidth x distance







Comparison of Multimode, GRIN and Single Mode

- Multi mode: high acceptance angle
- High dispersion ~20-100 Mhz km bandwidth product
- Can use LED's for emitters
- Easy to install used for short distance networks
- GRIN acceptance angle near multimode: easy to connect
- Medium dispersion ~300 MHz km bandwidth product
- LED or lasers as emitters
- Medium distance networks
- Single mode small acceptance angle
- Thus harder to interconnect fibers
- Low dispersion 100 GB/s in field
- Laser diodes needed for narrow wavelength

 TABLE 2.3
 Comparison of typical characteristics of multimode step-index, single-mode step-index, and graded-index fibers. (Typical values combined from various sources.)

Property	Multimode step-index fiber	Single-mode step-index fiber	Graded index fiber
$\Delta = (n_1 - n_2)/n_1$	0.02	0.003	0.015
Core diameter (µm)	100	$8.3 (\text{MFD} = 9.3 \mu\text{m})$	62.5
Cladding diameter (µm)	140	125	125
NA	0.3	0.1	0.26
Bandwidth \times distance or	20 – 100 MHz km.	$<3.5 \text{ ps km}^{-1} \text{ nm}^{-1} \text{ at } 1.3 \mu\text{m}$	300 MHz km - 3 GHz km
Dispersion		>100 Gb s ⁻¹ km in common	at 1.3 µm
		use	at 1.3 µm
Attenuation of light	$4 - 6 dB km^{-1}$ at 850 nm	1.8 dB km^{-1} at 850 nm	3 dB km ⁻¹ at 850 nm
	$0.7 - 1 \text{ dB km}^{-1}$ at 1.3 μm	0.34 dB km ⁻¹ at 1.3 μm	$0.6 - 1 dB km^{-1}$ at 1.3 μm
		0.2 dB km ⁻¹ at 1.55 μm	0.3 dB km ⁻¹ at 1.55 μm
Typical light source	Light emitting diode	Lasers, single mode	LED, lasers
	(LED)	injection lasers	
Typical applications	Short haul or subscriber	Long haul communications	Local and wide-area
	local network		networks. Medium haul
	communications		communications

Fiber Bending Loses

- When fiber is bent or has imperfection get increased loss
- Due to change in light angle to below critical angle
- Lose Total Internal Reflection
- The effective absorption coefficient α_B increases greatly
- Smaller radius of curvature, larger the light leaking out







FIGURE 2.33 Measured microbending loss for a 10 cm fiber bent by different amounts of radius of curvature *R*. Single mode fiber with a core diameter of 3.9 µm, cladding radius 48 µm, $\Delta = 0.004$, NA = 0.11, $V \approx 1.67$ and 2.08 (Data extracted and replotted with Δ correction from, A.J. Harris and P.F. Castle, *IEEE J. Light Wave Technology*, Vol. LT14, pp. 34–40, 1986; see original article for discussion of peaks in α_B vs. *R* at 790 nm.)

How Fiber is Made

- Fiber starts with rod of core and cladding
- Heated in drawing furnace- often rotated to make even
- Pulled into a narrow fiber
- Monitor fiber diameter adjusted with pull rate
- Add plastic coating on outside
- Rolled into spool



