Review: Complex Envelope

- \( x_p(t) \): pre-envelope (analytical signal), positive freq only.
- A **bandpass** signal \( x_p(t) \) can be shifted down to the DC. This lowpass version of \( x_p(t) \) is called complex envelope:

\[
\tilde{x}(t) = x_p(t)e^{-j2\pi f_c t},
\]

\[
x_p(t) = \tilde{x}(t)e^{j2\pi f_c t}
\]

- \( \tilde{x}(t) \) is generally complex, because \( \tilde{X}(f) \) is usually not conjugate symmetric.
- \( \tilde{x}(t) \) is a fictitious signal.
Review: Complex Envelope

\[ \tilde{x}(t) \text{ complex} \quad \Rightarrow \quad \tilde{x}(t) = x_I(t) + jx_Q(t) : \]

- \( x_I(t) \): In-phase component,
- \( x_Q(t) \): Quadrature component.

A bandpass deterministic signal can be written as

\[ x(t) = x_I(t) \cos 2\pi f_c t - x_Q(t) \sin 2\pi f_c t \]

Proof:

\[
x(t) = \text{Re}\{x_p(t)\} = \text{Re}\{\tilde{x}(t)e^{j2\pi f_c t}\} \\
= \text{Re}\{(x_I(t) + jx_Q(t))(\cos 2\pi f_c t + j \sin 2\pi f_c t)\} \\
= x_I(t) \cos 2\pi f_c t - x_Q(t) \sin 2\pi f_c t
\]
Definition of Bandpass or Narrowband Random Process

- Definition: A random process $X(t)$ is bandpass or narrowband random process if its power spectral density $S_X(f)$ is nonzero only in a small neighborhood of some high frequency $f_c$.

- The definition of bandpass random process is the generalization of bandpass deterministic signals.
  - Deterministic signals: defined by its Fourier transform
  - Random processes: defined by its power spectral density.

- Notes:
  1. Since $X(t)$ is bandpass, it has zero mean: $E[(X(t)] = 0$.
  2. $f_c$ needs not be the center of the signal bandwidth, or in the signal bandwidth at all (see Problem 8.38).
Narrowband Noise

- Representation of the narrowband noise:

  - psd of a NB noise
  - Example of a NB noise waveform
Narrowband Noise

- Generating $N_I(t)$ and $N_Q(t)$ from $N(t)$, and vice versa:

(See Fig. 3.25)
Narrowband Noise

- Some properties of narrowband noise:

  1. Mean: \[ \mathbb{E}[N_I(t)] = \mathbb{E}[N_Q(t)] = 0. \]

  2. If \( N(t) \) is Gaussian, \( N_I(t) \) and \( N_Q(t) \) are also Gaussian.

  3. If \( N(t) \) is stationary, \( N_I(t) \) and \( N_Q(t) \) are also stationary.
Narrowband Noise

4 Power spectral density: (Proved in HW7)

\[ S_{N_I}(f) = S_{N_Q}(f) = \begin{cases} 
S_N(f - f_c) + S_N(f + f_c), & -B \leq f \leq B. \\
0, & \text{otherwise.}
\end{cases} \]

Example:
Narrowband Noise

\[ S_{N_{I}}(f) = S_{N_{Q}}(f) = \begin{cases} S_{N}(f - f_{c}) + S_{N}(f + f_{c}), & -B \leq f \leq B. \\ 0, & \text{otherwise.} \end{cases} \]

Cautions:

- \( S_{N}(f) \) does not have to be symmetric around \( f_{c} \).
- \( f_{c} \) does not have to be the center of the \( S_{N}(f) \).
Narrowband Noise

5. Variance: \( \text{var}[N_I(t)] = \text{var}[N_Q(t)] = \text{var}[N(t)] \).

Proof: