ENSC327
Communications Systems
25: ISI and Pulse Shaping (Ch 6)

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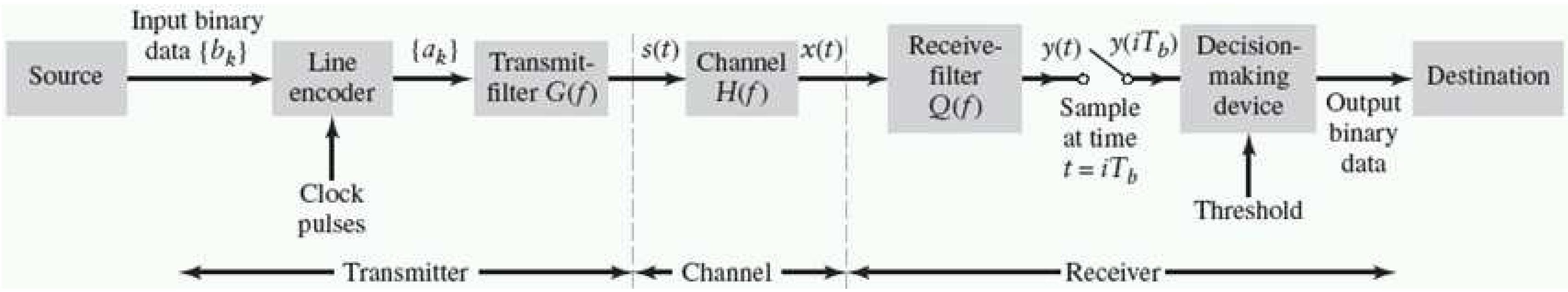
Outline

- Chapter 6: Baseband Data Transmission
  - No modulation, sending pulse sequences directly
  - Suitable for lowpass channels (eg, coaxial cables)
  - (Chap 7 will study digital bandpass modulation, for bandpass channels (eg, wireless) that need high-freq carrier)
- ISI and Pulse Shaping
  - ISI Definition
  - Zero ISI Condition
  - Nyquist Pulse Shaping Condition
  - Nyquist bandwidth
  - Nyquist channel

Introduction

- A digital communication system involves the following operations:
  - Transmitter: maps the digital information to analog electromagnetic energy.
  - Receiver: records the analog electromagnetic energy, and recover the digital information.
- The system can introduces two kinds of distortions:
  - Channel noise: due to random and unpredictable physical phenomena.
    - Studied in Chap. 9 and 10.
  - Intersymbol interference (ISI): due to imperfections in the frequency response of the channel.
    - The main issues is that previously transmitted symbols affect the current received symbol.
    - Studied in Chap. 6.

6.1 Baseband Transmission with PAM

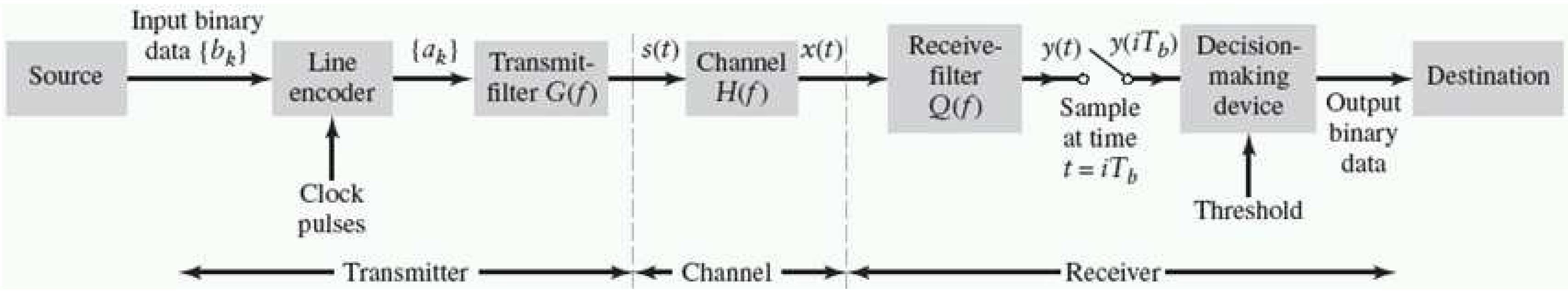


- Input: binary data, bk = 0 or 1, with duration T_b.
  - Bit rate: $1/T_b$ bits/second.

- Line encoder (Chap. 5): electrical representation of the binary sequences, e.g.,

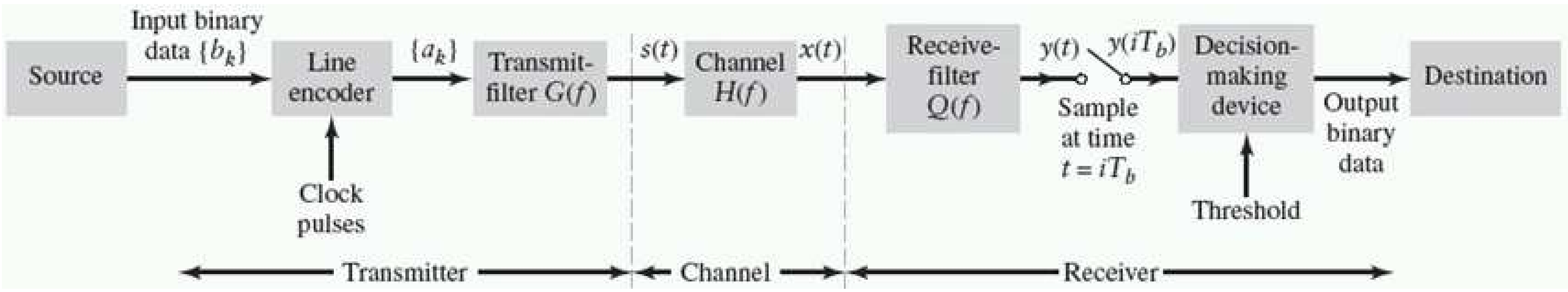
$$a_k = \begin{cases} +1, & b_k = 1, \\ -1, & b_k = 0. \end{cases}$$

6.1 Baseband Transmission with PAM



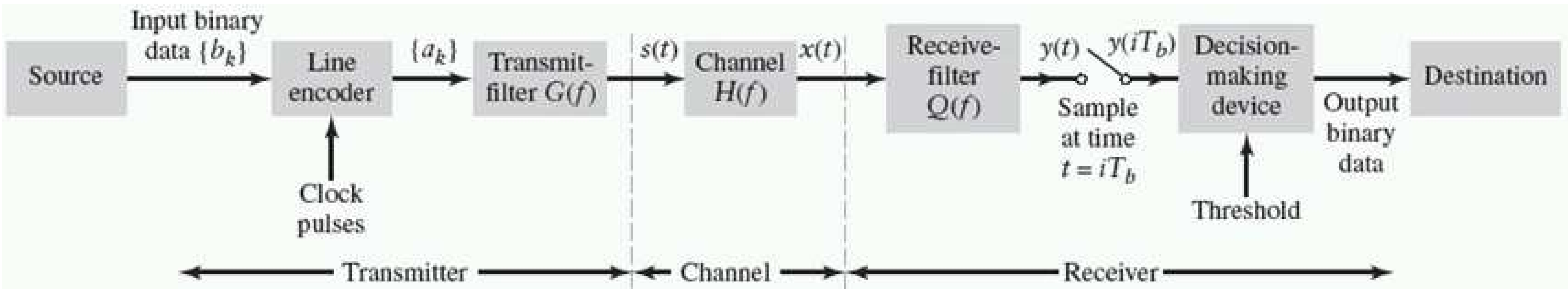
- Transmit filter: use pulses of different amplitudes to represent one or more binary bits.
- The basic shape is represented by a filter g(t) or G(f)
- The output discrete PAM signal from the transmitter:

6.1 Baseband Transmission with PAM



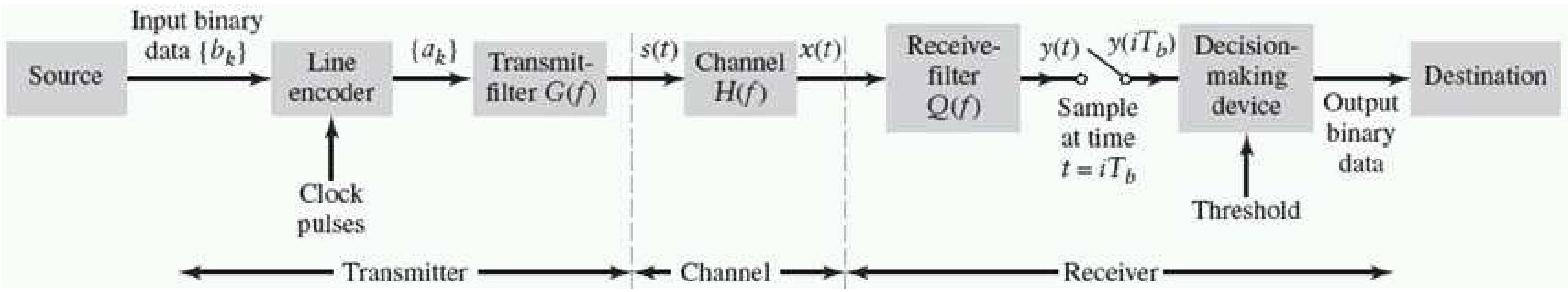
- Channel:
  - If the channel is ideal, no distortion will be introduced.
  - A practical channel can be represented by a linear, time invariant (LTI) filter.
- The channel output: the convolution of the transmitted signal and the channel impulse response. This introduces intersymbol interference (ISI)

6.1 Baseband Transmission with PAM



- Receiver can be represented by another filter:
  - To remove noise and cancel channel distortion.
- The output is
- The output is then sampled synchronously with the clock at the transmitter.
- Finally, a decision-making device is used to recover the binary bits. Different methods can be used, e.g.,
  - Threshold, equalization, maximal likelihood decoding

6.1 Baseband Transmission

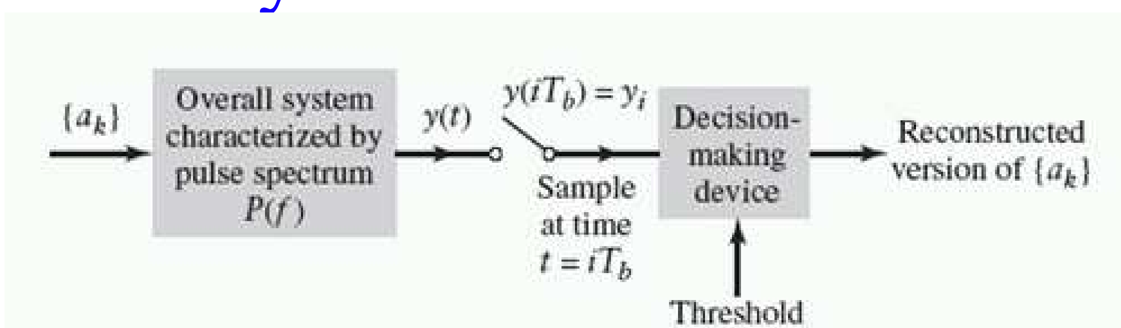


- The model can be used to study baseband system as well as bandpass system (via complex envelope)
- Objectives of pulse shaping:
  - Given h(t), design g(t) and q(t) to eliminate the intersymbol interference (ISI).
  - The transmitted signal should have small bandwidth to meet the bandwidth constraint of the channel.

Equivalent Model

- The joint effect of g(t), h(t), and q(t) is a composite filter:

➔ Equivalent system model:



- What is the condition on p(t) to eliminate ISI?

Mathematical Definition of ISI

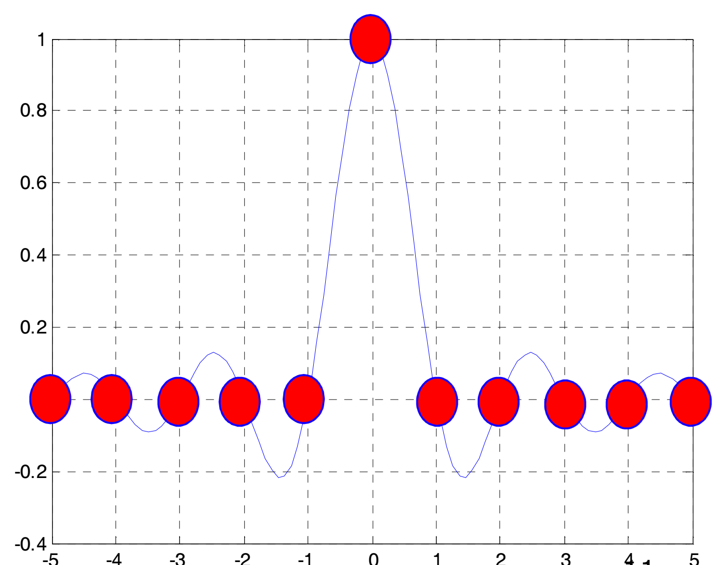
source $\sum_k a_k \delta(t - kT_b)$ $\Rightarrow$ $y(t) = \sum_k a_k p(t - kT_b)$

Zero ISI Condition

- To get $\sum_{k \neq i} a_k p_{i-k} = 0$ for any ak, the sample of p(t) should satisfy $$p_i = p(iT_b) = \begin{cases} \sqrt{E}, & \text{i} = 0, \\ 0, & \text{i} \neq 0. \end{cases}$$

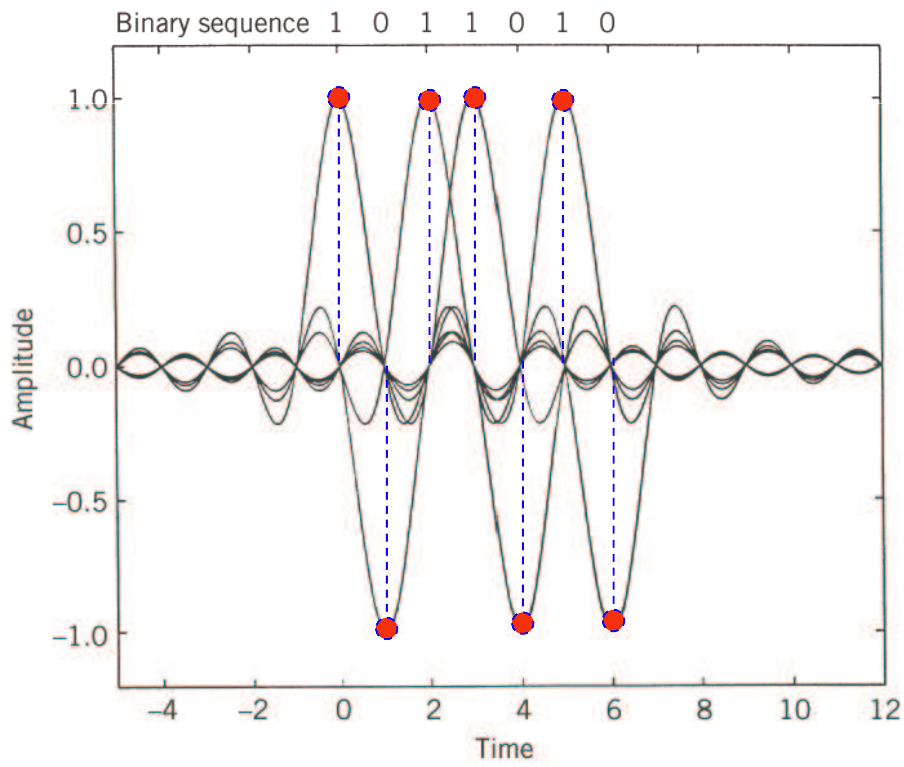
- This is true if, for example,

$$p(t) = \sqrt{E}\text{sinc}(\frac{t}{T_b}).$$

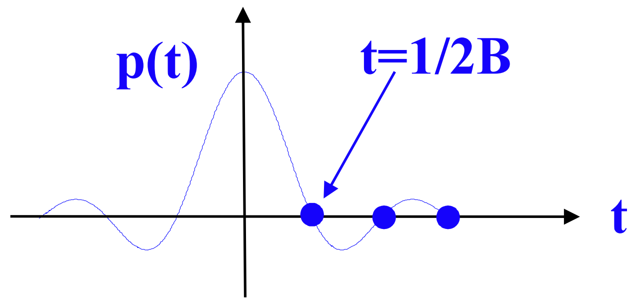


Zero ISI Condition

Example: A series of sinc pulses corresponding to the sequence 1011010. No ISI at sampling times.



Zero ISI Condition



- Is sinc function a good composite filter? No!
  - Infinite duration
  - Non-causal
  - A small timing jitter can cause large ISI.
- Any other possibilities for p(t)?
  - Yes. There are infinite number of solutions!
  - What is the general condition for p(t) to be ISI free?

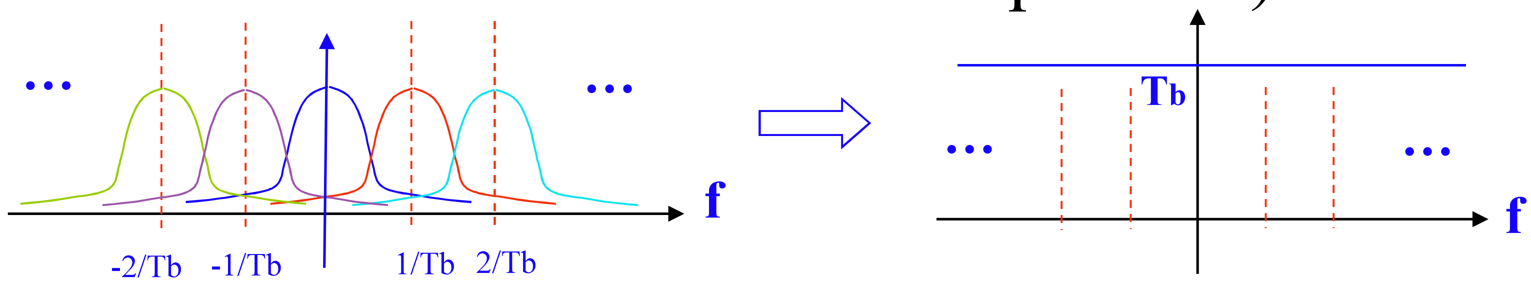
Nyquist Pulse Shaping Condition

The condition $p_i = p(iT_b) = \begin{cases} \sqrt{E}, & \mathrm{i} = 0, \\ 0, & \mathrm{i} \neq 0. \end{cases}$ is achieved

if the frequency response of p(t) satisfies

$$\sum_{k=-\infty}^{\infty} P(f + \frac{k}{T_b}) = T_b\sqrt{E}.$$

(The sum of all shifted versions of P(f) by multiple of $1 / T_b$ should be constant at all frequencies).



Nyquist Pulse Shaping Condition

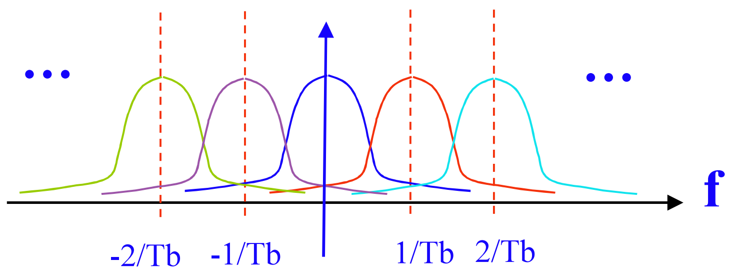
Proof:

Nyquist Pulse Shaping Condition

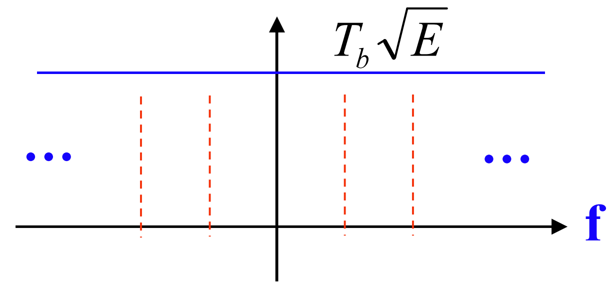
Nyquist Pulse Shaping Condition

Note that $\sum_{k=-\infty}^{\infty} P(f+\frac{k}{T_b})$ is periodic signal with period 1 / Tb $\Rightarrow$

The condition $\sum_{k=-\infty}^{\infty} P(f+\frac{k}{T_b}) = T_b\sqrt{\mathrm{E}}$ should be true for all f.



$\Rightarrow$



The condition is not difficult to meet (e.g., by raised cosine pulse in Chap 6.4)

Nyquist Channel and Nyquist Bandwidth

- Special case:
- If the bandwidth of P(f) is $B_0 = \dfrac{1}{2T_b}$ then to satisfy

$$\sum_{k=-\infty}^{\infty} P(f + \frac{k}{T_b}) = T_b \sqrt{E},$$

P(f) must be

Nyquist Pulse Shaping Condition

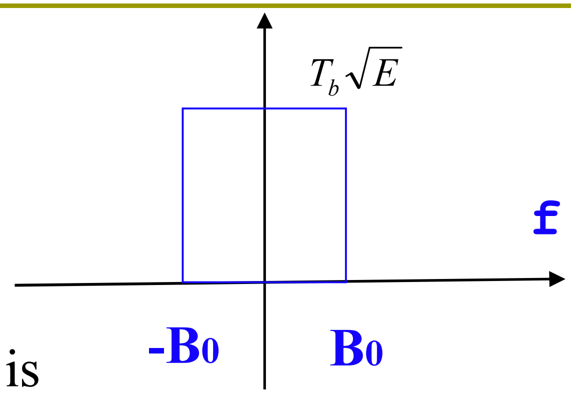
- If the bandwidth of P(f) is less than $1 / (2T_b)$,

Nyquist Pulse Shaping Condition

- If the bandwidth of P(f) is greater than $1 / (2T_b)$,

Nyquist Pulse Shaping Condition

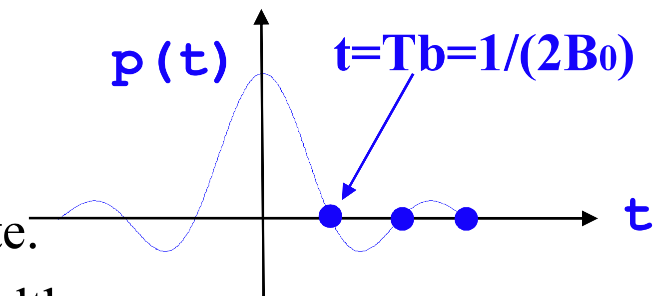
$$\sum_{k=-\infty}^{\infty} P(f+\frac{k}{T_b}) = T_b\sqrt{E}.$$



- Nyquist channel and Nyquist bandwidth:
  - If the bit duration is fixed at Tb, or bit rate Rb is 1/Tb, then the transmission bandwidth is

$$B_T \geq \frac{1}{2T_b} = \frac{1}{2}R_b$$

    so the minimal bandwidth is half of the bit rate.



  - On the other hand, if the transmission bandwidth is fixed at BT, then

$$R_b \leq 2B_T$$

  - the maximal transmitted bits per second (bit rate) is twice of the bandwidth.