ENSC327 Communication Systems 26: Raised Cosine Pulse and Eye Diagram

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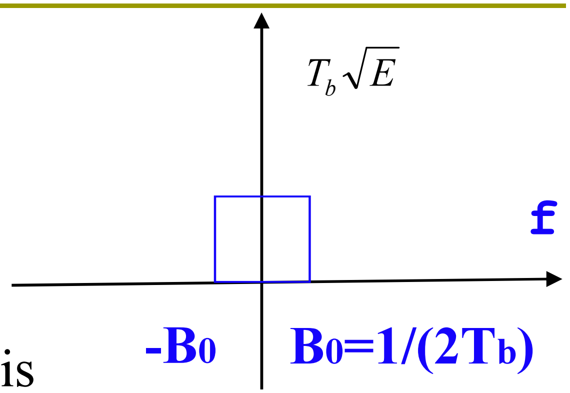
Outline

- 6.4 Raised cosine pulse spectrum
- 6.6 Eye Diagram

Nyquist Pulse Shaping Condition

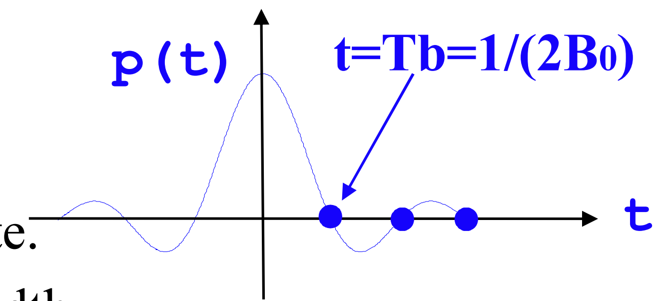
$$\sum_{k=-\infty}^{\infty} P(f + \frac{k}{T_b}) = T_b \sqrt{E}.$$



- Nyquist channel and Nyquist bandwidth:
  - If the bit duration is fixed at T_b, or bit rate R_b is $1/T_b$, then the transmission bandwidth is

$$B_T \geq B_0 = \frac{1}{2T_b} = \frac{1}{2} R_b$$

so the minimal bandwidth is half of the bit rate.



  - On the other hand, if the transmission bandwidth is fixed at B_T, then

$$R_b \leq 2B_T$$

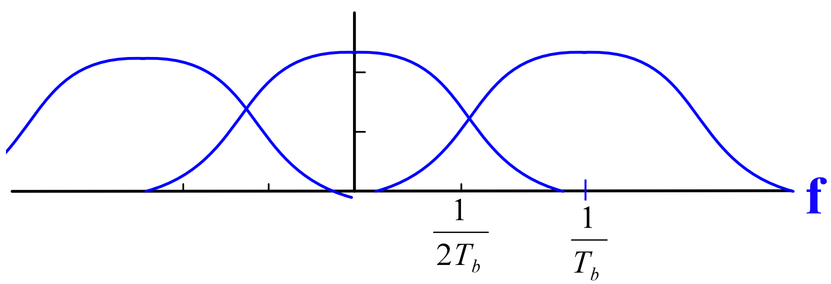
  - the maximal transmitted bits per second (bit rate) is twice of the bandwidth.

Examples of zero ISI spectrum

- If BW of P(f) is greater than 1/(2Tb), there are infinite possible solutions to satisfy

$$\sum_{k=-\infty}^{\infty} P(f+\frac{k}{T_b}) = T_b\sqrt{E}.$$

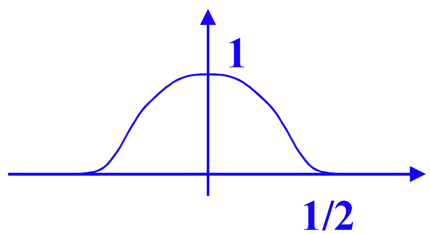
- In particular, if the BW of $P(f)$ is less than $1/Tb$, then the summation only involves two terms at each frequency, which can be easily satisfied:

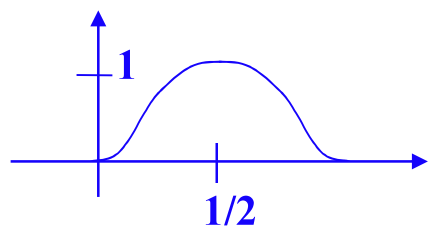


Raised Cosine Spectrum

- For example, the previous case can be achieved by using raised cosine function.
- If $T_b = 2$. Let

$$P(f) = 1/2\left(1 + \cos 2\pi f\right)$$





Many other functions also satisfy the requirement, for example,

P(f) can be a triangle. But the cosine function has some nice properties.

Raised Cosine Spectrum

- The previous example can be generalized.
- Raised Cosine Spectrum: Let $2B_0 = 1/T_b$, or $T_b\sqrt{E} = \dfrac{\sqrt{E}}{2B_0}$

f_1 : cut-off frequency.

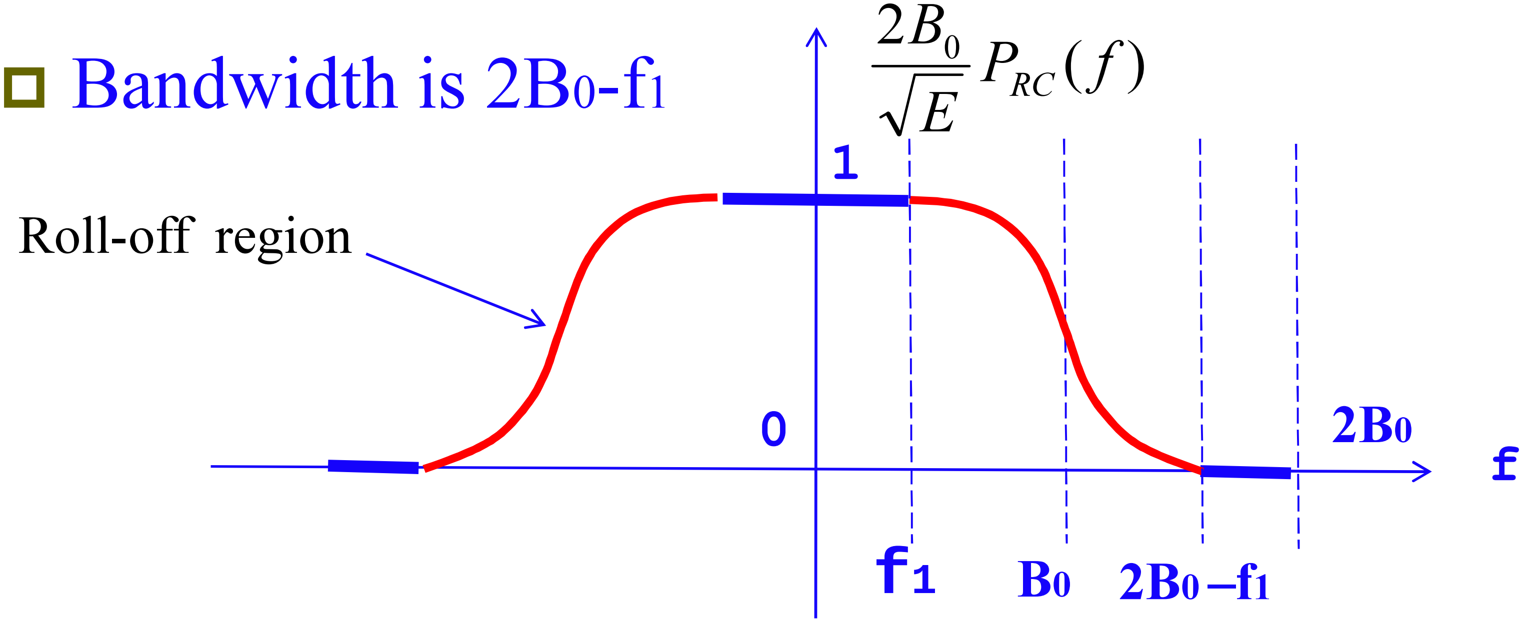
Matlab functions:

rcosfir( ): FIR

rcosine( ): FIR/IIR

Example: Raised Cosine Spectrum

- Bandwidth is $2B_0$-f_1



- Constant for $f < f1$, raised cosine for f in $[f_1, 2B_0 – f_1]$.

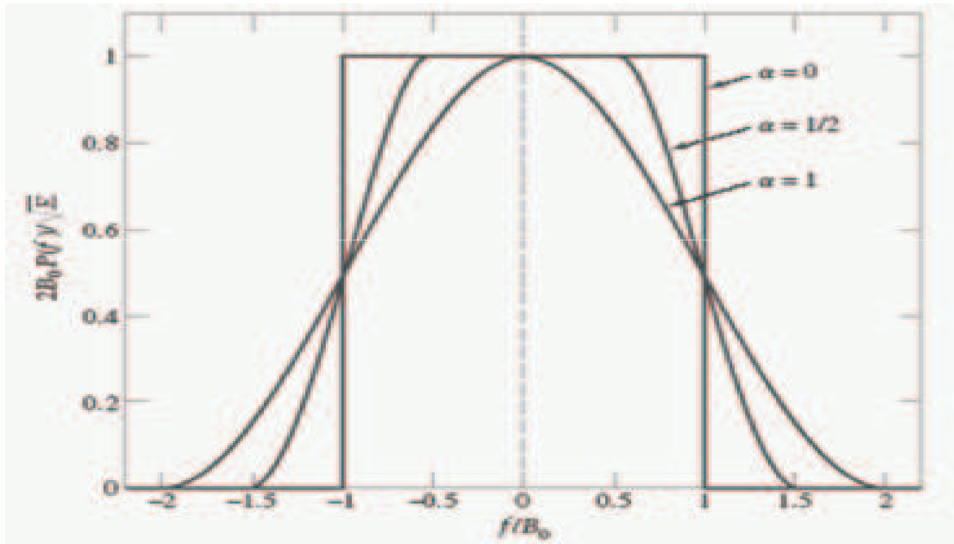
$$\frac{\pi(|f| - f_1)}{2(B_0 - f_1)}:$$ scales $[f_1, 2B_0 – f_1]$ to $[0, \pi]$.

$$1 + \cos\left(\frac{\pi(|f| - f_1)}{2(B_0 - f_1)}\right):$$ Mapped to $1 + \cos(x)$, x in $[0, \pi]$.

Example: Raised Cosine Spectrum

Roll-off factor, or excess-bandwidth factor (over Nyquist bw B_0):

Cut-off frequency:



Bandwidth:

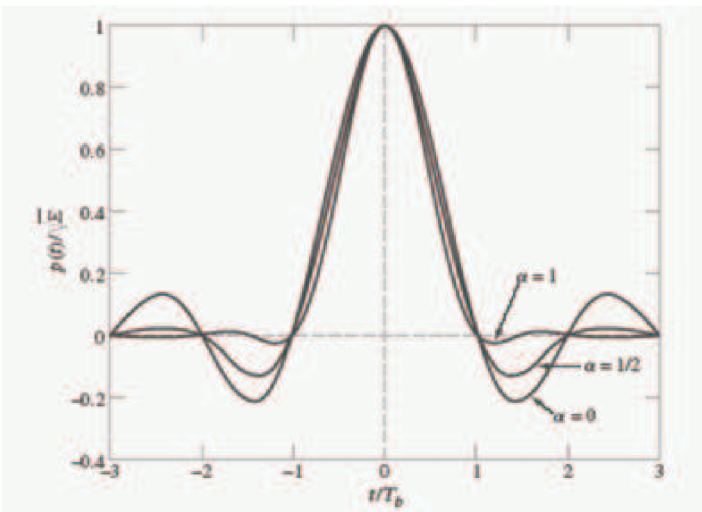
- f1 and α can be adjusted to control the trade-off between the bandwidth and length of the impulse response.
- α > 0: more bandwidth than Nyquist, but filter is shorter.

$$f_1 = (1-\alpha)B_0$$

Example: Raised Cosine Spectrum

A nice thing about raised cosine window is that its impulse response has closed-form expression:

This is a scaled sinc function: $a = 0: \rightarrow \mathrm{p(t)} = \sqrt{E}\mathrm{sinc}(2B_0 \mathrm{t}).$



- If rolloff factor α increases
  - f_1 decreases
  - Bandwidth increases
  - But p(t) is shorter ➔ More robust to timing error.

Root Raised Cosine Pulse

- If the raised cosine pulse is used, we have

$$p(t) = g(t) * h(t) * q(t) = \text{Raised cosine function}$$

- One way to achieve this is:

- Given the channel H(f), use the first equation to find transmitter filter G(f).
- Received filter is the root raised cosine.

Example: Bandwidth of T1 system

- T1 system: multiplexing 24 telephone inputs.
- Bit duration: $T_b = 0.647\mu s$
- ➔ Bit rate: $R_b = 1/T_b = 1.544$ Mb/s
- The Nyquist bandwidth is (minimal required BW):

- A more realistic choice is to use α = 1:

- Note: the unit of bandwidth is Hz,

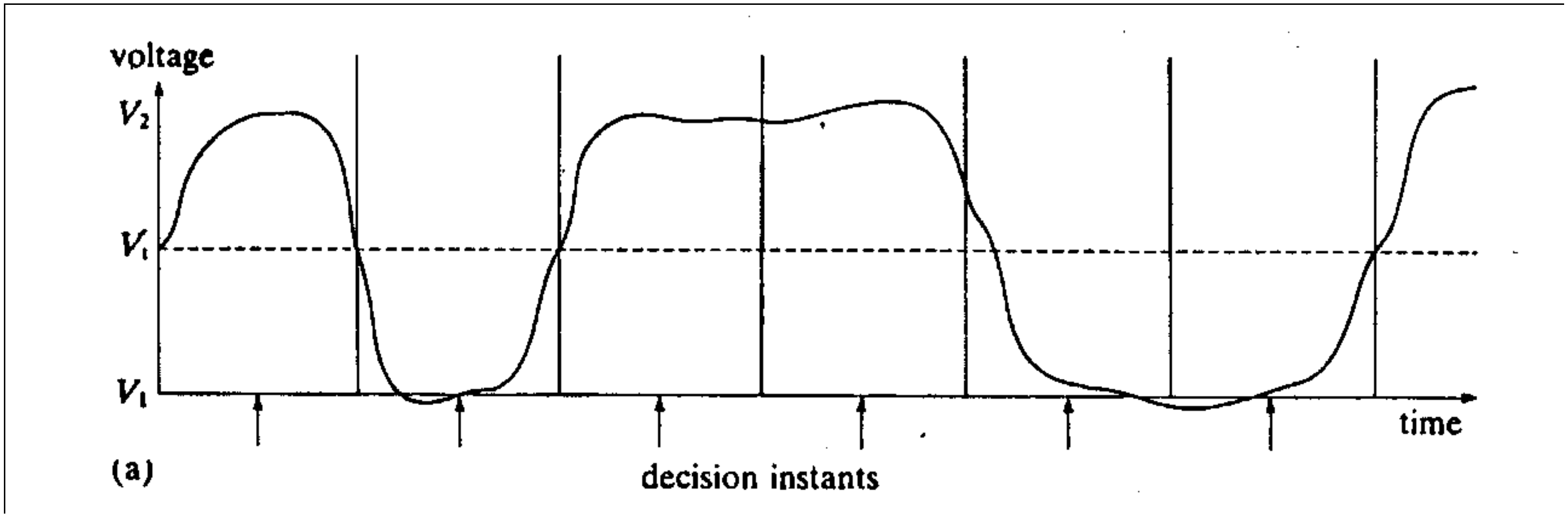
  and the unit of bit rate is bits/sec (b/s).

Outline

- 6.4 Raised cosine pulse spectrum
- 6.6 Eye Diagram

6.6 Eye Diagram

- An effective way to observe ISI



  - Extract one or more symbol periods
  - Superimpose all possible results
  - Can be easily obtained by oscilloscope

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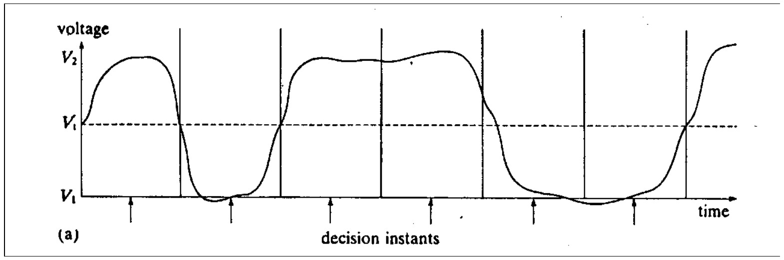
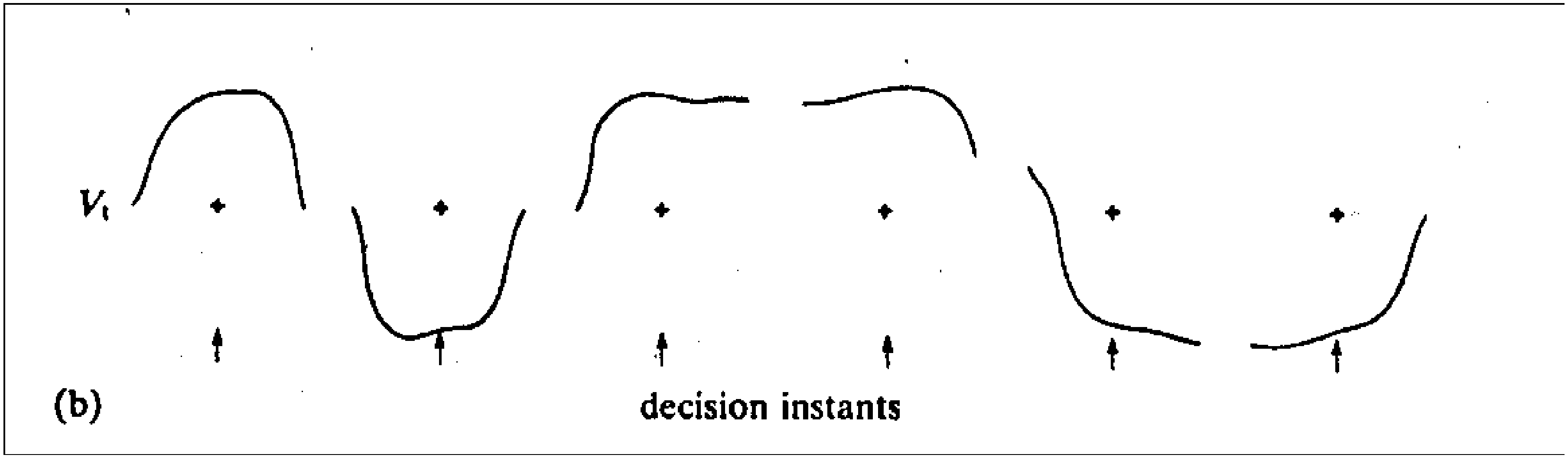
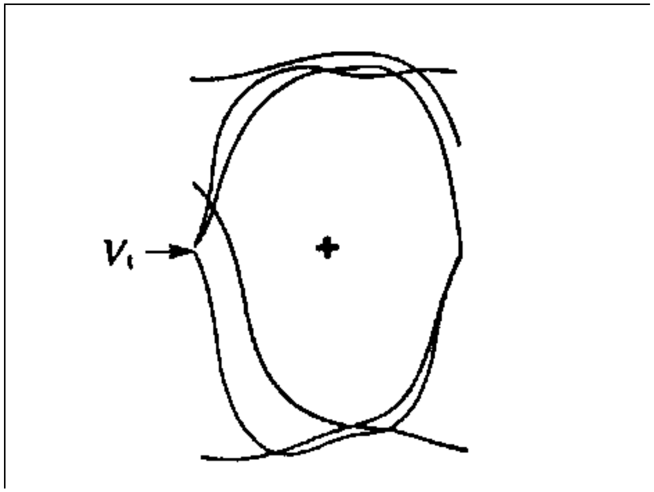


voltage
V_2
V_t
V_1
time
(a)
decision instants



V_t
(b)
decision instants



V_t

http://www.highfrequencyelectronics.com/Archives/Nov05/HFE1105_Tutorial.pdf

cont …

- Easy to show on a scope

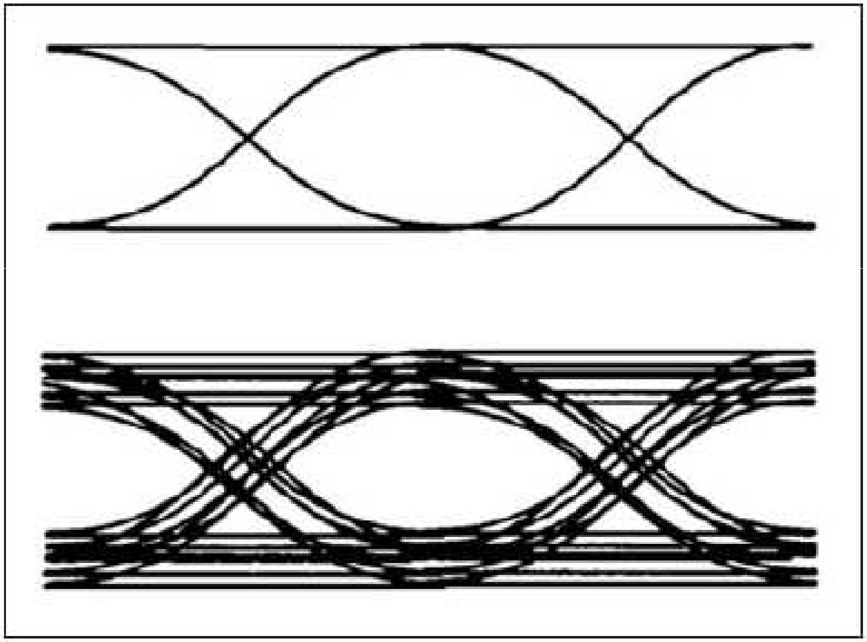
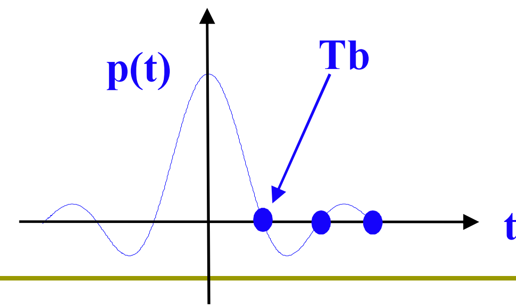
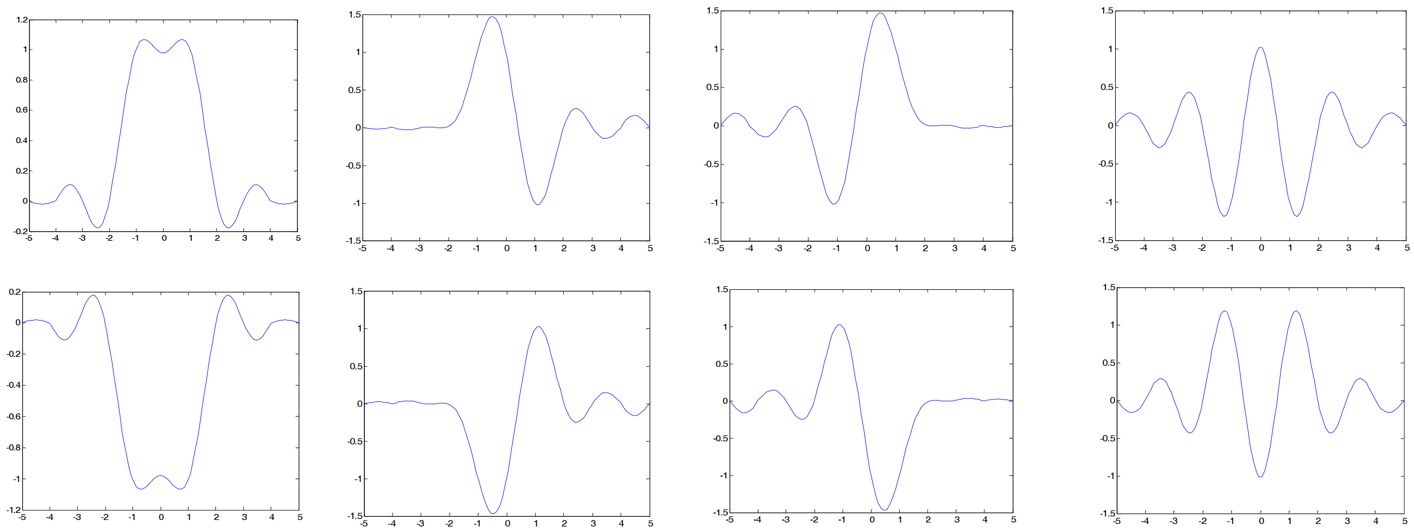


Figure 1 · At top is an undistorted eye diagram of a band-limited digital signal. The bottom eye pattern includes amplitude (noise) and phase (timing) errors. The various transition points can provide insight into the nature of the impairments.

Example: sinc pulse

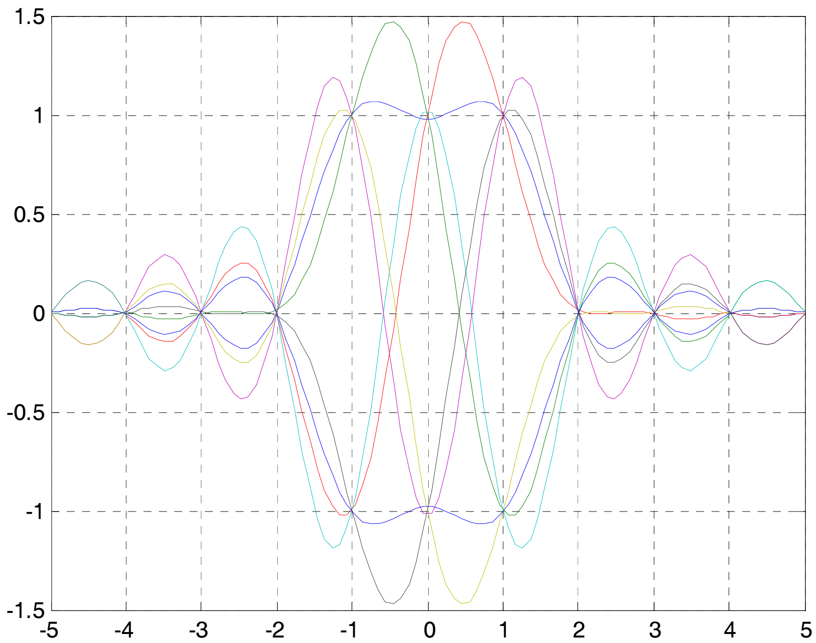


- If the composite filter p(t) is a sinc pulse:
- If only interferences from the immediate neighboring pulses are considered: ==> 8 possibilities
  - {1, 1, 1}, {1, 1, -1}, {-1, 1, 1}, {-1, 1, -1}
  - {-1, -1, -1}, {-1, -1, 1}, {1, -1, -1}, {1, -1, 1}

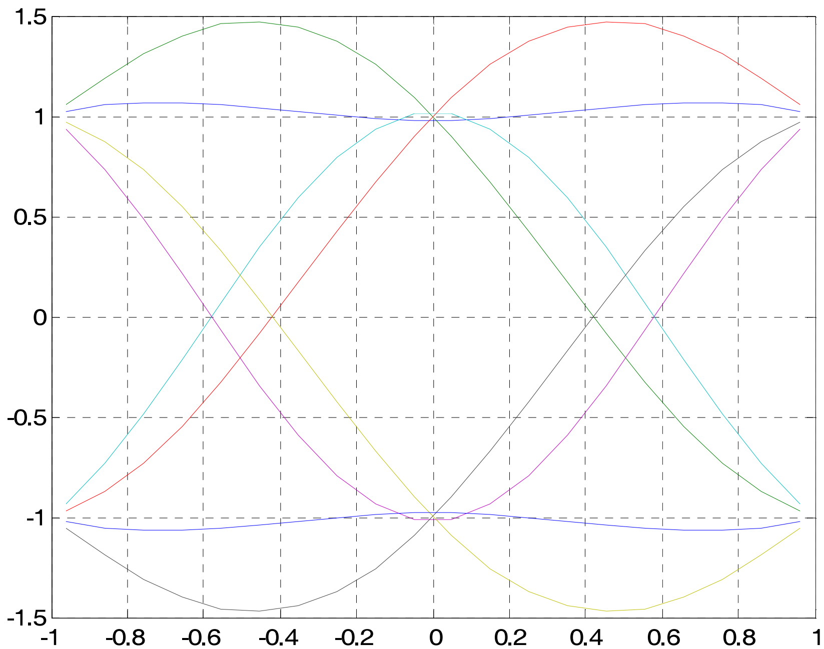


Example: sinc pulse

Superimposed results

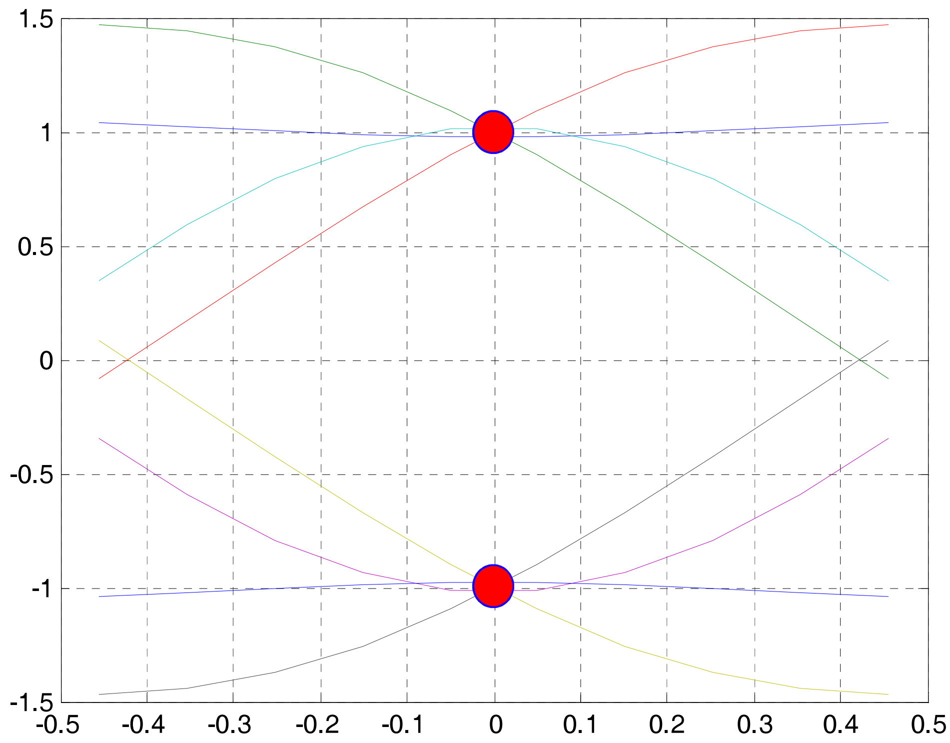


Details within [-Tb, Tb]



Example: sinc pulse

Details within [-Tb/2, Tb/2]



If there is no noise and no timing error, the data 1 or -1 can be perfected detected at time 0.

Eye Diagram Summary

- Practical eye diagrams have some errors:

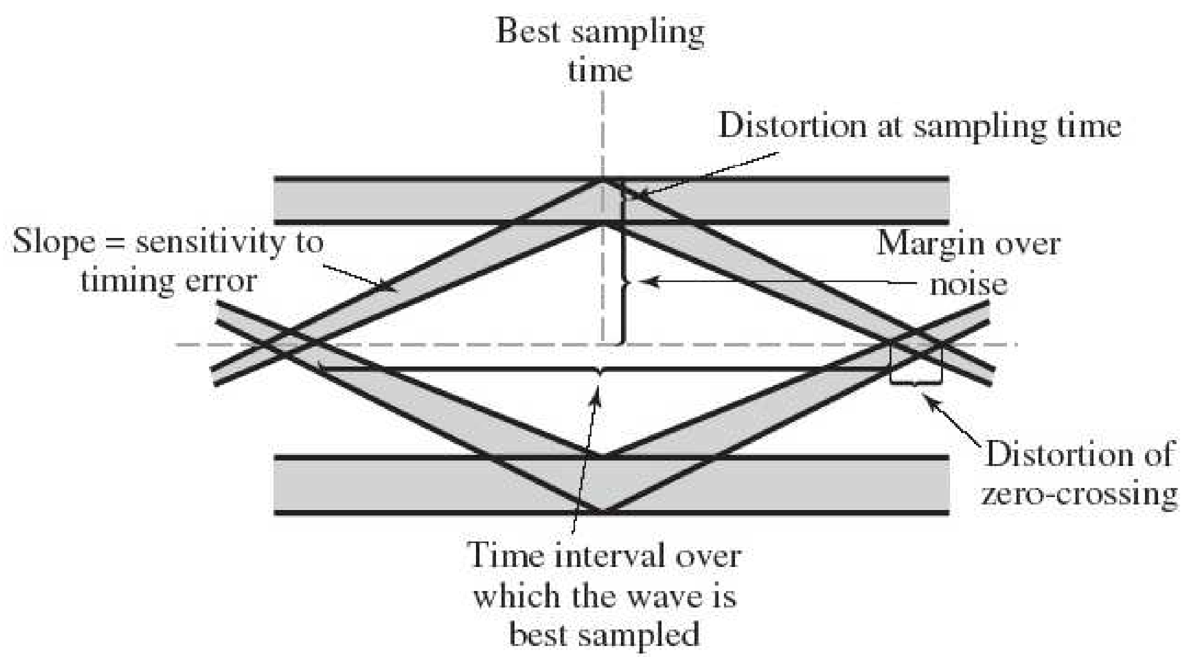


FIGURE 6.6 Interpretation of the eye pattern for a baseband binary data transmission system.