Question 1.

Go to the link below to install Matlab on your personal computer.

https://www.sfu.ca/itservices/technical/software/matlab/NamedUserLicense.html

Question 2.

Read (or re-familiarize yourself with) Matlab by reading the following introductory materials

1. the MIT Matlab Tutorial at http://www2.ensc.sfu.ca/people/faculty/ho/ENSC380/MIT-Matlab.pdf, and/or

2. the Matlab Primer at http://www2.ensc.sfu.ca/people/faculty/ho/ENSC380/Matlab-Primer.pdf

Question 3:

Using Matlab, store samples of the pulse

\[ p(t) = \begin{cases} \sin(\pi t), & 0 \leq t \leq 1; \\ 0, & \text{otherwise} \end{cases} \]

in an array \( P \). Use a sampling interval of \( \Delta t = 1/16 \).

Using Matlab’s “plot” command, plot \( p(t) \) in the interval \( 0 \leq t \leq 2 \). Label your axes and provide grid lines on your plot.

Question 4:

General the following binary, serially modulated signal using Matlab, where \( p(t) \) is the sine-pulse above, \( a_n \in \{+1,-1\} \) is the \( n \)-th (random) data bit (which is either +1 or -1), and \( N \) is the total number of bits generated.
\[ v(t) = \sum_{n=0}^{N} a_n p(t-n) \]

Just like in the last question, use a sampling interval of \( \Delta t = 1/16 \). Plot your \( v(t) \) for the interval \( 0 \leq t \leq 10 \). Provide a copy of your Matlab codes.

Some Matlab commands that you may need in this exercise are

1. Do loop based on the “for n=1:N”..... “end” format
2. Concatenation of sub-arrays to form a larger array
3. the uniform random generator function “rand”
4. the “if”....“then”....“else” function
5. the “sign” function

**Question 5: Circular Complex Random Variables**

Let \( x \) and \( y \) be two independent and identically distributed (iid) zero-mean real value random variables. Both are uniformly distributed in the interval \([-1, 1] \), and zero elsewhere

(a) Sketch the pdfs of the two random variables, as well as their joint pdf.
(b) Is the complex random variable \( z = x + jy \) proper? Is it circular complex?

**Question 6: Problem 2.5 from Textbook** (reproduced below)

2.5 Carry out the Gram-Schmidt orthogonalization of the signals in Figure 2.2–1(a) in the order \( s_4(t), s_3(t), s_1(t) \), and thus obtain a set of orthonormal functions \( \{ f_m(t) \} \). Then determine the vector representation of the signals \( \{ s_n(t) \} \) by using the orthonormal functions \( \{ f_m(t) \} \). Also determine the signal energies.
Question 7: Problem 2.10 (reproduced below)

Consider the three waveforms \( f_n(t) \) shown below.

(a) Show that these waveforms are orthonormal.
(b) Express the waveform \( x(t) \) as a linear combination of \( f_n(t), \ n = 1, 2, 3, \) if

\[
x(t) = \begin{cases} 
-1 & 0 \leq t < 1 \\
1 & 1 \leq t < 3 \\
-1 & 3 \leq t < 4 
\end{cases}
\]

and determine the weighting coefficients.
Solution

Question 3

The matlab codes and results are shown below. Note that I deliberately do not generate a sample at \( t = 1 \) because this will facilitate the generation of the modulated signal \( v(t) \) from the array \( P \) in the next question.

```matlab
% This program generate a half sine pulse
% t=[0:15]/16; % the array of sampling times; altogether 16 samples
p=sin(pi*t); % the half sine pulse is stored in this array "p"
% plot(t,p) % generate a linear plot of p(t) versus t
grid % generate grid lines on the plot
xlabel('Time t') % label the x-axis
ylabel('p(t)') % label the y-axis
axis([0,2,0,2]) % limit the plotting range to be 0<t<2, 0<p(t)<2
```

![Plot of half sine pulse](image)

Question 4

The matlab codes and results are shown below. The first few lines of the codes were taken from the previous question. This gives us the basic pulse shape, store in the array "p". The modulated signal \( s(t) \) is stored in the array "s", which is obtained by concatenating \( a_n \cdot p \), for \( n=0 \) to \( 10 \). The concatenation
is equivalent to the “\( p(t - n) \)” in \( s(t) \). Note that strict concatenation can be used here because the duration of \( p(t) \) is 1, the same as the pulse repetition period. If the pulse duration is longer than 1, then overlap and add is needed.

% This program generates a random binary antipodal signal using a half-sine pulse
% t=[0:15]/16; % the array of sampling times; altogether 16 samples
p=sin(pi*t); % the half sine pulse is stored in the array "p"
% s=[]; % initial "s" is the empty vector; it will eventually be the modulated signal
% for n=1:11 % Matlab does not allow the use of 0 as an array index
a(n)=2*round(rand)-1; % the n-th random data bit from the set \{+1,-1\}
s=[s,a(n)*p]; % concatenate current waveform segment with all previous ones
end
% t=[0:11*16-1]/16; % the array "t" now stores the sampling times of s(t)
plot(t,s) % generate a linear plot of p(t) versus t
grid % generate grid lines on the plot
xlabel('Time t') % label the x-axis
ylabel('s(t)') % label the y-axis
axis([0,10,-1.5,1.5]) % limit the plotting range to be 0<t<10, -1.5<s(t)<1/5

Modulated signal for a random pattern of (1 1 -1 1 -1 -1 1 1 1 1 -1)
Problem 5

(a) \( p(x) = \begin{cases} 
1/2, & -1 \leq x < 1 \\
0, & \text{elsewhere} 
\end{cases} \),

\( p(y) = \begin{cases} 
1/2, & -1 \leq y < 1 \\
0, & \text{elsewhere} 
\end{cases} \), and

\( p(x, y) = p(x) p(y) = \begin{cases} 
1/4, & -1 \leq x, y < 1 \\
0, & \text{elsewhere} 
\end{cases} \)

(b) The complex variable \( z = x + jy \) is proper because \( \sigma_x^2 = \sigma_y^2 = E[x^2] = \int_{-1}^{1} x^2 p(x) dx = \frac{2}{3} \) (i.e. identical variance) and \( E[xy] = E[x] E[y] = 0 \) (uncorrelated).

If it is not circular symmetric though, i.e. its pdf is not a function of \( |z| \) only, but also depends on individual values of \( x \) and \( y \). As an example, let the initial value of \( z \) is \( z_0 = 1 + j \). At this location, \( p(z_0) = 1/4 \). If we rotate \( z_0 \) by \( \pi/4 \) to obtain, \( z_1 = z_0 e^{j\pi/4} = j\sqrt{2} \), the pdf at this location is 0.
The first basis function is:
\[ g_4(t) = \frac{s_4(t)}{\sqrt{E_4}} = \frac{s_4(t)}{\sqrt{3}} = \begin{cases} -1/\sqrt{3}, & 0 \leq t \leq 3 \\ 0, & \text{o.w.} \end{cases} \]

Then, for the second basis function:
\[ c_{43} = \int_{-\infty}^{\infty} s_3(t)g_4(t)dt = -1/\sqrt{3} \Rightarrow \] 
\[ g_3(t) = s_3(t) - c_{43}g_4(t) = \begin{cases} 2/3, & 0 \leq t \leq 2 \\ -4/3, & 2 \leq t \leq 3 \\ 0, & \text{o.w.} \end{cases} \]

Hence:
\[ g_3(t) = \frac{g_3(t)}{\sqrt{E_3}} = \begin{cases} 1/\sqrt{6}, & 0 \leq t \leq 2 \\ -2/\sqrt{6}, & 2 \leq t \leq 3 \\ 0, & \text{o.w.} \end{cases} \]

where \( E_3 \) denotes the energy of \( g_3(t) \):
\[ E_3 = \int_0^3 (g_3(t))^2 dt = \frac{8}{3}. \]

For the third basis function:
\[ c_{42} = \int_{-\infty}^{\infty} s_2(t)g_4(t)dt = 0 \quad \text{and} \quad c_{32} = \int_{-\infty}^{\infty} s_2(t)g_3(t)dt = 0 \]

Hence:
\[ g_2(t) = s_2(t) - c_{42}g_4(t) - c_{32}g_3(t) = s_2(t) \]

and
\[ g_2(t) = \frac{g_2(t)}{\sqrt{E_2}} = \begin{cases} 1/\sqrt{2}, & 0 \leq t \leq 1 \\ -1/\sqrt{2}, & 1 \leq t \leq 2 \\ 0, & \text{o.w.} \end{cases} \]

where \( E_2 = \int_0^2 (s_2(t))^2 dt = 2 \).

Finally for the fourth basis function:
\[ c_{41} = \int_{-\infty}^{\infty} s_1(t)g_4(t)dt = -2/\sqrt{3}, \quad c_{31} = \int_{-\infty}^{\infty} s_1(t)g_3(t)dt = 2/\sqrt{6}, \quad c_{21} = 0 \]

Hence:
\[ g_1(t) = s_1(t) - c_{41}g_4(t) - c_{31}g_3(t) - c_{21}g_2(t) = 0 \Rightarrow g_1(t) = 0 \]

The last result is expected, since the dimensionality of the vector space generated by these signals is 3. Based on the basis functions \((g_2(t), g_3(t), g_4(t))\) the basis representation of the signals is:
\[ s_4 = (0, 0, \sqrt{3}) \Rightarrow E_4 = 3 \]
\[ s_3 = (0, \sqrt{8/3}, -1/\sqrt{3}) \Rightarrow E_3 = 3 \]
\[ s_2 = (\sqrt{2}, 0, 0) \Rightarrow E_2 = 2 \]
\[ s_1 = (2/\sqrt{6}, -2/\sqrt{3}, 0) \Rightarrow E_1 = 2 \]
Problem 2.10

a. To show that the waveforms $f_n(t)$, $n = 1, \ldots, 3$ are orthogonal we have to prove that:

$$\int_{-\infty}^{\infty} f_m(t)f_n(t)dt = 0, \quad m \neq n$$

Clearly:

$$c_{12} = \int_{-\infty}^{\infty} f_1(t)f_2(t)dt = \int_{0}^{4} f_1(t)f_2(t)dt$$

$$= \int_{0}^{4} f_1(t)f_2(t)dt + \int_{2}^{4} f_1(t)f_2(t)dt$$

$$= \frac{1}{4} \int_{0}^{2} dt - \frac{1}{4} \int_{2}^{4} dt = \frac{1}{4} \times 2 - \frac{1}{4} \times (4 - 2)$$

$$= 0$$

Similarly:

$$c_{13} = \int_{-\infty}^{\infty} f_1(t)f_3(t)dt = \int_{0}^{4} f_1(t)f_3(t)dt$$

$$= \frac{1}{4} \int_{0}^{1} dt - \frac{1}{4} \int_{1}^{2} dt - \frac{1}{4} \int_{2}^{3} dt + \frac{1}{4} \int_{3}^{4} dt$$

$$= 0$$

and:

$$c_{23} = \int_{-\infty}^{\infty} f_2(t)f_3(t)dt = \int_{0}^{4} f_2(t)f_3(t)dt$$

$$= \frac{1}{4} \int_{0}^{1} dt - \frac{1}{4} \int_{1}^{2} dt + \frac{1}{4} \int_{2}^{3} dt - \frac{1}{4} \int_{3}^{4} dt$$

$$= 0$$
Thus, the signals $f_n(t)$ are orthogonal. It is also straightforward to prove that the signals have unit energy:

$$\int_{-\infty}^{\infty} |f_i(t)|^2 dt = 1, \quad i = 1, 2, 3$$

Hence, they are orthonormal.

b. We first determine the weighting coefficients

$$x_n = \int_{-\infty}^{\infty} x(t) f_n(t) dt, \quad n = 1, 2, 3$$

\[
x_1 = \int_0^4 x(t) f_1(t) dt = -\frac{1}{2} \int_0^1 dt + \frac{1}{2} \int_1^2 dt - \frac{1}{2} \int_2^3 dt + \frac{1}{2} \int_3^4 dt = 0
\]

\[
x_2 = \int_0^4 x(t) f_2(t) dt = \frac{1}{2} \int_0^4 x(t) dt = 0
\]

\[
x_3 = \int_0^4 x(t) f_3(t) dt = -\frac{1}{2} \int_0^1 dt - \frac{1}{2} \int_1^2 dt + \frac{1}{2} \int_2^3 dt + \frac{1}{2} \int_3^4 dt = 0
\]

As it is observed, $x(t)$ is orthogonal to the signal waveforms $f_n(t), n = 1, 2, 3$ and thus it cannot be represented as a linear combination of these functions.