



# A discrete-time model of TCP with Active Queue Management

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# Roadmap

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- Motivation
- Background
- **S-RED**: a discrete-time model of **TCP Reno** with **RED**
- Model validation of **S-RED**
- Comparison of TCP/RED models
- **S-RED**: a modification
- **S-ARED**: extension to **S-RED**
- Conclusions
- Future work



# Motivation

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- Modeling TCP Reno with RED:
  - examine the interactions between TCP and RED
  - understand and predict the network dynamic behavior
  - analyze the impact of system parameters

RED: Random Early Detection Gateways for Congestion Avoidance

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# Roadmap

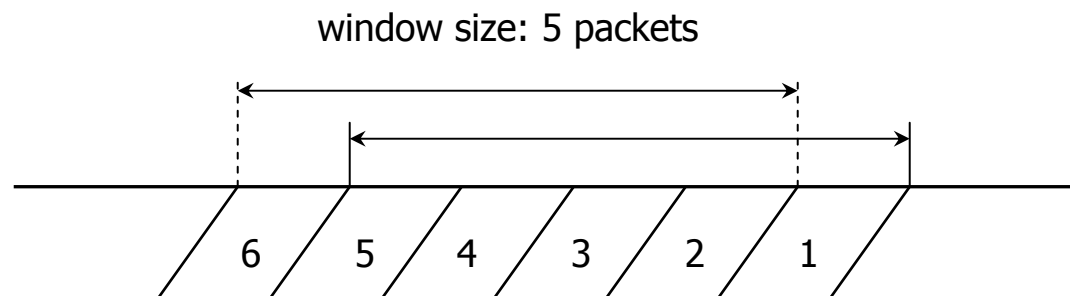
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- Motivation
- **Background**
- S-RED: a discrete-time model of TCP Reno with RED
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# TCP

- TCP: Transmission Control Protocol
- Fourth layer of the OSI model
- Connection oriented, reliable, and byte-stream service
- Employs window based flow and congestion control algorithms



OSI: Open System Interconnection reference model

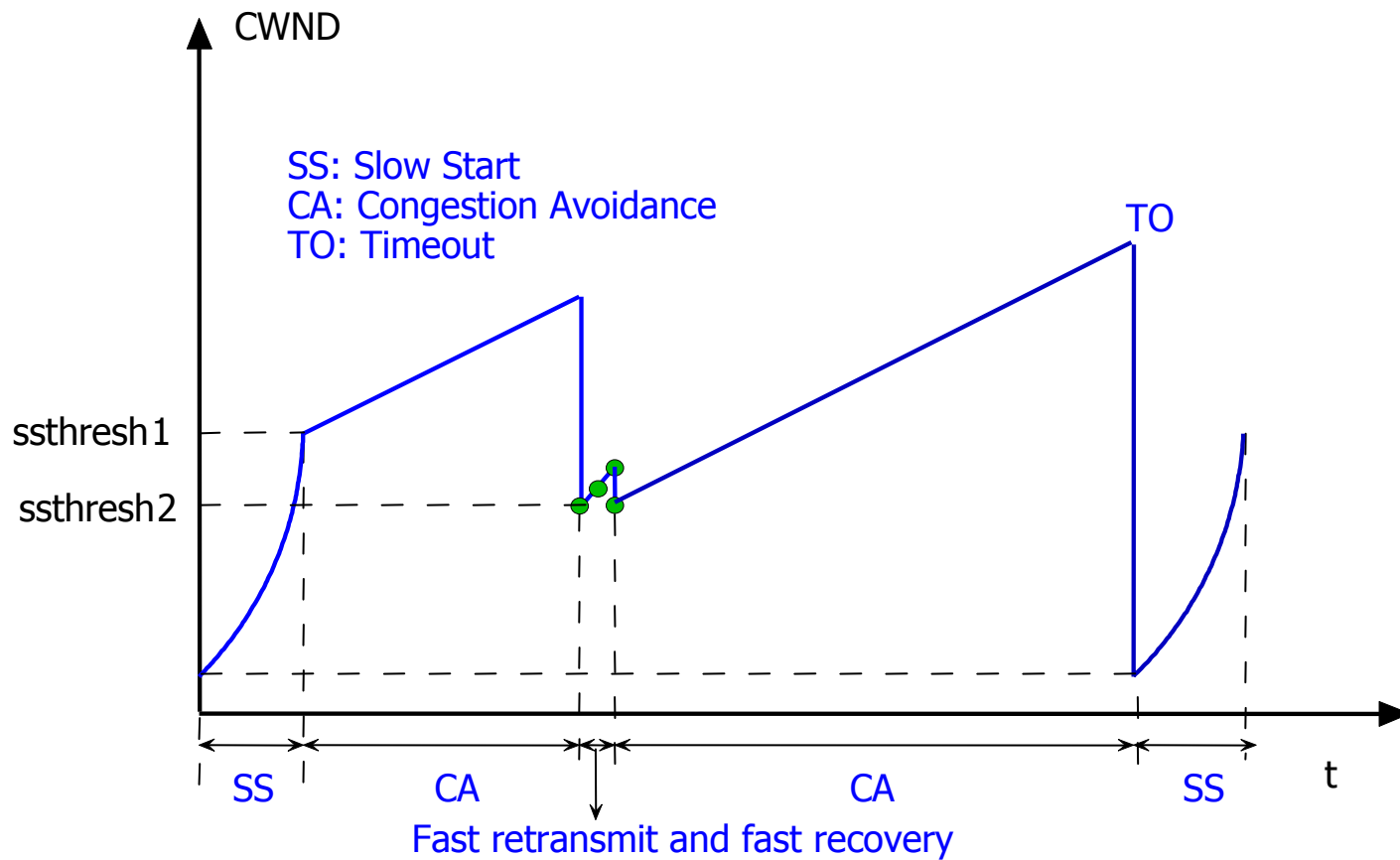


# TCP

- Several flavors of TCP:
  - Tahoe: 4.3 BSD Tahoe (~ 1988)
    - slow start, congestion avoidance, and fast retransmit (RFC 793, RFC 2001)
  - Reno: 4.3 BSD Reno (~ 1990)
    - slow start, congestion avoidance, fast retransmit, and fast recovery (RFC 2001, RFC 2581)
  - NewReno (~ 1996, RFC 2582)
    - new fast recovery algorithm
  - SACK (~ 1996, RFC 2018)



# TCP Reno





# TCP Reno: slow start and congestion avoidance

- Slow start:
  - $cwnd = IW$  (1 or 2 packets)
  - when  $cwnd < ssthresh$   
 $cwnd = cwnd + 1$  for each received *ACK*
- Congestion avoidance:
  - when  $cwnd > ssthresh$   
 $cwnd = cwnd + 1/cwnd$  for each *ACK*

*cwnd*: congestion window size

*IW*: initial window size

*ssthresh*: slow start threshold

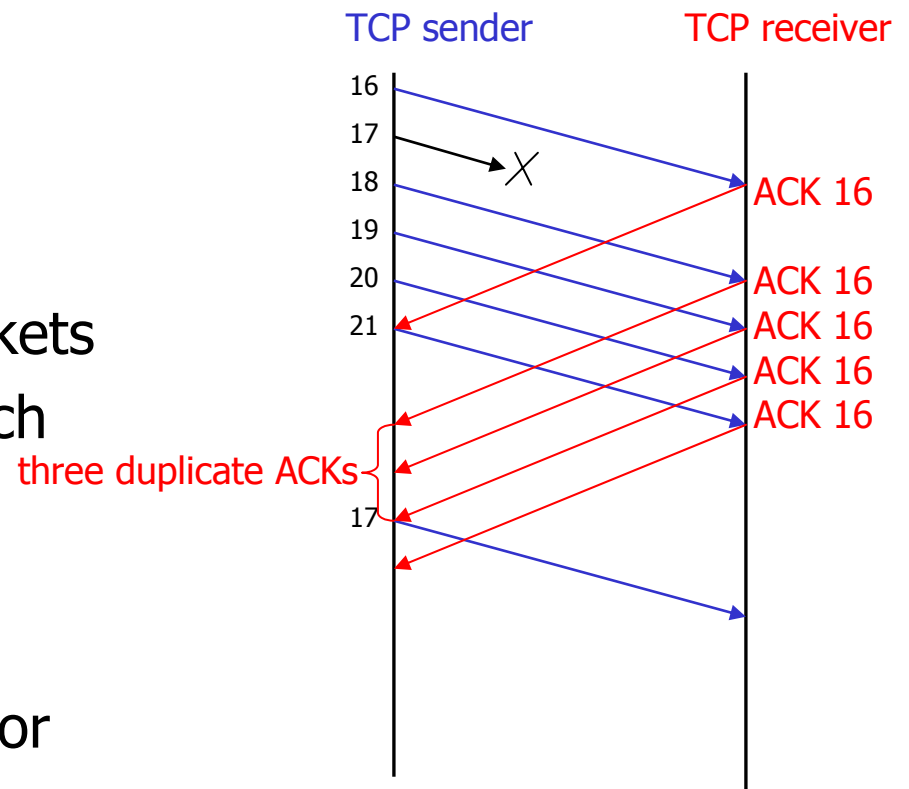
*ACK*: acknowledgement

*RTT*: round trip time



# TCP Reno: fast retransmit and fast recovery

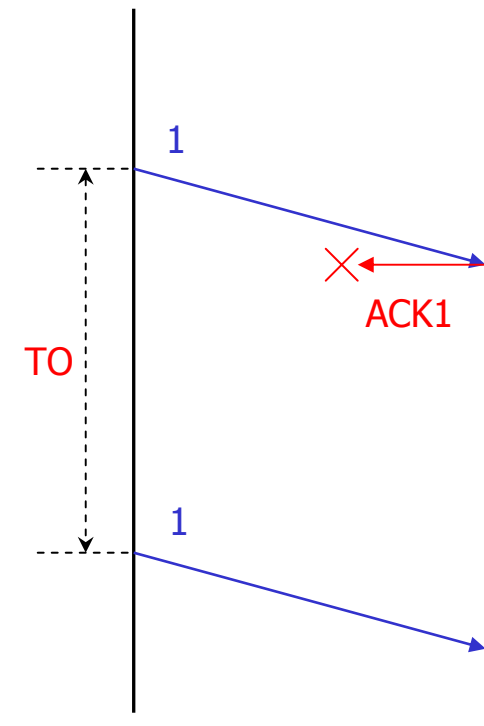
- Three duplicate *ACKs* are received
- Retransmit the packet
- $ssthresh = cwnd/2$ ,  
 $cwnd = ssthresh + 3$  packets
- $cwnd = cwnd + 1$ , for each additional duplicate ACK
- Transmit the new data, if  $cwnd$  allows
- $cwnd = ssthresh$ , if ACK for new data is received





# TCP Reno: timeout

- TCP maintains a **retransmission timer**
- The duration of the timer is called **retransmission timeout**
- Timeout occurs when the ACK for the delivered data is not received before the **retransmission timer** expires
- TCP sender retransmits the lost packet
- $ssthresh = cwnd/2$   
 $cwnd = 1$  or 2 packets





# AQM: Active Queue Management

- **AQM** (RFC 2309):
  - reduces bursty packet drops in routers
  - provides lower-delay interactive service
  - avoids the “lock-out” problem
  - reacts to the incipient congestion before buffers overflow
- AQM algorithms:
  - **RED** (RFC 2309)
  - **ARED**, **CHOKe**, and **BLUE**, ...



# RED

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- Random Early Detection Gateways for Congestion Avoidance
  - Proposed by S. Floyd and V. Jacobson, LBN, 1993.  
S. Floyd and V. Jacobson, "Random early detection gateways for congestion avoidance," *IEEE/ACM Trans. Networking*, vol. 1, no. 4, pp. 397-413, Aug. 1993.
  - Main concept: drop packets **before** the queue becomes full



# RED variables and parameters

- Main variables and parameters:
  - average queue size:  $\bar{q}$
  - instantaneous queue size:  $q$
  - drop probability:  $p_a$
  - queue weight:  $w_q$
  - maximum drop probability:  $p_{\max}$
  - queue thresholds:  $q_{\min}$  and  $q_{\max}$



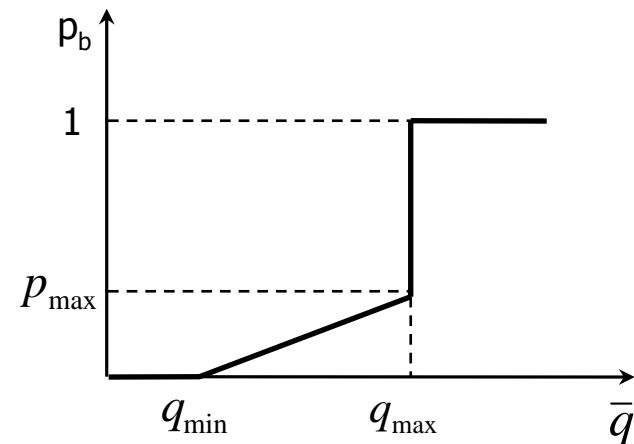
# RED algorithm

Calculate:

- average queue size for each packet arrival

$$\bar{q} = (1 - w_q)\bar{q} + w_q q$$

- drop probability





# RED algorithm: drop probability

- If  $(q_{\min} < \bar{q} < q_{\max})$

$$p_b = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}} \quad p_a = \frac{p_b}{1 - count \times p_b}$$

*count*: number of packets that arrived since the last packet drop

- Else if  $(\bar{q} > q_{\max})$

$$p_a = 1$$

- Else  $(\bar{q} < q_{\min})$

$$p_a = 0$$

- Mark or drop the arriving packet with probability  $p_a$



# ARED

- Adaptive RED aims to improve the robustness of RED with minimal modifications:
  - S. Floyd, R. Gummadi, and S. Shenker, "Adaptive RED: an algorithm for increasing the robustness of RED's Active Queue Management," Aug. 2001: <http://www.icir.org/floyd/papers/>.
- Maintain a stable average queue size (target)
- Adjust  $p_{max}$  in response to dynamical changes of the average queue size



# ARED algorithm

- Update  $p_{\max}$   
For every interval
  - If  $\bar{q} > target$   
$$p_{\max} = p_{\max} + \alpha$$

- Else if  $\bar{q} < target$   
$$p_{\max} = p_{\max} \times \beta$$

$target : [q_{\min} + 0.4(q_{\max} - q_{\min}), q_{\min} + 0.6(q_{\max} - q_{\min})]$

$\alpha : \min(0.01, p_{\max}/4)$

$\beta : 0.9$



## Simulation tool: ns-2

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- ns-2 is a discrete event network simulator  
<http://www.isi.edu/nsnam/ns>
- Supports simulation of TCP, routing, and multicast protocols over wired and wireless networks
- We used ns-2 to validate the proposed **S-RED** model

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# Modeling methodology

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- Categories of TCP models:
  - averaged and **discrete-time** models
  - short-lived and **long-lived TCP** connections
- **S-RED** model:
  - **discrete-time** model with a **long-lived** connection
- State variables:
  - **window size** (TCP)
  - **average queue size** (RED)



# S-RED model

- Key properties of the proposed **S-RED** model:
  - slow start, congestion avoidance, fast retransmit, and fast recovery (simplified)
  - **Timeout:**

J. Padhye, V. Firoiu, and D. F. Towsley, "Modeling TCP Reno performance: a simple model and its empirical validation," *IEEE/ACM Trans. Networking*, vol. 8, no. 2, pp. 133-145, Apr. 2000.
  - Captures the basic **RED** algorithm



# Assumptions

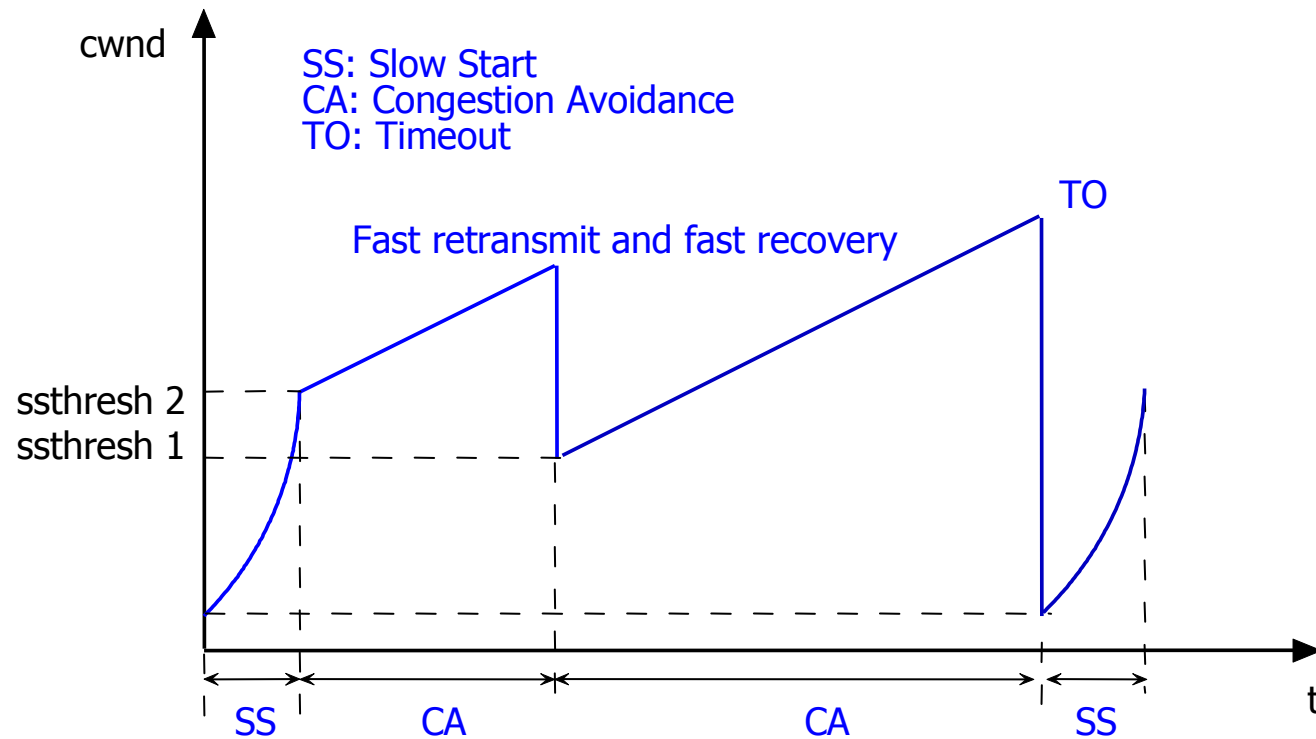
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- Long-lived TCP connection
- Constant propagation delay between the source and the destination
- Constant packet size
- Timeout occurs only due to packet loss
- The system is sampled at the end of every RTT



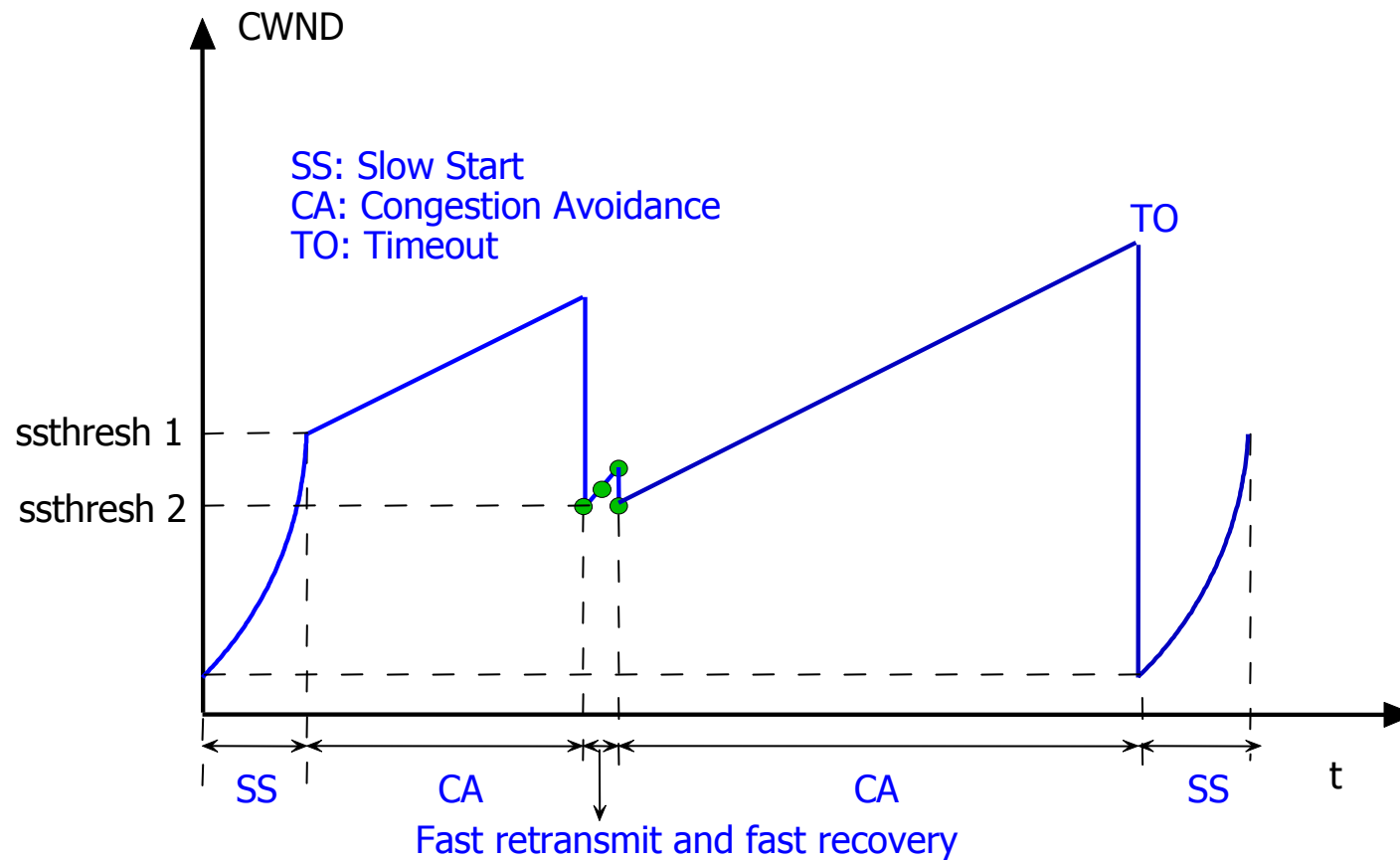
# S-RED model simplifications

- Simplified fast recovery





# TCP Reno: fast recovery





# S-RED model simplifications

- TO = 5 RTT

V. Firoiu and M. Borden, "A study of active queue management for congestion control," in *Proc. of IEEE INFOCOM 2000*, vol. 3, pp. 1435-1444, Tel-Aviv, Israel, Mar. 2000.

- RED: *count* not used

If  $(q_{\min} < \bar{q} < q_{\max})$

$$p_b = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}} \xrightarrow{p_a = p_b}$$

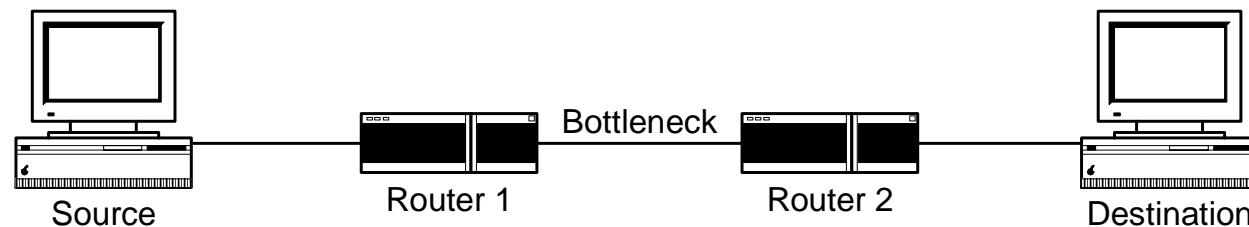
$$p_a = \frac{p_b}{1 - \text{count} \times p_b}$$

If  $(q_{\min} < \bar{q} < q_{\max})$

$$p_a = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}}$$



# Network topology



- Components: one source, two routers, and one destination
- The link between routers 1 and 2 is the only bottleneck
- RED algorithm is deployed in router 1



# S-RED: parameters and variables

- Variables:

$w$ : window size

$\bar{q}$ : average queue size

$p$ : drop probability

$q$ : instantaneous queue size

- Parameters:

$q_{max}$ : maximum queue threshold

$q_{min}$ : minimum queue threshold

$p_{max}$ : maximum drop probability

$w_q$ : queue weight

$d$ : propagation delay

$M$ : packet size

$C$ : link capacity

# S-RED: a discrete-time model for TCP Reno with RED



- Calculate the **average queue size**:

$$\begin{aligned}q_{k+1} &= q_k + W_{k+1} - \frac{C}{M} \left( d + \frac{q_k \times M}{C} \right) \\ &= W_{k+1} - \frac{C \cdot d}{M} \quad (1)\end{aligned}$$

$$\bar{q}_{k+1} = (1 - w_q) \bar{q}_k + w_q q_{k+1} \quad (2)$$

- the **average queue size** is updated after each packet arrival
- $\bar{q}_{k+1}$  is updated  $W_{k+1}$  times in  $k+1$ -th round

From (1) and (2):

$$\bar{q}_{k+1} = (1 - w_q)^{W_{k+1}} \cdot \bar{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max\left(W_{k+1} - \frac{C \cdot d}{M}, 0\right)$$



# S-RED model: drop probability

- Calculate the drop probability:

$$p_{k+1} = \begin{cases} 0 & \text{if } \bar{q}_{k+1} \leq q_{\min} \\ 1 & \text{if } \bar{q}_{k+1} \geq q_{\max} \\ \frac{\bar{q}_{k+1} - q_{\min}}{q_{\max} - q_{\min}} p_{\max} & \text{otherwise} \end{cases}$$



## S-RED model: three cases

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- No packet lost:
  - slow start
  - congestion avoidance
- Single packet lost:
  - fast retransmit
  - fast recovery
- At least two packets lost:
  - timeout



## S-RED model: case 1

- **No packet lost:**  $p_k \cdot W_k < 0.5$

$$W_{k+1} = \begin{cases} \min(2W_k, ssthresh) & \text{if } W_k < ssthresh \\ \min(W_k + 1, rwnd) & \text{if } W_k \geq ssthresh \end{cases}$$

$$\bar{q}_{k+1} = (1 - w_q)^{W_{k+1}} \cdot \bar{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max\left(W_{k+1} - \frac{C \cdot d}{M}, 0\right)$$

ssthresh: slow start threshold

rwnd: receiver's advertised window size



## S-RED model: cases 2 and 3

- **One packet lost:**  $0.5 \leq p_k \cdot W_k < 1.5$

$$W_{k+1} = \frac{1}{2} W_k$$

$$\bar{q}_{k+1} = (1 - w_q)^{W_{k+1}} \cdot \bar{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max\left(W_{k+1} - \frac{C \cdot d}{M}, 0\right)$$

- **At least two packets lost:**  $p_k \cdot W_k \geq 1.5$

$$W_{k+1} = 0$$

$$\bar{q}_{k+1} = \bar{q}_k$$

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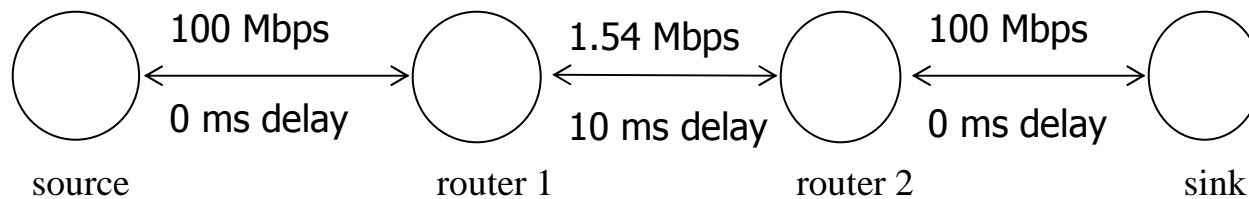
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# Simulation scenario



- source to router1:
  - link capacity: 100 Mbps with 0 ms delay
- router 1 to router 2: the only bottleneck in the network
  - link capacity: 1.54 Mbps with 10 ms delay
- router 2 to sink:
  - link capacity: 100 Mbps with 0 ms delay



# RED: default parameters

- RED parameters:

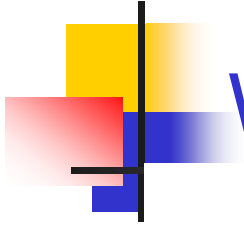
S. Floyd, "RED: Discussions of Setting Parameters," Nov. 1997:  
<http://www.icir.org/floyd/REDparameters.txt>

Queue weight ( $w_q$ )	0.002
Maximum drop probability ( $p_{\max}$ )	0.1
Minimum queue threshold ( $q_{\min}$ )	5 (packets)
Maximum queue threshold ( $q_{\max}$ )	15 (packets)

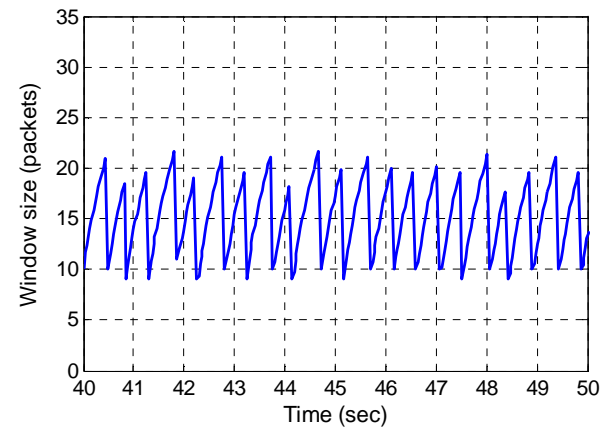
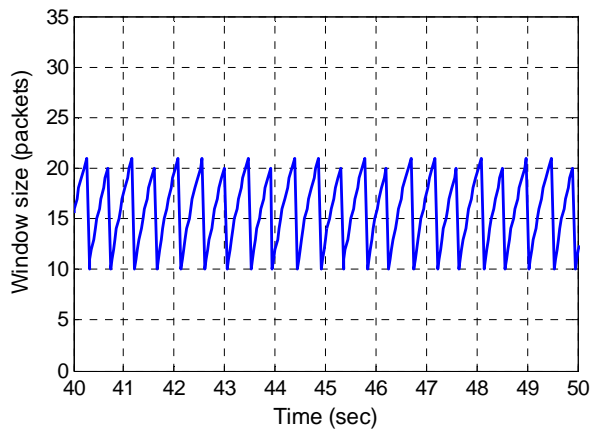
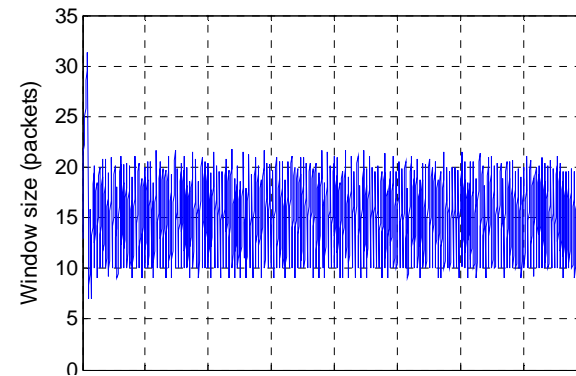
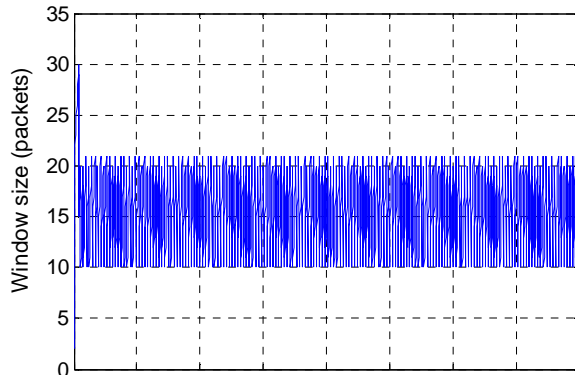


# S-RED model validation

- Waveforms of the state variables with default parameters:
  - window size
  - average queue size
- Validation for various values of the system parameters:
  - queue weight:  $w_q$
  - maximum drop probability:  $p_{\max}$
  - queue thresholds:  $q_{\min}$  and  $q_{\max}$ ,  $q_{\max}/q_{\min} = 3$



# Window size: waveforms

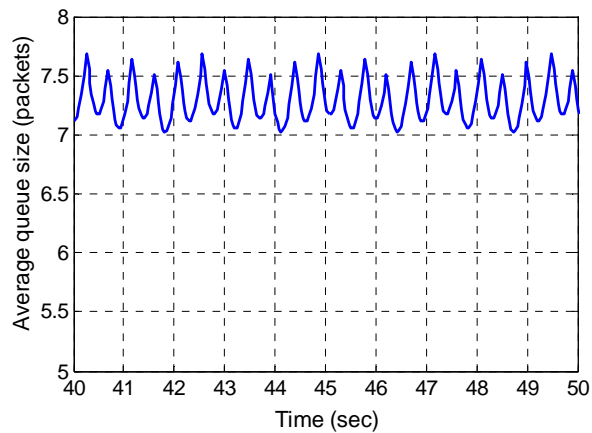
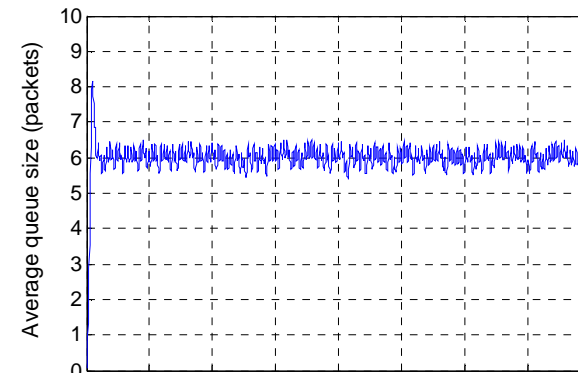
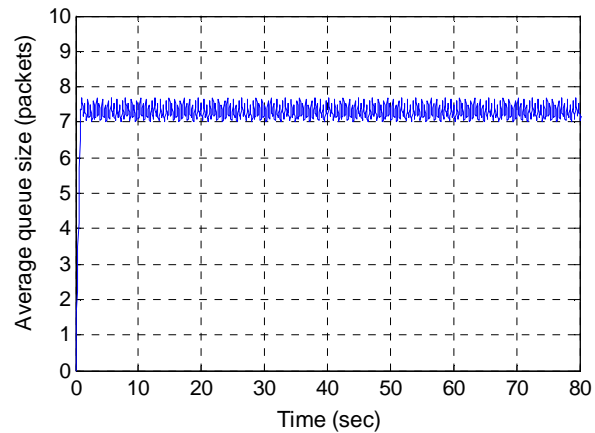


S-RED model

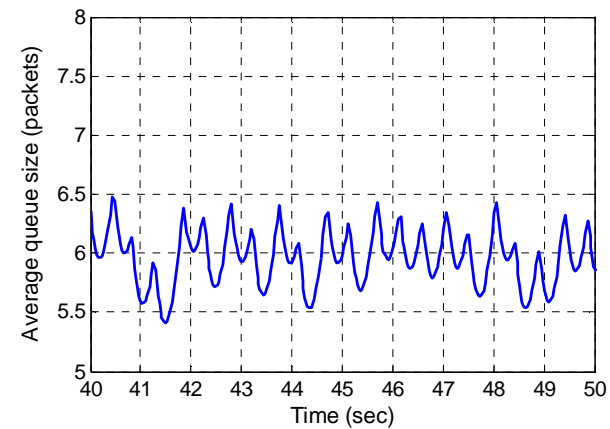
ns-2



# Average queue size: waveforms



S-RED model



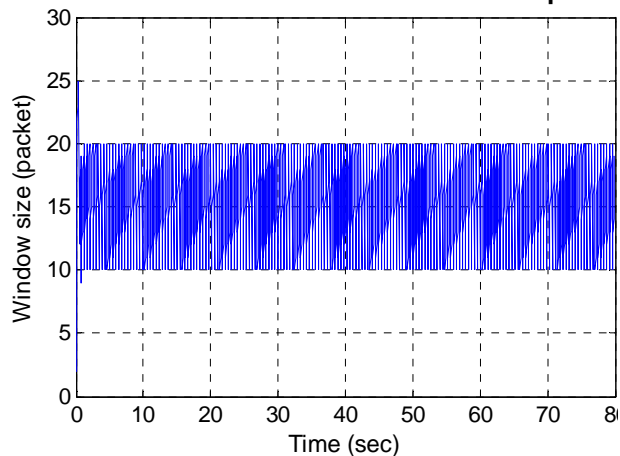
ns-2



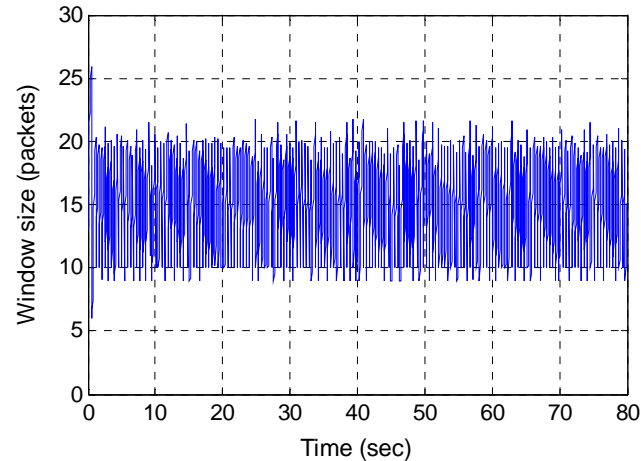
# Model validation: $w_q$

- $w_q = [0.001, 0.01]$ , with other parameters default

- window size:  $w_q = 0.006$



S-RED

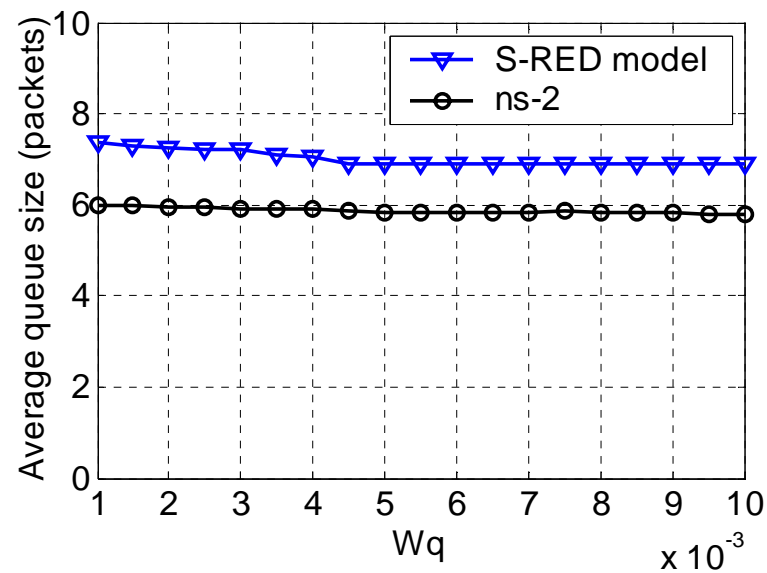


ns-2



# Model validation: $w_q$

- average queue size during steady state:





# Model validation: $w_q$

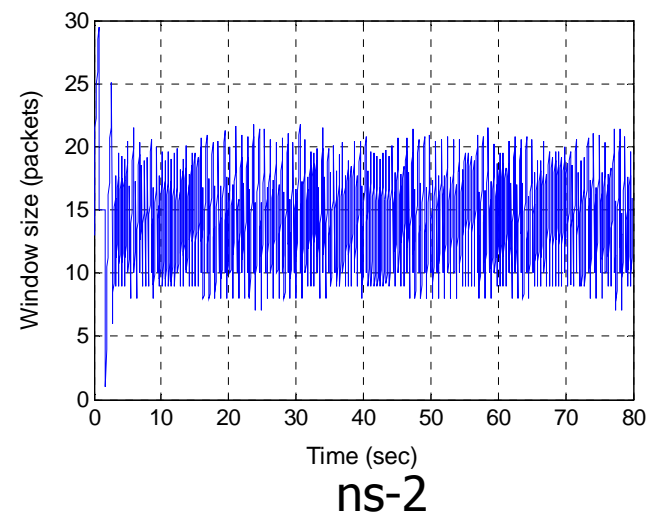
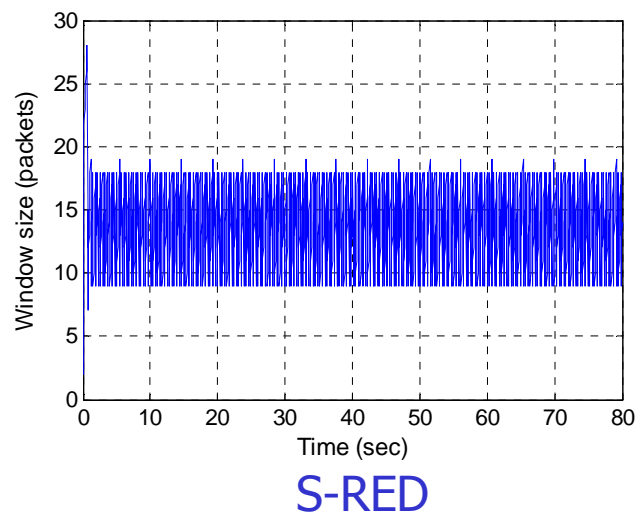
- Comparison of system variables:

Parameters	Average RTT (msec)			Sending rate (packets/sec)			Drop rate (%)		
	S-RED model	ns-2	$\Delta$ (%)	S-RED model	ns-2	$\Delta$ (%)	S-RED model	ns-2	$\Delta$ (%)
weight ( $w_q$ )									
0.001	40.3	36.1	11.63	384.99	384.71	0.073	0.55	0.54	1.29
0.002	39.9	36.0	10.83	384.98	384.77	0.056	0.56	0.55	2.56
0.004	39.4	36.2	8.80	385.11	384.79	0.083	0.59	0.56	6.12
0.006	39.0	35.8	8.93	385.08	384.73	0.093	0.60	0.56	7.91
0.008	39.0	35.8	8.90	385.10	384.68	0.109	0.61	0.55	11.11
0.010	38.9	35.7	8.96	385.02	384.70	0.083	0.61	0.55	11.72



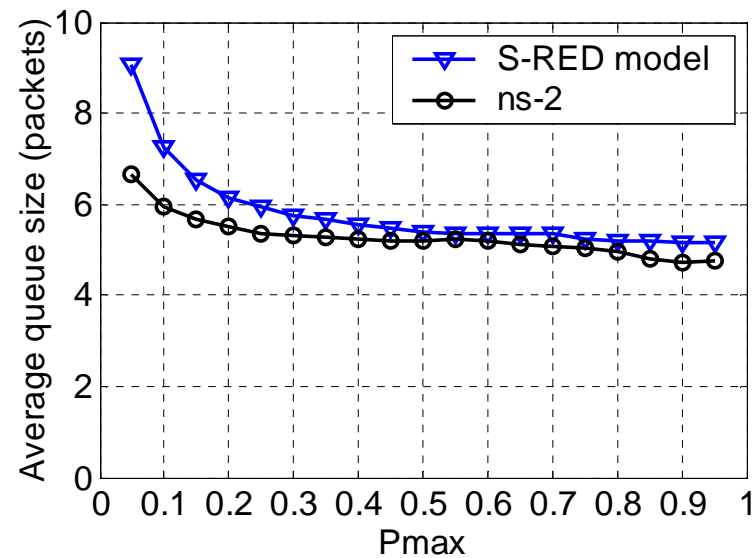
# Model validation: $p_{\max}$

- $p_{\max} = [0.05, 0.95]$ , with other parameters default
- window size: waveforms,  $p_{\max} = 0.5$



# Model validation: $p_{\max}$

- average queue size during steady state:





# Model validation: $p_{max}$

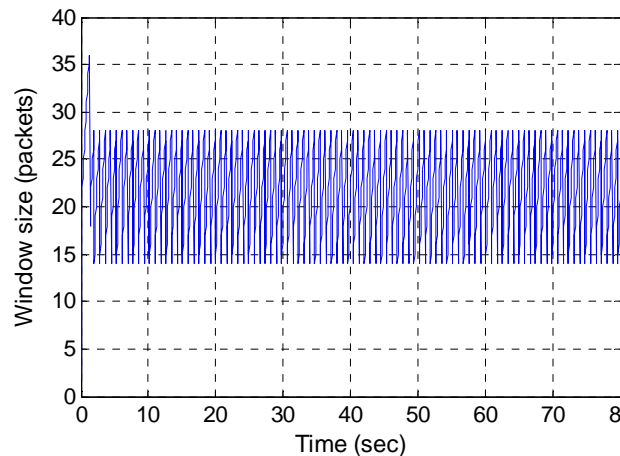
- Comparison of system variables:

$p_{max}$	Average RTT (msec)			Sending rate (packets/sec)			Drop rate (%)		
	S-RED model	ns-2	$\Delta$ (%)	S-RED model	ns-2	$\Delta$ (%)	S-RED model	ns-2	$\Delta$ (%)
0.05	44.3	38.1	16.27	385.13	384.70	0.11	0.45	0.51	-11.76
0.10	39.9	36.0	10.83	384.98	384.77	0.06	0.56	0.55	2.56
0.25	36.5	34.5	5.80	384.93	384.73	0.05	0.65	0.59	11.28
0.50	35.3	34.0	3.80	384.98	379.37	1.48	0.73	0.61	19.09
0.75	34.8	35.1	-0.85	384.63	357.55	7.60	0.74	0.65	14.37

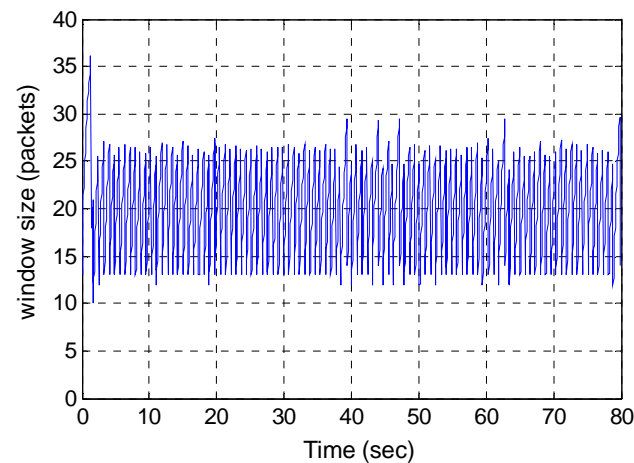


# Model validation: $q_{\min}$ and $q_{\max}$

- $q_{\min} = [1 - 20]$  packets,  $q_{\max}/q_{\min} = 3$ , with other parameters default
  - window size: waveforms,  $q_{\min} = 10$  packets



S-RED

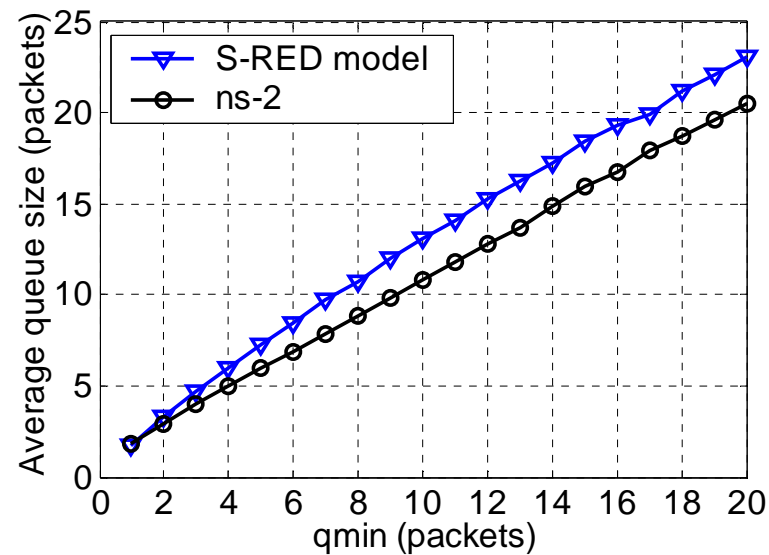


ns-2



# Model validation: $q_{\min}$ and $q_{\max}$

- average queue size during steady state:





# Model validation: $q_{\min}$ and $q_{\max}$

- Comparison of system variables:

$q_{\min}$ (packets)	Average RTT (msec)			Sending rate (packets/sec)			Drop rate (%)		
	S-RED model	ns-2	$\Delta$ (%)	S-RED model	ns-2	$\Delta$ (%)	S-RED model	ns-2	$\Delta$ (%)
3	33.4	31.1	7.4	383.22	382.44	0.20	0.78	0.71	10.01
5	39.9	36.0	10.83	384.98	384.77	0.06	0.56	0.55	2.56
10	54.7	48.1	13.72	385.10	384.85	0.06	0.31	0.33	-6.34
15	67.7	60.3	12.27	385.06	384.83	0.06	0.20	0.22	-10.71
20	79.1	73.0	8.36	385.30	384.95	0.09	0.15	0.16	-5.66



# S-RED: model evaluation

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- Waveforms of the **window size**:
  - match the ns-2 simulation results
- The **average queue size**:
  - mismatch, but similar trend
- System variables RTT, sending rate, and drop rate:
  - reasonable agreement with ns-2 simulation results, depending on the system parameters

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# Comparison: S-RED model vs. M-model

- **M-model:**

A discrete nonlinear dynamical model of **TCP Reno** with **RED** proposed by a research group from University of Maryland

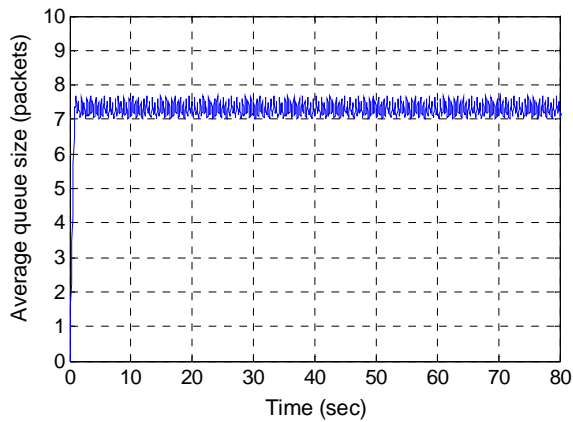
P. Ranjan, E. H. Abed, and Richard J. La, "Nonlinear instabilities in TCP-RED," in *Proc. IEEE INFOCOM 2002*, New York, NY, USA, June 2002, vol. 1, pp. 249-258

- One state variable: **average queue size**

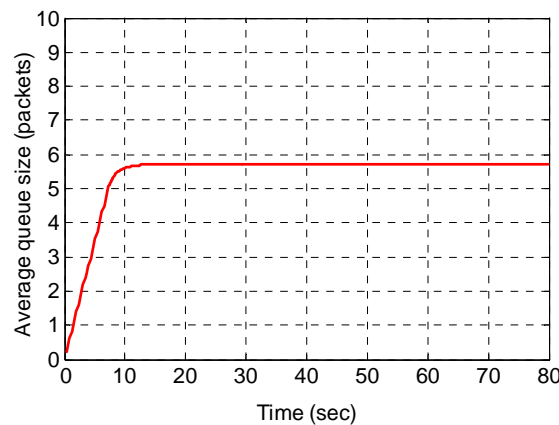


# Model comparison: default parameters

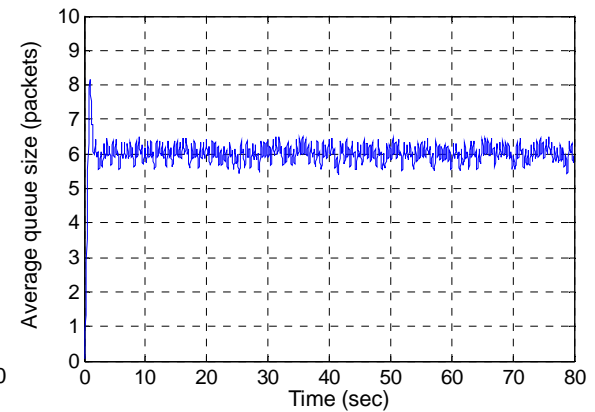
- The waveform of the **average queue size** with default **RED** parameters:



S-RED



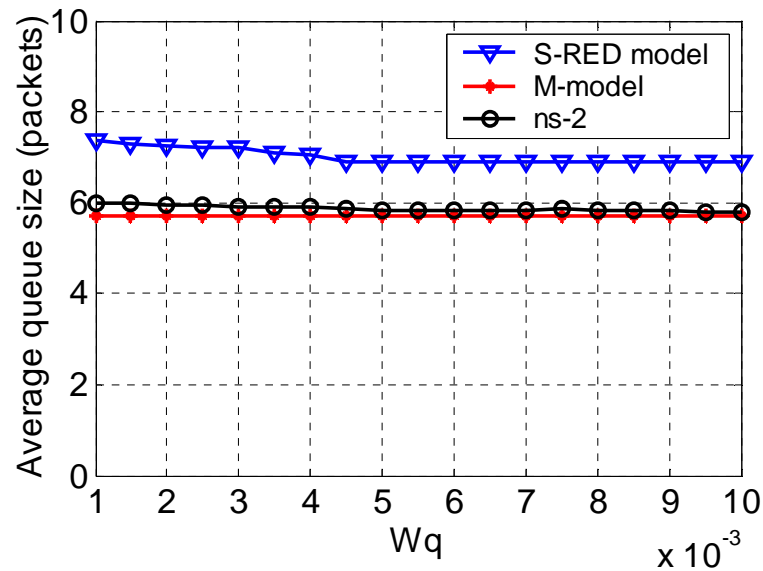
M-model



ns-2

# Model comparisons: $w_q$

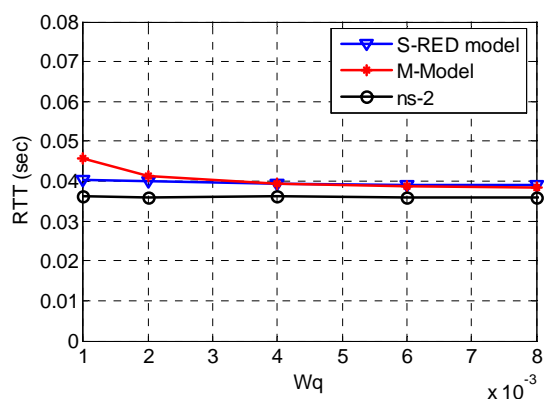
- $w_q = [0.001, 0.01]$ , with other parameters default
  - **average queue size** during steady state:



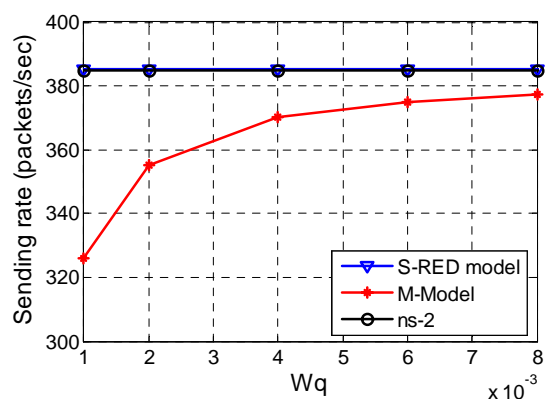


# Model comparisons: $w_q$

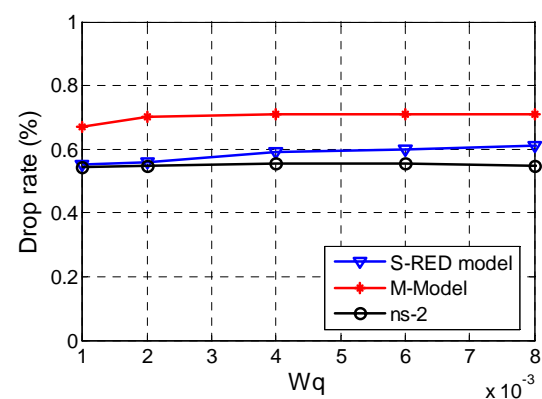
## ■ system variables:



RTT



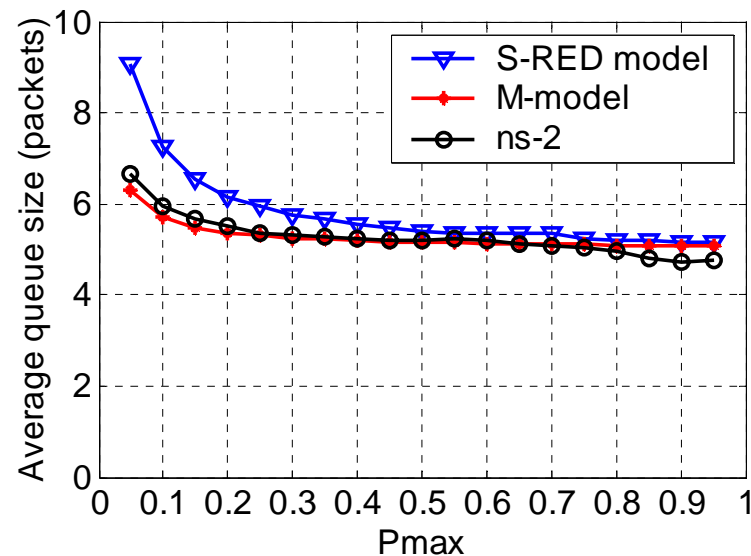
sending rate



drop rate

## Model comparisons: $p_{\max}$

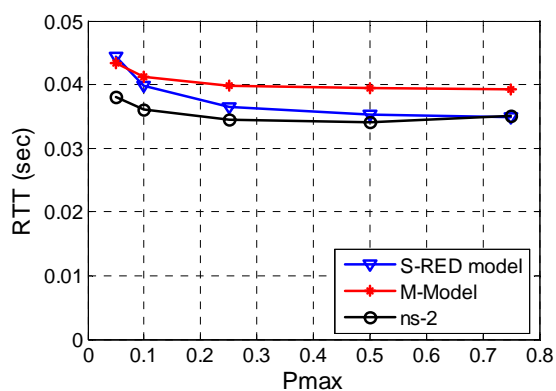
- $p_{\max} = [0.05, 0.95]$ , with other parameters default
  - **average queue size** during steady state:



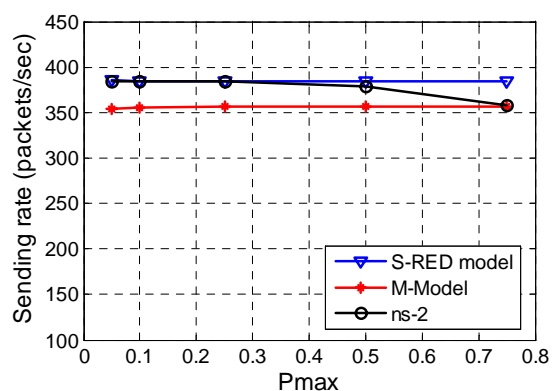


# Model comparisons: $p_{\max}$

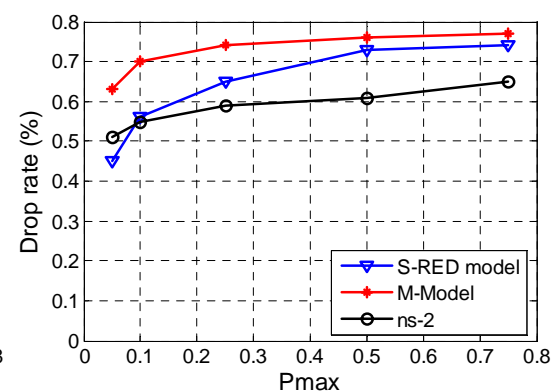
## ■ system variables:



RTT



sending rate

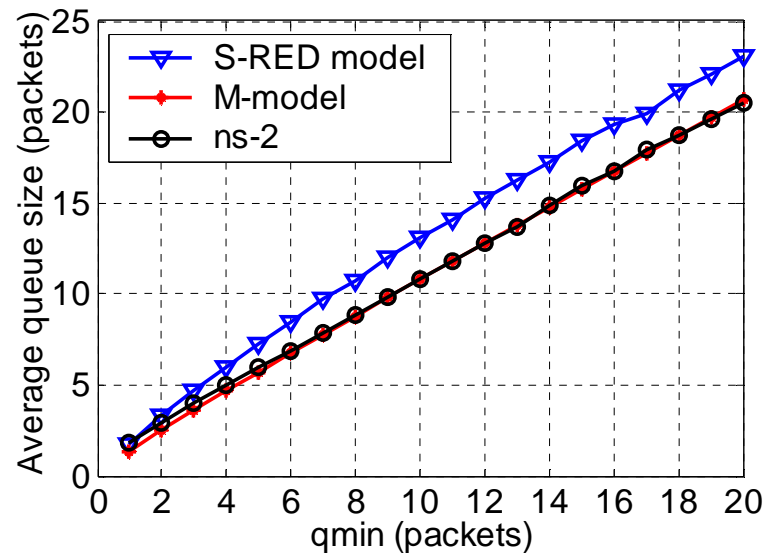


drop rate



## Model comparisons: $q_{\min}$ and $q_{\max}$

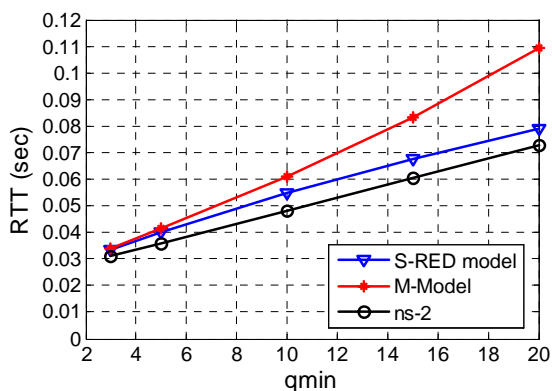
- $q_{\min} = [1 - 20]$  packets,  $q_{\max}/q_{\min} = 3$ , with other parameters default
  - average queue size during steady state:



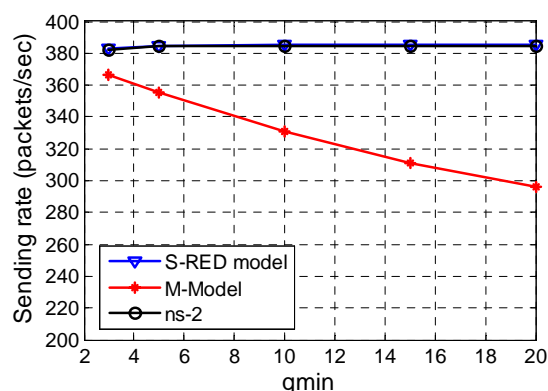


# Model comparisons: $q_{\min}$ and $q_{\max}$

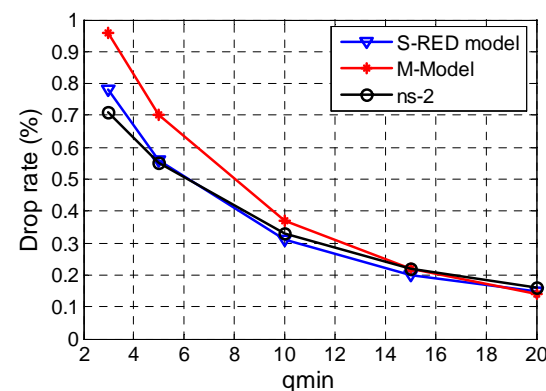
■ system variables:



RTT



sending rate



drop rate



## Model comparison: summary

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- **S-RED** model captures dynamical details of **TCP/RED**
- RTT, sending rate, and drop rate: **S-RED** model, in general, matches the ns-2 simulation results better than the **M-model**
- **M-model**: average queue size
  - constant during steady-state
  - matches better the ns-2 simulation results

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# Roadmap

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- Motivation
- Background
- S-RED: a discrete-time model of TCP Reno with RED
- Model validation of S-RED
- Comparison of TCP/RED models
- **S-RED: a modification**
- S-ARED: extension to S-RED
- Conclusions
- Future work



## S-RED: modification

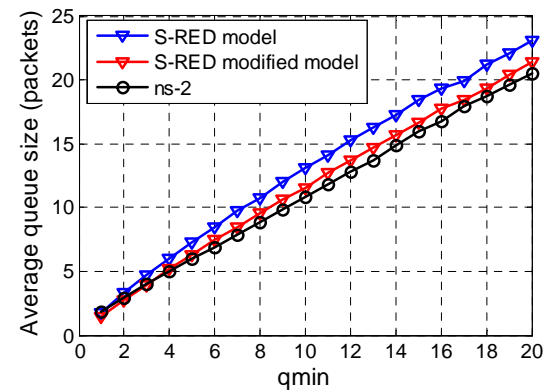
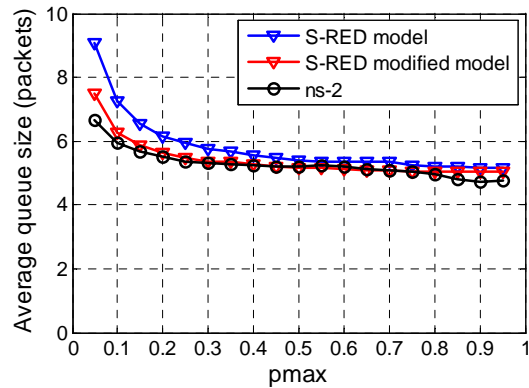
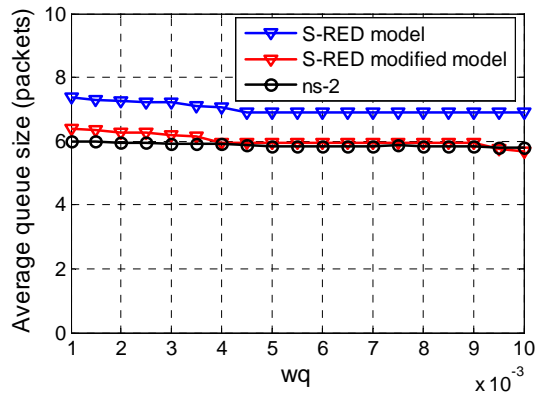
- The difference in the **average queue size** between S-RED model and ns-2 is due to the simplification of  $P$  :

$$\begin{array}{l} \text{If } (q_{\min} < \bar{q} < q_{\max}) \\ p_b = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}} \end{array} \quad \xrightarrow{p_a = p_b} \quad \begin{array}{l} \text{If } (q_{\min} < \bar{q} < q_{\max}) \\ p_a = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}} \end{array}$$
$$p_a = \frac{p_b}{1 - \text{count} \times p_b}$$

- Modification  $p_a$ :  $p_a = \alpha \cdot p_b$  ( $\alpha > 1$ )

# Modified S-RED model

- Modification:  $\alpha = 1.8$





# Roadmap

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# S-ARED model

- **ARED** algorithm:
  - extension to the **RED** algorithm
  - designed to improve **RED**'s robustness
  - achieves a specified target value for the **average queue size** by dynamically updating  $P_{\max}$
- **S-ARED** model:
  - developed based on **S-RED** model
  - keeps all the assumptions of **S-RED** model
  - state variables: **window size** and **average queue size**
  - interval for update  $P_{\max}$  :  $C \cdot \text{RTT}$  ( $C = 10$ )



# S-ARED: model validation

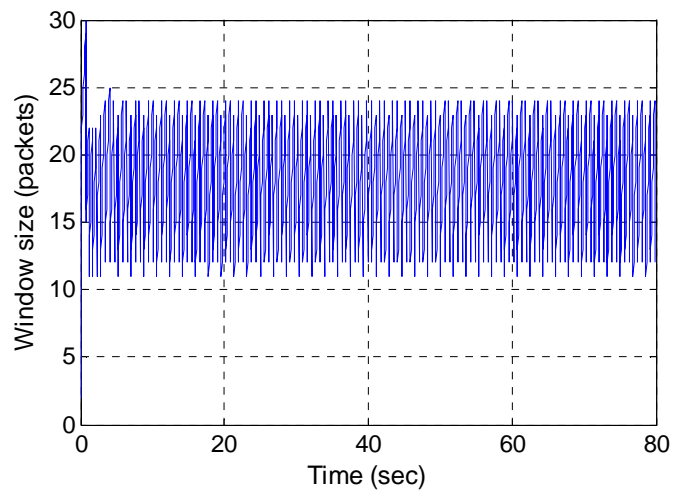
- The simulation scenario used to validate S-ARED model is identical to the scenario used for S-RED model
- Parameters:

Packet size $M$ (bytes)	500
Maximum drop probability $p_{max}$	0.1
Minimum queue threshold $q_{min}$ (packets)	5
Maximum queue threshold $q_{max}$ (packets)	15
Queue weight $w_q$	0.002
Sample_interval $C$	10

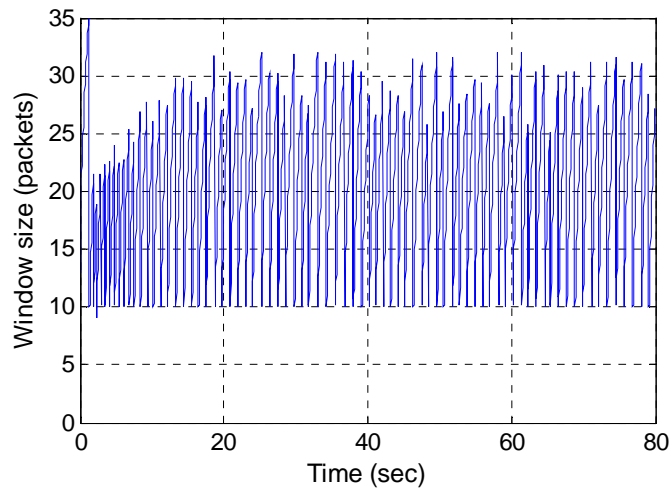


# S-ARED: validation

- window size: waveforms



S-ARED

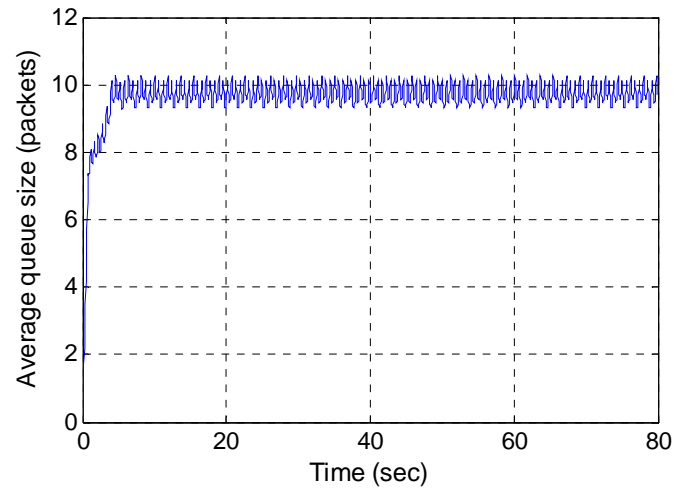


ns-2

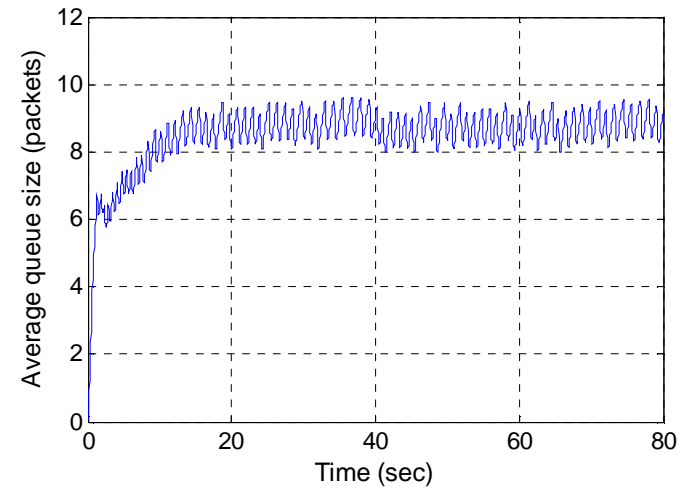


# S-ARED: validation

- average queue size: waveforms



S-ARED



ns-2



# S-ARED: validation

## S-ARED:

- captures the basic TCP/ARED system
- window size and average queue size:  $\sim 10\%$  difference with the ns-2 simulation results
- RTT and sending rate match well the ns-2 simulation results
- drop probability shows large mismatch with ns-2

	S-ARED model	ns-2	$\Delta$ (%)
Window size (packets)	17.66	20.10	-12.13
Average queue size (packets)	9.69	8.69	11.50
RTT (ms)	45.9	44.2	3.84
Sending rate (packets/s)	385.03	377.41	2.02
Drop probability (%)	0.44	0.26	69

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# Conclusions

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- We developed a second-order discrete-time model for TCP Reno with RED
- The S-RED model includes slow start, congestion avoidance, fast retransmit, timeout, elements of fast recovery, and RED
- The model is validated by comparing its performance to ns-2 simulations
- The S-RED model can capture the main features of the dynamic behavior of TCP Reno with RED
- We compared S-RED model with the M-model



# Conclusions

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- After modifying the drop probability, the **S-RED** model shows a better agreement with ns-2 simulation results
- We also introduced the **S-ARED** model, an extension to the **S-RED** model
- **S-RED** model may be used for performance evaluation of **TCP/RED** based **AQM** schemes



## Future work

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- Analyze nonlinear phenomena of TCP/RED:
  - chaos and bifurcation
- Evaluate performance of other TCP/RED based AQM schemes:
  - CHOKe
- Develop new AQM algorithms



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Thank You!  
Questions?



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