Performance Enhancement Through Joint Detection of Cochannel Signals Using Diversity Arrays

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Abstract— **Joint detection based on exploiting differences among the channels employed by several users allows a receiver to distinguish cochannel signals without reliance on spectrum spreading. This paper makes a number of new contributions to the topic: it provides an analytical expression for the union bound on average symbol error rate for an arbitrary number of users and diversity antennas in a fading environment, for both perfect and imperfect CSI, and it compares the performance of joint detection with diversity antennas against classical MMSE combining. The performance is remarkable. With accurate CSI, several users can experience good performance with only a single antenna; moreover, for perfect CSI, only a 2 dB penalty is incurred for each additional user. With several antennas, many more users than the number of antennas may be supported with a slow degradation in performance for each additional user. Furthermore, high accuracy is not required from the channel estimation process. In all cases, the performance of joint detection exceeds that of MMSE combining by orders of magnitude.**

I. Introduction

Joint detection is a method whereby the central receiver in a communication system exploits differences among several cochannel users' signals in order to make a simultaneous decision on all of the users' data. In this way, several signals can occupy the same frequency and time slot, leading to improved spectrum efficiency and system capacity.

Recently, much attention has been focused on joint detection for CDMA systems [1], [2], [3] where the users' orthogonal spreading sequences are used to distinguish the signals. An alternative method, though, is to exploit differences in the channels between each user and the central receiver. This method does not rely on spectrum spreading, therefore it can be applied to narrowband systems such as FDMA or TDMA. Since the number of users sharing the same slot in a such systems is likely to be much less than in a CDMA system, the computational complexity of the joint detection algorithm may not be as significant an issue.

Joint detection based solely on channel differences has received only limited attention in the literature, e.g. [4], [5], [6]. The present paper makes a number of new contributions. It appears to be the first to address multiuser detection based on channel differences in the context of fading channels. It provides an analytical expression for the union bound on average symbol error rate for an arbitrary number of users and diversity antennas, for both perfect and imperfect CSI. In addition, it compares the performance of joint detection with diversity antennas against MMSE antenna combining—a classical approach for suppressing cochannel interference when making single user decisions [7].

II. System Model

This paper focuses on the transmission of M cochannel signals over frequency-flat Rayleigh fading channels. The signals are assumed to be PSK modulated and synchronized by symbol, with the m-th user's transmitted signal given by

$$s_m(t) = A_m \sum_k c_m(k)\, p(t - kT) \qquad (1)$$

where $c_m(k)$ is the m-th user's data symbol during the k-th signalling interval, $p(t)$ is a root Nyquist pulse, T is the symbol period; and $A_m = \sqrt{2P_m}$ where P_m is the average power in $s_m(t)$. Although the model has been simplified by requiring users to be synchronized by symbol, this study provides the motivation for investigation of asynchronous performance.

L-fold antenna diversity is employed at the receiver with the antenna elements spaced far enough apart to ensure independent fading (Fig. 1). After matched filtering and symbol rate sampling the received signal vector is given by

$$\mathbf{r}(k) = \sum_{m=1}^{M} A_m \mathbf{g}_m(k)\, c_m(k) + \mathbf{n}(k). \qquad (2)$$

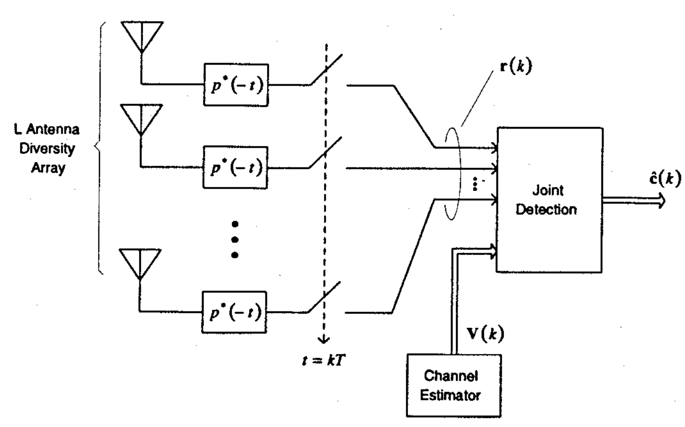

Fig. 1. Joint detection receiver model.

The elements of the complex Gaussian noise vector $\mathbf{n}(k)$ and the m-th user's complex Gaussian channel gain vector $\mathbf{g}_m(k)$ are assumed to be i.i.d. and zero-mean with variance N_o and $\sigma^2_{g_m}$ respectively. Furthermore, the users are assumed to be spaced far enough apart (a few wavelengths) such that the M users' channel gain vectors are mutually independent. The output of the detector is the vector of symbol decisions $\widehat{\mathbf{c}}(k)$ which is an estimate of the transmitted data vector $\mathbf{c}(k) = (c_1(k), c_2(k), \ldots, c_M(k))$.

It is assumed in this study that estimates of all users' channel gains are available at the receiver. To keep the treatment general, we have not prescribed a specific channel estimation scheme, although estimates are typically obtained through the use of embedded references such as pilot tones or pilot symbols [8], [9]. It is assumed that the elements of the estimate of $\mathbf{g}_m(k)$ (denoted $\mathbf{v}_m(k)$) are i.i.d., zero-mean, complex Gaussian random variables with variance $\sigma^2_{v_m}$. Furthermore, the corresponding elements of $\mathbf{v}_m(k)$ and $\mathbf{g}_m(k)$ are correlated with correlation coefficient ρ_m. It is important to emphasize that the ρ_m's collectively reflect the quality of channel estimation, with the limiting case of perfect CSI occuring for $\rho_m = 1$ for all users.

III. Joint Detection Metric

Let $C = \{(c_{i1}, c_{i2}, \ldots, c_{iM})\}$ be the set of all possible transmitted data vectors where the time dependence of all variables has been dropped in the subsequent analysis for convenience. For M users and a PSK constellation size of Q, the number of possible data vectors is Q^M. The joint detection metric is derived starting from the observation that the detector should select the vector $\mathbf{c}_i$ from the set C for which the probability $p(\mathbf{r} \mid \mathbf{c}_i, \mathbf{V})$ is maximum, where the matrix $\mathbf{V}$ is used simply to denote the M channel estimate vectors.

Observing (2), it can be seen that $\mathbf{r}$ conditioned on $\mathbf{c}_i$ and $\mathbf{V}$ is Gaussian. Thus,

$$p(\mathbf{r} \mid \mathbf{c}_i, \mathbf{V}) \sim \exp\left[-\frac{\left(\mathbf{r} - \boldsymbol{\mu}_{\mathbf{r}|\mathbf{c}_i,\mathbf{V}}\right)^{\dagger} \mathbf{R}^{-1}_{\mathbf{r}|\mathbf{c}_i,\mathbf{V}} \left(\mathbf{r} - \boldsymbol{\mu}_{\mathbf{r}|\mathbf{c}_i,\mathbf{V}}\right)}{2}\right] \tag{3}$$

where the conditional mean and covariance matrix of $\mathbf{r}$ are given by

$$\boldsymbol{\mu}_{\mathbf{r}|\mathbf{c}_i,\mathbf{V}} = \sum_{m=1}^{M} A_m c_{im} \boldsymbol{\mu}_{\mathbf{g}_m|\mathbf{v}_m} \tag{4}$$

and

$$\mathbf{R}_{\mathbf{r}|\mathbf{c}_i,\mathbf{V}} = \sum_{m=1}^{M} A_m^2 \mathbf{R}_{\mathbf{g}_m|\mathbf{v}_m} + N_o \mathbf{I} \tag{5}$$

respectively, where $\mathbf{I}$ is the $L \times L$ identity matrix. The conditional mean and covariance matrix of $\mathbf{g}_m$ are determined as follows: first define the length-$2L$ vector $\mathbf{x}_m = (g_{1m}, v_{1m}, g_{2m}, v_{2m}, \ldots, g_{Lm}, v_{Lm})^T$, i.e. the elements of $\mathbf{g}_m$ and $\mathbf{v}_m$ interlaced. The conditional pdf of $\mathbf{g}_m$ is then

$$p(\mathbf{g}_m \mid \mathbf{v}_m) = \frac{\left(\sigma^2_{v_m}\right)^L}{(2\pi)^L |\mathbf{R}_{\mathbf{x}_m}|} \frac{\exp\left[-\frac{1}{2}\mathbf{x}_m^{\dagger}\mathbf{R}^{-1}_{\mathbf{x}_m}\mathbf{x}_m\right]}{\exp\left[-\frac{1}{2\sigma^2_{v_m}}\mathbf{v}_m^{\dagger}\mathbf{v}_m\right]} \tag{6}$$

where the covariance matrix of $\mathbf{x}_m$ is a $2L \times 2L$ block diagonal matrix due to independent fading across the antenna array. The 2×2 blocks on the main diagonal are given by

$$\mathbf{R}_m = \begin{bmatrix} \sigma^2_{g_m} & \rho_m \sigma_{g_m} \sigma_{v_m} \\ \rho_m^* \sigma_{g_m} \sigma_{v_m} & \sigma^2_{v_m} \end{bmatrix}. \tag{7}$$

Expanding (6) and cancelling terms yields

$$p(\mathbf{g}_m \mid \mathbf{v}_m) = \frac{1}{\left(2\pi\sigma^2_{e_m}\right)^L} \exp\left[-\frac{1}{2\sigma^2_{e_m}}\mathbf{e}_m^{\dagger}\mathbf{e}_m\right] \tag{8}$$

where

$$\mathbf{e}_m = \mathbf{g}_m - \beta_m \mathbf{v}_m, \tag{9}$$

$$\beta_m = \rho_m \frac{\sigma_{g_m}}{\sigma_{v_m}}, \tag{10}$$

and

$$\sigma^2_{e_m} = \sigma^2_{g_m}\left(1 - |\rho_m|^2\right). \tag{11}$$

Consequently, $\boldsymbol{\mu}_{\mathbf{g}_m|\mathbf{v}_m} = \beta_m \mathbf{v}_m$ and $\mathbf{R}_{\mathbf{g}_m|\mathbf{v}_m} = \sigma^2_{e_m}\mathbf{I}$. From basic estimation theory [10], the optimal estimate of $\mathbf{g}_m$ based on $\mathbf{v}_m$ is given by the conditional mean $\boldsymbol{\mu}_{\mathbf{g}_m|\mathbf{v}_m}$; thus, (9) gives the error in the estimate of $\mathbf{g}_m$, and (11) gives the estimation error variance. Furthermore, $\mathbf{e}_m$ and $\mathbf{v}_m$ are uncorrelated.

Substituting the results for $\boldsymbol{\mu}_{\mathbf{g}_m|\mathbf{v}_m}$ and $\mathbf{R}_{\mathbf{g}_m|\mathbf{v}_m}$ into (4) and (5) and the resulting expressions into (3) yields the joint detection metric

$$\Lambda_i = \sum_{l=1}^{L} \left| r_l - \sum_{m=1}^{M} A_m \beta_m c_{im} v_{lm} \right|^2 \tag{12}$$

which must be minimized over the set C.

Evidently, the receiver requires knowledge of the product $A_m v_{lm}$ for every user. Fortunately, this quantity is generated explicitly in a pilot-based channel estimator, e.g. [8]. The other required parameter, β_m, is determined at design time; however, if the true channel statistics differ from the design statistics, a bias is introduced. We have assumed the bias to be zero, and focused the analysis on the random channel estimation errors.

IV. Error Probability Analysis

This section contains an analysis of the error performance of joint detection based on the metric derived above. For reference, an analysis of the error performance of the well known MMSE combining receiver [7] with the addition of channel estimates is presented as well. In contrast to joint detection which exploits knowledge of other users' channels to make a single joint decision, the MMSE combining receiver makes separate decisions on each user's data while attempting to suppress the interference from other users.

A. Joint Detection

Let the transmitted data vector be $\mathbf{c}_j = (c_{j1}, c_{j2}, \ldots, c_{jM})$. According to (12), the detector chooses the erroneous data vector $\mathbf{c}_i = (c_{i1}, c_{i2}, \ldots, c_{iM})$ instead of $\mathbf{c}_j$ if $\Lambda_i < \Lambda_j$. The probability of this pairwise error event is denoted $P(D_{ij} < 0 \mid \mathbf{c}_j)$ where $D_{ij} = \Lambda_i - \Lambda_j$. The union bound on the probability of symbol error for the m-th user is given by

$$P_{s_m} \leq \sum_i P(D_{ij} < 0 \mid \mathbf{c}_j) \tag{13}$$

where i indexes the subset of vectors in C that differ in their m-th position from $\mathbf{c}_j$. The pairwise error probability $P(D_{ij} < 0 \mid \mathbf{c}_j)$ is determined in the following manner. First, using an alternative form of the metric defined in (12) the random variable D_{ij} can be written as the sum of L Hermitian quadratic forms in $M+1$ zero-mean complex Gaussian random variates:

$$D_{ij} = \sum_{l=1}^{L} \mathbf{z}_l^{\dagger} \mathbf{F}_{ij} \mathbf{z}_l \tag{14}$$

where the length $M+1$ vector $\mathbf{z}_l$ is defined as

$$\mathbf{z}_l = (r_l, v_{l1}, v_{l2}, \ldots, v_{lM})^T \tag{15}$$

with covariance matrix $\mathbf{R} = \frac{1}{2} E\left[\mathbf{z}_l \mathbf{z}_l^{\dagger} \mid \mathbf{c}_j\right]$. Note that due to independent fading across the antenna array, the $\mathbf{z}_l$'s in (14) are independent. Furthermore, $\mathbf{R}$ is independent of l. The rank 2 Hermitian matrix $\mathbf{F}_{ij}$ is defined as

$$\mathbf{F}_{ij} = \left(\mathbf{u}_i \mathbf{u}_i^{\dagger} - \mathbf{u}_j \mathbf{u}_j^{\dagger}\right)^* \tag{16}$$

where

$$\mathbf{u}_i = (1, -A_1\beta_1 c_{i1}, -A_2\beta_2 c_{i2}, \ldots, -A_M\beta_M c_{iM})^T \tag{17}$$

and

$$\mathbf{u}_j = (1, -A_1\beta_1 c_{j1}, -A_2\beta_2 c_{j2}, \ldots, -A_M\beta_M c_{jM})^T \tag{18}$$

According to [11, eq. (B-3-21)] the two-sided Laplace transform of the pdf of D_{ij} is

$$\begin{aligned} \Phi_{D_{ij}}(s) &= \left[\frac{1}{\det(\mathbf{I} + 2s\mathbf{R}\mathbf{F}_{ij})}\right]^L \\ &= \left[\frac{1}{\prod_{k=1}^{M+1}(1 + s\lambda_{ijk})}\right]^L \end{aligned} \tag{19}$$

where λ_{ijk} is the k-th eigenvalue of $2\mathbf{R}\mathbf{F}_{ij}$.

Since $\mathbf{F}_{ij}$ is only rank 2, the matrix $2\mathbf{R}\mathbf{F}_{ij}$ has only two non-zero eigenvalues denoted λ_{ij1} and λ_{ij2} (one positive and one negative). Thus (19) reduces to a form similar to [12, eq. (4B.7)]. With suitable modifications, the sought after pairwise error probability $P(D_{ij} < 0 \mid \mathbf{c}_j)$ is easily determined. The resulting symbol error probability for the m-th user is then

$$P_{s_m} \leq \sum_i \frac{1}{\left(1 - \frac{\lambda_{ij1}}{\lambda_{ij2}}\right)^{2L-1}} \sum_{k=0}^{L-1} \binom{2L-1}{k} \left(-\frac{\lambda_{ij1}}{\lambda_{ij2}}\right)^k \tag{20}$$

where the convention is used that the eigenvalue λ_{ij1} is positive and λ_{ij2} is negative.

B. MMSE Combining

As is well known [7], the MMSE combining receiver weights the received signal vector $\mathbf{r}(k)$ by the weight vector $\mathbf{w}_m(k)$ and combines the weighted signals to form an estimate of the m-th user's symbol $c_m(k)$. The output of the detector is the symbol decision $\widehat{c}_m(k)$ which is chosen to be that symbol closest in Euclidean distance to the combiner output $\widetilde{c}_m(k) = \mathbf{w}_m^{\dagger}(k)\mathbf{r}(k)$. With the addition of channel estimates, the optimal weight vector for the m-th user is given by

$$\begin{aligned} \mathbf{w}_m^{opt} &= \left(E\left[\mathbf{r}\mathbf{r}^{\dagger} \mid c_m, \mathbf{V}\right]\right)^{-1} E\left[c_m^* \mathbf{r} \mid c_m, \mathbf{V}\right] \\ &= \left(\sum_{n=1}^{M} A_n^2 \left(\mathbf{v}_n \mathbf{v}_n^{\dagger} + 2\sigma_{e_n}^2 \mathbf{I}\right) + 2N_o \mathbf{I}\right)^{-1} A_m \beta_m \mathbf{v}_m. \end{aligned} \tag{21}$$

The expectation in (21) is taken over the joint ensemble of channel estimation errors, noise, and the $M-1$ interfering users' symbols. This result extends that in [7] to include the effects of imperfect channel estimation. For the case of perfect CSI though, (21) and [7, eq. (9)] are equivalent.

For the special case of BPSK modulation, the probability of bit error for the m-th user, denoted P_{b_m} is

$$P_{b_m} = P\left[\mathrm{Re}\left[\widetilde{c}_m\right] < 0 \mid c_m = +1\right]. \tag{22}$$

Unfortunately, the pdf of $\widetilde{c}_m$ is difficult to obtain for $M > 1$ due to the matrix inversion in (21). Thus, in order to determine bit error rates for arbitrary M, simulation is required.

V. Performance Results

In this section we provide some numerical results for BPSK modulation that follow from the analysis in the previous section. For the results concerning perfect CSI, $\rho_m = 1$ and $\sigma_{e_m}^2 = 0$ for all users.

Fig. 2 shows the performance of joint detection of equipower signals using only a single antenna. Observing the perfect CSI curves, the performance degrades by only about 2 dB for each additional user. This value decreases to approximately 0.2 dB with 4 antennas. In the case of imperfect CSI, an error floor results in the high SNR region similar to that observed in systems employing differential detection. As can be seen, though, up to 4 users may be supported while still maintaining an error rate below 10^{-2}—a striking result for only a single antenna.

These results are quite general in that we have assumed an arbitrary—but fixed—ρ_m with no reference to

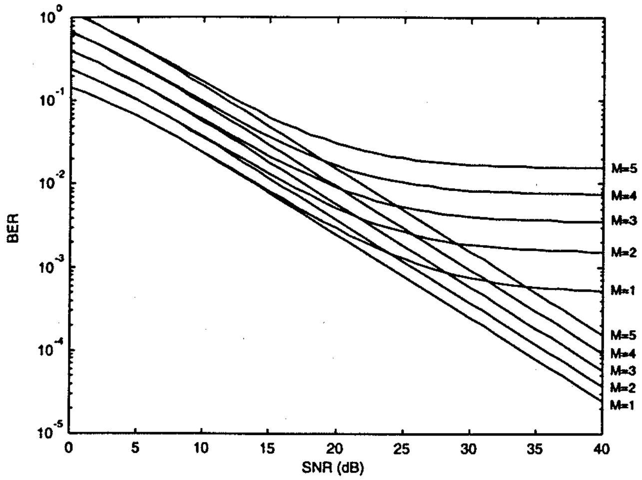

Fig. 2. Performance of joint detection for a single antenna, equipower users, and both perfect CSI (lower curves) and imperfect CSI (upper curves) with $\rho_m = 0.999$ for all users.

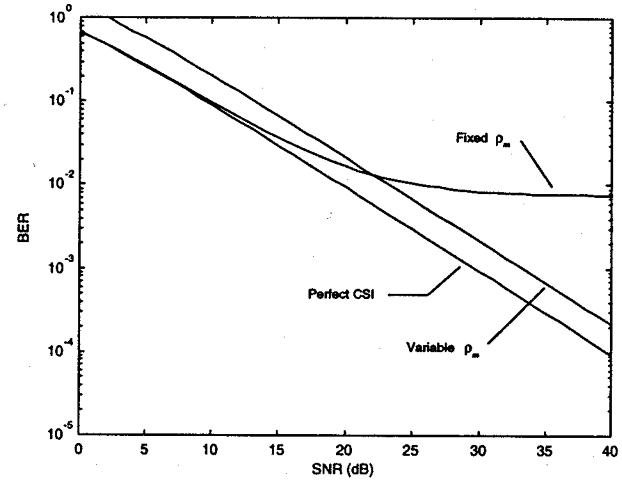

Fig. 3. Comparison of performance using fixed and variable ρ_m models for the detection of $M = 4$ equipower users using a single antenna.

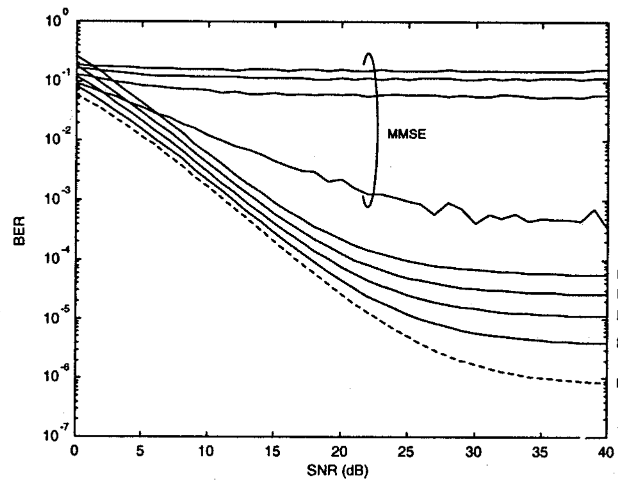

Fig. 4. Performance of joint detection compared to MMSE combining for $L = 2$ antennas, and imperfect CSI with $\rho_m = 0.999$ for all users.

the actual channel estimation scheme used. In a future paper, multiuser channel estimation schemes will be investigated. A fixed value of ρ_m may give pessimistic results, though, since in typical systems employing pilot symbols, e.g. [8], channel estimation accuracy improves with increasing SNR.

For illustrative purposes, results comparing the performance using variable (SNR dependent) and fixed ρ_m values are presented in Fig. 3. The model used for variable ρ_m is derived from [8] assuming typical parameters used in the PSAM scheme. Note that at the midpoint of the SNR range (20 dB) the variable and fixed values of ρ_m are equivalent (0.999). Evidently use of the variable ρ_m model eliminates the error floor and results in performance degraded by a constant amount from the perfect CSI case over the whole SNR range. This behaviour is typical of what we have observed in other situations described in this paper.

Fig. 4 compares the performance of both joint detection and MMSE combining for 2 antennas assuming imperfect CSI and equipower users. Clearly joint detection outperforms MMSE combining by a very large margin. Evidently joint detection can support many more users than the number of antennas. This fact is clearly illustrated in Fig. 5 in which the error performance of both joint detection and MMSE combining are plotted against number of users at a fixed SNR. As can be seen, the performance of joint detection degrades slowly as the number of users increases. In contrast, the performance of MMSE combining degrades very quickly and saturates at an unacceptably high error rate for $M > L$.

To illustrate the channel estimation accuracy required to achieve a given performance for various numbers of users and antennas, Fig. 6 shows the correlation coefficient ρ_m required to achieve an error floor of 10^{-3}. As can be seen, the accuracy requirements relax as each additional antenna is added. For example, for 2 antennas, the correlation coefficient must be no less than about 0.995 for the detection of 4 users, whereas for 4 antennas, the correlation coefficient must be no less than about 0.95.

Fig. 7 presents results for two different non-equipower distributions. Note that the BER of each user is plotted against its *own* SNR. This implies that at a given SNR, the noise level is different for each user: smaller for the weak users, larger for the strong users. Consequently, the performance of the weak users appears to be better than that of the strong users. To make a comparison at the same noise level, i.e. under the same operating conditions, one must mentally shift the weak users' curves to the right by an amount equivalent to the power difference between the users (10 dB in this case). Comparison under the same operating conditions reveals that the performance of the strong users is better than that of the weak users by about

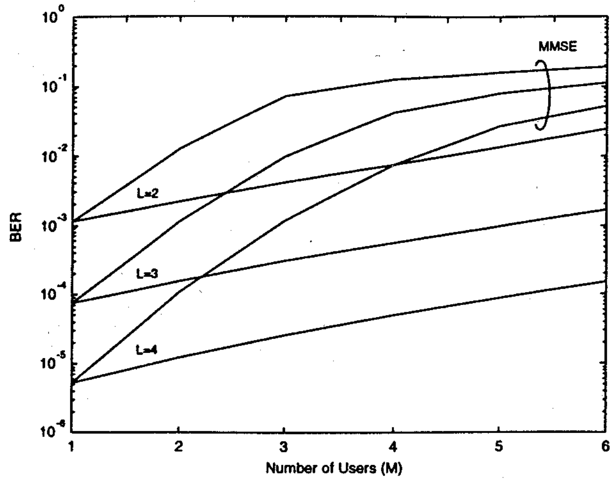

Fig. 5. Performance of joint detection compared to MMSE combining for a fixed SNR of 12 dB, equipower users, and imperfect CSI with $\rho_m = 0.99$ for all users.

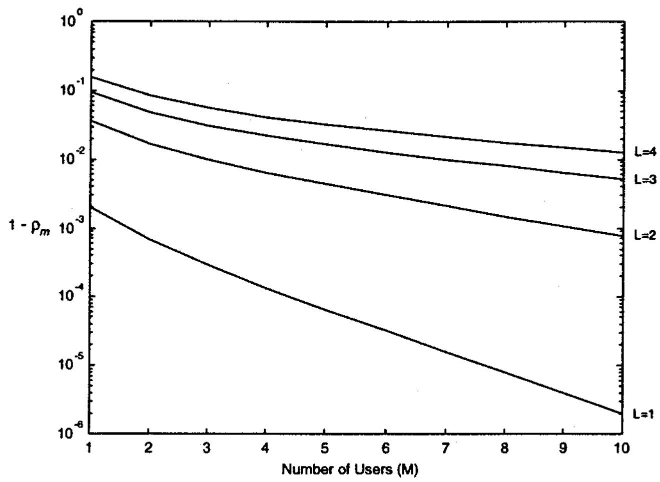

Fig. 6. Channel estimation correlation coefficient required to achieve error floor of 10^{-3} for equipower users.

7-8 dB. Moreover, the performance of all users is degraded from the equipower case, indicating that in an operational system, some degree of power control may be desirable.

VI. Conclusions

In this paper we have considered the joint detection of multiple cochannel, symbol synchronous, PSK signals using a diversity antenna array in a system with channel estimates available at the receiver. Results show that performance is very good: within the limits of achievable channel estimation accuracy, it is shown that several users may be supported using only a single antenna while maintaining the error floor below 10^{-2}—a commonly accepted threshold value. In the case of diversity reception, many more users than the number of antennas may be supported with a slow degradation in performance with each additional user. For all combinations of numbers of users and antennas, joint detection shows orders of magnitude improvement over MMSE combining.

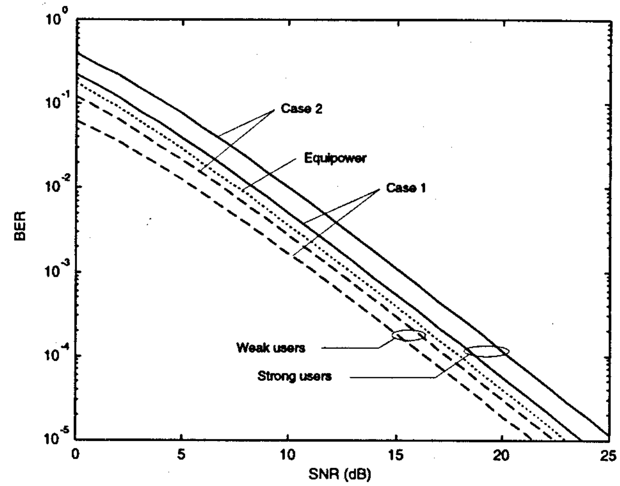

Fig. 7. Error performance of both strong and weak users (10 dB power difference) for $L = 2$ antennas, $M = 4$ users, and perfect CSI. Case 1: 3 strong/1 weak, Case 2: 1 strong/3 weak.

Useful bounds are presented which indicate the channel estimation accuracy required in order to achieve a given level of performance. Generally, the accuracy requirements relax significantly as the number of antennas is increased.

Unequal power distributions are investigated and it is found that both the weak and strong users performance is degraded from the equipower case, indicating that in a practical system some degree of power control may be desirable.

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