

Analytical Calculation of Outage Probability for a General Cellular Mobile Radio System

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Abstract—Outage probability in the presence of fading, shadowing, and path loss is a useful measure of the capacity of a cellular mobile radio system. Due to the complicated nature of the spatial average, though, previous capacity studies have resorted to numerical techniques such as Monte Carlo simulation. In this paper, we develop a fully analytical and computationally straightforward technique for the computation of outage probability through a simple, yet accurate, geometric approximation in the spatial average. An attractive feature of the technique is that it does not rely upon a hexagonal cell layout, thus enabling the study of more generalized systems in which the cochannel cells are of arbitrary size and location.

I. INTRODUCTION

In previous cellular system capacity studies, outage probability has been determined through time-consuming Monte Carlo simulation of outage events or by a variety of numerical techniques aimed at generating the distribution of the signal-to-interference ratio (SIR), e.g. [1],[2],[3]. This is primarily due to the complicated integrals involved in the spatial averaging of the interfering mobile's position-dependent path loss. Furthermore, these studies usually assumed a strict hexagonal cell layout in order to simplify the spatial averaging process.

In contrast, this paper presents a fully analytical technique for the calculation of outage probability used for the assessment of the uplink capacity of a general cellular system in which the cochannel cells are of arbitrary size and location. The system is assumed to be operating in a flat Rayleigh fading/log-normal shadowing environment with cochannel interference. The key feature of the technique is the analytical calculation of the moments of the inverse of SIR, in which the spatial averaging is performed analytically by making a simple geometric approximation which leads to a closed-form result. The technique is demonstrated for a typical cellular system layout with 120° sectorization and a base station employing diversity reception. Results comparing simulation and analysis show that the analytical technique achieves very good accuracy.

II. SYSTEM MODEL

Fig. 1 shows a few cells of the general cellular system studied in this paper. The central cell containing the desired user is surrounded by an arbitrary number of cochannel cells, each of radius R_i and at distance D_i . Cell sectorization is easily modeled by considering only those cochannel cells that fall within the angular width of a particular sector. This model goes beyond the rigid hexagonal cell layout used in previous capacity

studies [1],[2],[5] by modeling additional realism through arbitrary R_i and D_i . Furthermore, the model, and the resulting analysis in this paper, may easily be extended to multiuser systems where several intracell users are allowed to share the same time-frequency slot in order to increase capacity (see [4] in these proceedings).

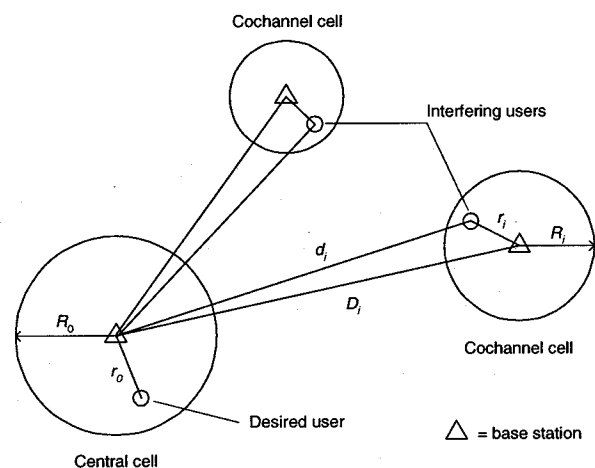


Fig. 1. Generalized cell layout. The distance D_i between the base stations of a particular cochannel cell and the central cell is arbitrary, as are the cell radii R_o and R_i .

The received signal on a particular diversity branch of the base station receiver in the central cell is given by

$$r(t) = r_0(t) + \sum_{i=1}^N r_i(t) + z(t) \quad (1)$$

where the first term is the signal received from the desired user in the central cell, the second term is the sum of the signals received from N interfering users in the surrounding cochannel cells, and the third term is additive white Gaussian noise with power spectral density N_o . All signals are represented by their complex baseband equivalents.

The transmitted signal from each mobile undergoes three effects, namely log-normal shadowing, path loss, and frequency-flat Rayleigh fading. The combined effects of log-normal shadowing and path loss are modeled by an attenuation (gain) of x dB superimposed on a path loss of $\gamma 10 \log_{10} [d]$ dB. Here x is a zero mean Gaussian random variable (RV) with standard deviation σ_x (in units of dB), d is the distance from the mobile to the central base, and γ is the path loss coefficient. For typical cellular systems, σ_x is usually between 6 and 8 dB [1], and γ is between 3 and 4 [2]. Frequency-flat Rayleigh

fading is modeled by the zero mean complex Gaussian random process $g(t)$ with variance σ_g^2 . Accordingly, the received signals from the desired and each interfering user are, respectively,

$$r_0(t) = 10^{x_0/20} r_0^{-\gamma_0/2} g_0(t) s_0(t) \quad (2a)$$

$$r_i(t) = 10^{x_i/20} d_i^{-\gamma_i/2} g_i(t) s_i(t - \tau_i) \quad (2b)$$

where $s_0(t)$ and $s_i(t)$ are the corresponding transmitted signals. The relative delay τ_i is included to reflect the fact that the interfering signals do not, in general, arrive synchronously.

Assuming a linear modulation format, the transmitted signal of the desired user is

$$s_0(t) = \sqrt{2E_{s_0}} \sum_k c_0(k) p(t - kT) \quad (3)$$

where $p(t)$ is a root-Nyquist pulse shape normalized to unit energy with autocorrelation function $q(t) = p(t) \otimes p^*(-t)$ (\otimes denotes convolution), T is the symbol period, E_{s_0} is the average transmitted energy per symbol, and $c_0(k)$ is the k th symbol drawn from the symbol sequence \mathbf{c}_0 . For convenience, the data symbols are normalized such that $E[|c_0(k)|^2] = 1$. For the interfering users' transmitted signals, the subscript '0' in (3) is replaced by 'i'.

In this paper, perfect power control is assumed, and, as in other capacity studies [2],[3], each mobile is assumed to be power controlled by the geographically closest base station, rather than by the base with the strongest received pilot. Consequently, at all times, $r_0 \leq R_0$ and $r_i \leq R_i$. With perfect power control, the base station in the central cell sets the average power in the transmitted signal $s_0(t)$ to exactly compensate for shadowing and path loss such that the average power of the received signal $r_0(t)$ at the base station is constant regardless of shadowing and mobile position. Thus,, $(2E_{s_0}/T) \sigma_{g_0}^2 10^{x_0/10} r_0^{-\gamma_0} = P_o$ where P_o is the constant received power. The base station in each cochannel cell performs the same function, except that the path loss is determined by the distance r_i from the interfering user to its *own* base rather than the distance to the central base. Furthermore, the shadowing is described by a different zero mean Gaussian random variable y_i (independent of x_i) with standard deviation σ_{y_i} . Consequently, the power control law for interfering mobiles is $(2E_{s_i}/T) \sigma_{g_i}^2 10^{y_i/10} r_i^{-\gamma_i} = P_o$.

In this paper, the signal-to-interference-plus-noise ratio (SINR) is defined at the output of the matched filter as follows

$$\Gamma = \frac{\frac{1}{2} E[|y_0(t)|^2]}{\sum_{i=1}^N \frac{1}{2} E[|y_i(t)|^2] + \frac{1}{2} E[|n(t)|^2]} \quad (4)$$

where $y_0(t)$, $y_i(t)$, and $n(t)$ are given by the convolution of the matched filter response $p^*(-t)$ with $r_0(t)$, $r_i(t)$, and $z(t)$ respectively. In the case of the interference component, the expectation is taken over the

fading ensembles only. In the case of the desired component, the expectation is taken over both the fading and symbol ensembles. Recall, though, that $y_0(t)$ is a cyclostationary process due to the linear modulation format. As a result, the power of the desired component is periodic with period T ; thus, the numerator of (4) should be interpreted as the *peak* power of $y_0(t)$.

Performing the expectations in (4) and using the power control laws discussed above gives the SINR as $\Gamma = 1/(\Gamma_I^{-1} + \Gamma_N^{-1})$ where

$$\Gamma_I^{-1} = \sum_{i=1}^N 10^{w_i/10} \left(\frac{r_i}{d_i} \right)^{\gamma_i} \left| \sum_k c_i(k) q(t - kT - \tau_i) \right|^2 \quad (5)$$

is the interference-to-signal ratio (ISR), and $\Gamma_N = P_o T / N_o$ is the signal-to-noise ratio (SNR). It is assumed here that the fading variance $\sigma_{g_0}^2 = \sigma_{g_i}^2$ for all interferers. Note that, the shadowing RV in (5) is simply $w_i = x_i - y_i$ with variance $\sigma_{w_i}^2 = \sigma_{x_i}^2 + \sigma_{y_i}^2$. Although Γ is defined here as a *power* ratio at the output of the matched filter, one can see that without interference, Γ is equivalent to the commonly defined *energy* ratio E_s/N_o , where E_s is, as usual, the average energy per symbol in the received signal $r_0(t)$.

III. OUTAGE PROBABILITY

As can be seen from above, the SINR Γ is a random variable governed by the set of shadowing random variables, mobile positions, symbol sequences, and relative delays of the interferers, that is $\{w_i, r_i, d_i, c_i, \tau_i\}$. Consequently, the symbol error rate experienced by the desired user in the central cell is also a random variable. Outage probability is defined as the probability that the random symbol error rate is greater than a certain threshold (typically 10^{-2} or 10^{-3}). Since the symbol error rate typically decreases monotonically with increasing Γ , outage probability is given by

$$P_{out} = P[\Gamma < \Gamma_t] = P[\Gamma_I^{-1} > \Gamma_t^{-1} - \Gamma_N^{-1}] \quad (6)$$

where Γ_t is the SINR for which the symbol error rate is equal to the threshold. The threshold SINR Γ_t depends on the detection technique as well as the diversity order of the system.

Clearly, an exact assessment of outage probability requires knowledge of the complementary cumulative distribution function (ccdf) of the ISR Γ_I^{-1} . Unfortunately, the expression for Γ_I^{-1} involves the sum of weighted log-normal variates for which the distribution is unknown. As is common in many capacity studies, though, the distribution is approximated by a convenient two-parameter distribution, e.g. log-normal [1],[7],[8] or Gaussian [2].

Assuming a log-normal distribution, outage probability is approximated as follows. First, define a log-normal random variable $v = 10^{x/10}$, where x is Gaussian with mean μ_x and standard deviation σ_x . The pdf of v is given by [8]

$$f_V(v) = \frac{1}{\sqrt{2\pi}\sigma v} e^{-\frac{1}{2\sigma^2} \ln^2\left(\frac{v}{\mu}\right)} \quad (7)$$

where $\mu = 10^{\mu_w/10}$ and $\sigma = [\ln(10)/10]\sigma_x$. The complementary cumulative distribution of v is then

$$F_V(v) = \int_v^\infty f_V(t) dt = Q\left[\frac{1}{\sigma} \ln\left(\frac{v}{\mu}\right)\right] \quad (8)$$

where $Q(v)$ is the Gaussian Q-function. Now make the approximation $\Gamma_I^{-1} \approx v$ which results in

$$P_{out} \approx Q\left[\frac{1}{\sigma} \ln\left(\frac{\Gamma_t^{-1} - \Gamma_N^{-1}}{\mu}\right)\right]. \quad (9)$$

The two parameters μ and σ may be obtained through Wilkinson's method [7], whereby the first two moments of Γ_I^{-1} , denoted m_1 and m_2 , are matched with the first two moments of v , given by $E[v] = \mu e^{\sigma^2/2}$ and $E[v^2] = \mu^2 e^{2\sigma^2}$ [8]. Solving for μ and σ then gives $\mu = m_1^2/\sqrt{m_2}$ and $\sigma = \sqrt{\ln[m_2/m_1^2]}$. In the next section, closed-form analytical expressions are developed for m_1 and m_2 .

IV. MOMENTS OF ISR

Observing (5) one can see that the ISR Γ_I^{-1} is simply the sum of N independent random variables. Consequently, the first and second moments of ISR may be expressed as

$$E[\Gamma_I^{-1}] = \sum_{i=1}^N E[z_i] \quad (10)$$

and

$$E[\Gamma_I^{-2}] = \sum_{i=1}^N E[z_i^2] + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N E[z_i] E[z_j] \quad (11)$$

where z_i is the i th term in the summation of (5). Clearly, the calculation of the moments requires the calculation of $E[z_i^n]$ for $n = 1, 2$ for all interferers.

Due to the independence of the individual random variables in z_i , one can see that $E[z_i^n]$ factors into three separate expectations. The first, namely the expectation over the shadowing ensemble, is easily performed using the moments of a log-normal random variable discussed previously. The result is

$$E[10^{nw_i/10}] = \exp\left[\frac{1}{2}n^2\left(\frac{\ln 10}{10}\sigma_{w_i}\right)^2\right]. \quad (12)$$

The two remaining averages are a spatial average, and an average over the symbol and delay ensembles. These are calculated in the following two subsections.

A. Spatial Average

Since spatial averaging is key to this and other capacity studies, we define the following function which we call the "position moment"

$$\Pi_p(R'_i) \triangleq E\left[\left(\frac{r_i}{d_i}\right)^p\right] \quad (13)$$

where $R'_i = R_i/D_i$. Fig. 2 shows the coordinate system used to calculate the position moment. By definition,

the expectation is calculated by weighting the quantity $(r_i/d_i)^p$ by the probability that a mobile occupies the infinitesimal area $dxdy$ at (x, y) and then adding up (integrating) the result over the whole cell. Assuming user i may be located at any point in the cell with equal probability, the probability that the user occupies the infinitesimal area is simply $dxdy$ divided by the total area of the cell.

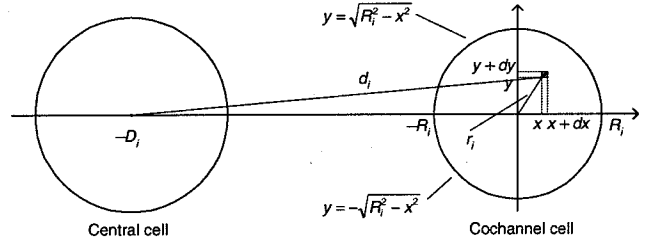


Fig. 2. Co-ordinate system used to calculate the position moment $\Pi_p(R'_i)$.

In the coordinate system of Fig. 2, the distance between the interferer and its own base is $r_i = \sqrt{x^2 + y^2}$, and the distance between the interferer and the central base is $d_i = \sqrt{(D_i + x)^2 + y^2}$. Consequently, the position moment is given by

$$\Pi_p(R'_i) = \frac{1}{\pi R_i'^2} \int_{-R'_i}^{R'_i} \int_{-\sqrt{R_i'^2 - u^2}}^{\sqrt{R_i'^2 - u^2}} \left(\frac{u^2 + v^2}{(1 + u)^2 + v^2}\right)^{\frac{p}{2}} dv du \quad (14)$$

where the transformation $u = x/D_i$ and $v = y/D_i$ is employed to make all variables dimensionless quantities. Furthermore, the transformation highlights the fact the position moment is a function of the ratio $R'_i = R_i/D_i$ and not R_i and D_i individually. To the authors' knowledge, no closed form solution of (14) exists, which is why, in previous capacity studies [1], [2], spatial averaging has been performed by either numerical integration or time consuming Monte Carlo simulation. In this paper, though, we make a simple, and virtually harmless, geometric approximation that allows us to derive a closed form analytical solution of the double integral, thereby avoiding long simulation runs.

The geometric approximation is based on the observation that for sufficiently small R'_i (distant cells), $d_i \approx D_i + x$ which results in the following expression for the approximate position moment after transformation to dimensionless quantities:

$$\Pi_p(R'_i) \approx \frac{1}{\pi R_i'^2} \int_{-R'_i}^{R'_i} \frac{du}{(1 + u)^p} \int_{-\sqrt{R_i'^2 - u^2}}^{\sqrt{R_i'^2 - u^2}} (u^2 + v^2)^{p/2} dv. \quad (15)$$

A closed-form analytical solution of this double integral for integer values of p is provided in the appendix. Note that the restriction to integer values of p implies that the path loss coefficient γ_i takes on integer values. For non-integer values of γ_i , the position moment may be obtained by interpolation. In order to demonstrate the

validity of the geometric approximation, the exact and approximate position moments are compared in Fig. 3. As expected, the accuracy of the approximations improves as R'_i decreases (increasingly distant cochannel cells). More importantly though, even for $R'_i = 0.5$ (equal radius contiguous cochannel cells), the approximations result in very good accuracy.

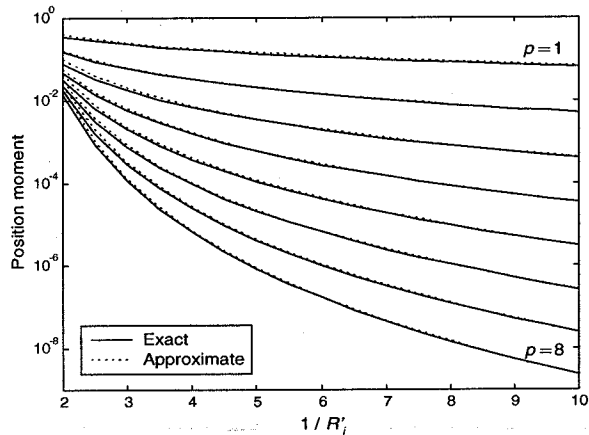


Fig. 3. Comparison of exact and approximate position moments.

B. Data/Delay Average

The remaining average to be calculated is

$$\Upsilon_i(n) = E \left[\left| \sum_k c_i(k) q(t - kT - \tau_i) \right|^{2n} \right]. \quad (16)$$

First, (16) is expanded in a multiple summation, and the average over the data ensemble is performed noting that the sequence c_i is white. Due to the complicated nature of the full Nyquist pulse $q(t)$, the average over the delay τ_i is then calculated by numerical integration. For BPSK modulation, pulse rolloff $\beta = 0.5$, and τ_i uniformly distributed on $[-T/2, T/2]$, the data/delay averages for $n = 1$ and 2 are $\Upsilon_i(1) = 0.8750$ and $\Upsilon_i(2) = 1.0657$ respectively. Note that there is no dependence on t because of the uniform distribution of delay. Furthermore, (16) does not usually depend on i , since all interferers likely use the same modulation format; consequently, the data/delay average need only be computed once and stored for later use.

V. RESULTS

Fig. 4 illustrates the accuracy of the log-normal approximation and Wilkinson's moment matching method discussed in Section III. In this plot, the lognormal cdf $F_V(v)$ is compared with the empirical cdf of Γ_I^{-1} generated via direct calculation of equation (5) in a Monte Carlo simulation. Although our analytical technique for the computation of the moments of the ISR is valid for arbitrary R_i and D_i , for familiarity, the results are generated for a layout of circular cells with cell centres identical to that of a standard hexagonal cell layout with three tiers of cochannel cells. Furthermore, ideal 120° antenna sectorization is assumed resulting in a total of

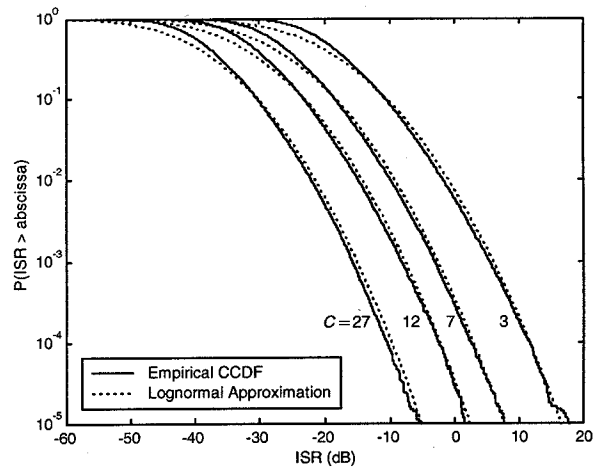


Fig. 4. Complementary cumulative distribution function (ccdf) of the ISR Γ_I^{-1} for $\gamma_i = 4 \forall i$ and $\sigma_{w_i} = \sqrt{2} \cdot 6$ dB $\forall i$. BPSK modulation with pulse rolloff $\beta = 0.5$ is assumed. C is the cluster size of the hexagonal cell layout. The empirical cdf is generated via Monte Carlo simulation with 10^6 points.

12 cochannel cells (2 in the first tier, 4 in the second, and 6 in the third). The radius of each circular cell is scaled such that its area is the same as the hexagonal cell shape.

As can be seen in Fig. 4, the log-normal cdf is a very good approximation to the empirical cdf for outage probabilities less than about 10%. The same accuracy was observed for several different combinations of path loss coefficients in the range 3-4 and shadowing standard deviations in the range 6-8 dB. This fortunate result indicates that even though the expression for the ISR Γ_I^{-1} involves the sum of randomly weighted log-normal random variables (see (5)), rather than pure log-normal RVs as studied in [7], the sum is still closely log-normal.

In Fig. 5, outage probability calculated via (9) is plotted versus cluster size assuming coherent detection of BPSK with L diversity antennas. The threshold SINR Γ_t is calculated for each value of L using the results of [9] assuming a threshold BER of 10^{-2} . The SNR Γ_N is arbitrarily chosen to be that corresponding to a BER of 10^{-3} . For $L = 1, 2, 3$, and 4 , the threshold ISRs in (9), i.e. $\Gamma_t^{-1} - \Gamma_N^{-1}$, are given by -14.3, -6.84, -4.13, and -2.53 dB respectively. As can be seen, the analytical calculation of outage probability results in values very close to that from Monte Carlo simulation. As expected, increasing the number of antennas reduces the outage probability dramatically. Equivalently, for a fixed outage probability, increasing the number of antennas allows the use of a smaller cluster sizes, hence increasing spectrum efficiency.

VI. CONCLUSIONS

In this paper, a new, fully analytical technique has been presented for the calculation of outage probability for a general cellular mobile radio system. The technique is based on the analytical calculation of the moments of the interference-to-signal ratio (ISR). The key contribution is the closed-form geometric approximation for the spatial average of the interfering mobile's

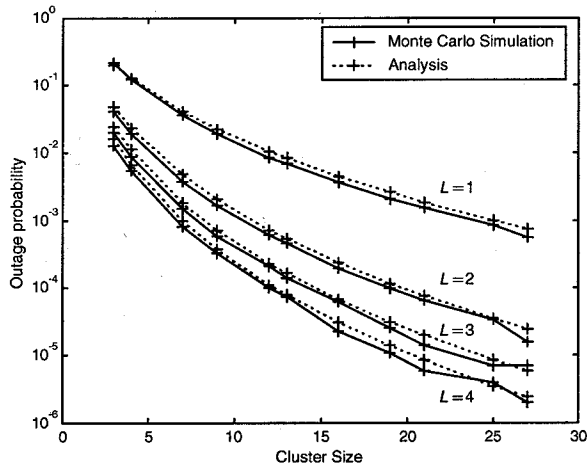


Fig. 5. Outage probability for the coherent detection of BPSK with L diversity antennas. System parameters are the same as in Fig. 2.

position dependent path loss which we term the “position moment.” The moments of ISR are used with Wilkinson’s moment matching method to approximate the complementary cumulative distribution function of the ISR which is used to calculate outage probability directly. This approximation along with the geometric approximation in the position moment is shown to result in very good accuracy for typical system parameters.

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APPENDIX

CALCULATION OF APPROXIMATE POSITION MOMENT

In this appendix, an analytical solution is provided for the approximate position moment

$$\Pi_p(R) = \frac{1}{\pi R^2} \int_{-R}^R \frac{du}{(1+u)^p} \int_{-\sqrt{R^2-u^2}}^{\sqrt{R^2-u^2}} (u^2+v^2)^{p/2} dv \quad (17)$$

where $R < 1$ and p is an integer. For the case of p even, the solution of the inner integral [10], denoted $f(u)$, is

$$f(u) = 2\sqrt{R^2-u^2} \sum_{k=0}^{p/2} \frac{\binom{p/2}{k}}{2k+1} u^{p-2k} (R^2-u^2)^k. \quad (18)$$

Expanding the term $(R^2-u^2)^k$ in a binomial series and substituting the result into (17) gives

$$\Pi_p(R) = \frac{2}{\pi R^2} \sum_{k=0}^{p/2} \frac{\binom{p/2}{k}}{2k+1} \sum_{m=0}^k (-1)^m \binom{k}{m} \cdot R^{2(k-m)} \phi_{p-2(k-m),p}(R) \quad (19)$$

where

$$\begin{aligned} \phi_{m,n}(R) &= \int_{-R}^R \frac{u^m \sqrt{R^2-u^2}}{(1+u)^n} du \\ &= \sum_{l=0}^m (-1)^l \binom{m}{l} I_{n-m+l}(R) \end{aligned} \quad (20)$$

and

$$I_j(R) = \int_{1-R}^{1+R} \frac{\sqrt{R^2-(w-1)^2}}{w^j} dw. \quad (21)$$

The latter equality in (20) is found by making the substitution $w = 1 + u$ and then expanding the term $u^m = (w-1)^m$ in the numerator in a binomial series. Note that in (20), $m \leq n$, so that in (21), $j \geq 0$.

The solution of $I_j(R)$ for $j \geq 0$ is given by the difference equation [10]

$$\begin{aligned} I_j(R) &= \left(\frac{2j-5}{j-1} \right) \left(\frac{1}{1-R^2} \right) I_{j-1}(R) \\ &+ \left(\frac{4-j}{j-1} \right) \left(\frac{1}{1-R^2} \right) I_{j-2}(R) \end{aligned} \quad (22)$$

with initial conditions

$$I_0(R) = \frac{1}{2} \pi R^2 \quad (23a)$$

$$I_1(R) = \pi \left(1 - \sqrt{1-R^2} \right). \quad (23b)$$

Thus, starting with $I_0(R)$ and $I_1(R)$, the integral $I_j(R)$ may be found in a recursive manner for any j .

For the case of p odd, the solution of the inner integral of (17) involves a complicated logarithmic function. Consequently, the complete solution of (17) for p odd is analytically intractable. Fortunately, though, numerical evaluation of (17) shows that the position moment for p odd is approximated very well by the geometric mean of the position moments for $p-1$ and $p+1$; that is,

$$\Pi_p(R) \approx \sqrt{\Pi_{p-1}(R) \Pi_{p+1}(R)}. \quad (24)$$

Consequently, the solution of $\Pi_p(R)$ for p even, given in (19), is sufficient for calculating the position moment for all p , whether even or odd.

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