Increased Uplink Capacity for TDMA Systems Through Joint Detection with Diversity Arrays

Stephen J. Grant* and James K. Cavers
School of Engineering Science, Simon Fraser University, Burnaby, B.C., Canada V5A 1S6
*Email: grantq@sfu.ca Web: www.sfu.ca/~grantq Tel: (604) 291-5898

Abstract— **Joint detection of Rayleigh faded cochannel signals has recently been shown to produce a new mode of reception behaviour, in which each additional signal costs only a small dB penalty with no loss of diversity order at the base station. Many simultaneous users can be supported in the same time-frequency slot, which results in a large increase in capacity. In this paper, we quantify the capacity increase on a system-wide basis while accounting for the increase in interference from cochannel cells due to the increased number of users. We demonstrate a soft capacity limit, similar to CDMA, and show that with several users per cell, the relative capacity improvement over a single-user TDMA system approaches M (the number of users) as the number of antennas is increased. We also analyze a system that uses conventional MMSE combining of base station antennas, and show that system capacity increases by only a small factor and actually declines as more users are added.**

I. Introduction

TDMA systems are based on keeping simultaneous intracell users orthogonal in both time and frequency, while treating user signals in other cochannel cells as noise. The result is a hard capacity limit, in contrast to CDMA which exhibits a graceful degradation with additional users.

Recently, the use of base station diversity arrays to extend the TDMA capacity limit has been examined [1]-[4], where the objective is to allow several intracell cochannel signals in a form of spatial division multiple access (SDMA). The most common way to use these diversity receivers is minimum mean square error (MMSE) combining [2]. Although this technique does allow multiple cochannel users per time/frequency slot, and therefore an increase in capacity, it has two significant drawbacks: (1) the number of simultaneous users is limited to the number of antennas elements, which leads to a hard capacity limit; and (2) before the cell reaches that limit, each additional user reduces the order of diversity by one for all users—a critical consideration in fading channels.

In this paper, we examine joint detection (JD) of intracell cochannel users with a diversity array. Recently, JD has been shown to produce a new mode of reception behaviour within a cell [3], [4]. In contrast to MMSE combining, its salient features are the following: (1) the number of simultaneous users can greatly exceed the number of antennas; and (2) each additional user costs all users a small dB penalty in SNR, but has no effect on the order of diversity. This behaviour is illustrated in Fig. 1 where the bit error rate for JD is compared with that for MMSE combining for M cochannel users and two antennas (results are from [3]). For JD, two antenna elements continue to provide all users with dual diversity as the number of users is increased, whereas for MMSE combining with 2 users, both users experience only single diversity; for $M > 2$, MMSE combining fails. Note that for $M = 1$, the performance of MMSE and JD is identical.

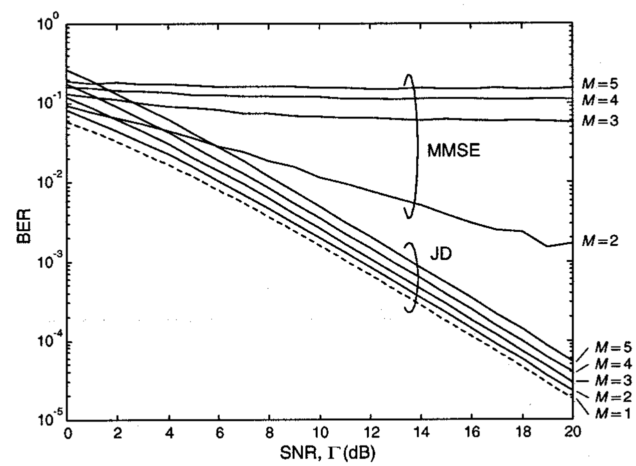

Fig. 1. BER for both joint detection (JD) and MMSE combining for several equipower users, 2 antennas, BPSK modulation, and perfect channel state information.

Partially offsetting the capacity gain from an increased number of intracell users is the fact that all cochannel cells have a similarly increased number of users. The result is a proportional increase in the level of cochannel interference (CCI) from the cochannel cells, necessitating an increase in reuse distance. Increasing the reuse distance dilutes the channel set (number of channels) available to each cell, so that system capacity does not increase in direct proportion to M.

This paper is the first analysis and first comparison of the effects of JD and of MMSE combining on uplink capacity, taking into account the additional CCI. The analysis accounts for shadowing and the random location of users in cochannel cells, and draws on the analytical method for determining cellular system performance presented in [5]. We show that JD can produce a significant increase in system-wide uplink capacity. In contrast, the alternative of MMSE combining results in only a modest initial increase, followed by

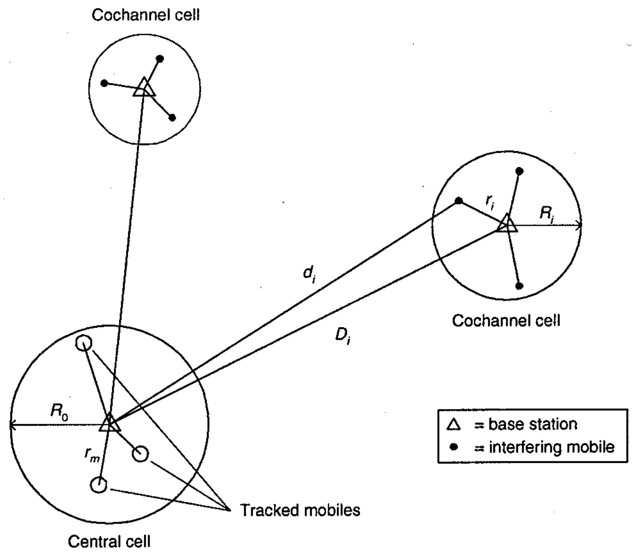

Fig. 2. Generalized cell layout. The distance D_i between the base stations of a particular cochannel cell and the central cell is arbitrary, as are the cell radii R_o and R_i. All cells are populated by M cochannel users.

decreased capacity as more users are added.

II. System Model

In order to assess uplink capacity, a cellular layout similar to that introduced in [5] is used. Fig. 2 illustrates a few cells of this generalized layout where the only difference between this figure and that in [5] is that both the central and cochannel cells are loaded with M cochannel users rather than just one. As can be seen, the central cell is surrounded by an arbitrary number of cochannel cells each of radius R_i and at distance D_i. Although this layout allows the study of more generalized systems in which the cochannel cells are of arbitrary size and location, in this paper, the cell centres are assumed to be located at the same positions as in a three tier hexagonal layout with cluster size C, and the radii of each circular cell is scaled such that its area is the same as the hexagonal cell shape. Furthermore, ideal 120° antenna sectorization is assumed, resulting in a total of 12 cochannel cells and a total of $12M$ interferers.

The received signal on a particular diversity branch of the base station receiver in the central cell is given by

$$r(t) = \sum_{m=1}^{M} r_m(t) + \sum_{i=1}^{N} r_i(t) + z(t) \tag{1}$$

where the first term is the sum of the signals received from the tracked users in the central cell, the second term is the sum of N interfering signals received from users in the surrounding cochannel cells ($N = 12M$), and the third term is additive white Gaussian noise with power spectral density N_o.

The transmitted signal $s_m(t)$ from each tracked mobile undergoes the effects of path loss, log-normal shadowing, and frequency-flat Rayleigh fading. Accordingly, the received signal from each tracked mobile is given by

$$r_m(t) = 10^{x_m/20} r_m^{-\gamma_m/2} g_m(t) s_m(t) \tag{2}$$

where r_m is the distance between the tracked mobile and the central base, γ_m is the path loss coefficient, x_m is a zero mean Gaussian random variable with standard deviation σ_{x_m} that models shadowing, and $g_m(t)$ is a complex Gaussian random process with variance $\sigma^2_{g_m}$ that models frequency-flat Rayleigh. Typical values for γ_m are between 3 and 4, and for σ_{x_m} are between 6 and 8 dB [6],[7]. Similarly, the received signal from each interfering mobile is given by

$$r_i(t) = 10^{x_i/20} d_i^{-\gamma_i/2} g_i(t) s_i(t - \tau_i) \tag{3}$$

where d_i is the distance between the interferer and the central base. The delay τ_i is included to reflect the fact that the interfering signals do not arrive synchronously. In contrast, the tracked mobiles in the central cell are assumed to by synchronized by symbol, which allows the results of [3] to be used for a comparison of joint detection and MMSE combining. Furthermore, a PSK modulation format is assumed for both $s_m(t)$ and $s_i(t)$.

As in [7], it is assumed that mobiles in each cell are power controlled by the geographically closest base station, and that the power control algorithm is able to exactly compensate for path loss and shadowing. Consequently, all tracked signals in the central cell arrive at the base with the same (constant) time-average power

$$\begin{aligned} P_o &= \frac{1}{2} E\left[|r_m(t)|^2\right] \\ &= 2 P_{s_m} \sigma^2_{g_m} 10^{x_m/10} r_m^{-\gamma_m} \end{aligned} \tag{4}$$

where P_{s_m} is the (controlled) time-average power of the transmitted signal $s_m(t)$. A similar power control law applies to the cochannel cells with the subscript 'm' replaced by 'i'. The only difference is that a shadowing random variable (RV) x_i must be replaced by a different RV y_i that models the shadowing between the interfering mobile and its *own* base. For convenience, a new shadowing RV is defined as $w_i = x_i - y_i$ which has variance $\sigma^2_{w_i} = \sigma^2_{x_i} + \sigma^2_{y_i}$ since x_i and y_i are independent.

Due to power control, the signal-to-interference-plus-noise ratio (SINR), and hence the symbol error rate, for each of the M tracked users in the central cell is the same. Consequently, the SINR for user m is defined in the same fashion as in [5], and is given by $\Gamma = 1/\left(\Gamma_I^{-1} + \Gamma_N^{-1}\right)$ where

$$\Gamma_I^{-1} = \sum_{i=1}^{N} 10^{w_i/10} \left(\frac{r_i}{d_i}\right)^{\gamma_i} \left|\sum_k c_i(k)\, q(t - kT - \tau_i)\right|^2 \tag{5}$$

is the interference-to-signal ratio (ISR) and $\Gamma_N = P_o T/N_o$ is the signal-to-noise ratio (SNR). Here $c_i(k)$ is the PSK symbol sequence for user i, $q(t)$ is a full Nyquist pulse given by the convolution of the transmit pulse shape $p(t)$ and the matched filter $p^*(-t)$, and T is the symbol period. The rolloff parameter of $p(t)$ is denoted by β (e.g., $\beta = 0.5$). In (5) it is assumed that the fading variance $\sigma^2_{g_i} = \sigma^2_{g_m}$ for all interferers. Clearly, Γ is a random variable governed by the set of shadowing random variables, mobile positions, symbol sequences, and relative delays of the interferers, that is $\{w_i, r_i, d_i, \mathbf{c}_i, \tau_i\}$. Consequently, the symbol error rate experienced by the tracked users is also a random variable.

III. System Capacity

In general, the capacity of a cellular mobile radio system is determined via outage probability, which is defined as the probability that the random symbol error rate is greater than a certain threshold (typically 10^{-2} or 10^{-3}). Since the symbol error rate typically decreases monotonically with increasing Γ, outage probability is defined as

$$P_{out} = P[\Gamma < \Gamma_t] = P\left[\Gamma_I^{-1} > \Gamma_t^{-1} - \Gamma_N^{-1}\right] \quad (6)$$

where Γ_t is the SINR for which the symbol error rate is equal to the threshold. The threshold SINR Γ_t depends on the detection technique as well as the number of intracell cochannel users M and the number of diversity antennas L.

In [5] an all-analytical technique is developed for approximating outage probability. The technique is based on the observation that the complementary cumulative distribution function (ccdf) of the ISR Γ_I^{-1} is approximated very well by the ccdf of a log-normal random variable for outage probabilities less than about 10%. The parameters of the log-normal distribution, namely the mean μ and the standard deviation σ, are determined by matching the moments of Γ_I^{-1} (calculated analytically) to the moments of the log-normal random variable. The result is

$$P_{out} \approx Q\left[\frac{1}{\sigma}\ln\left(\frac{\Gamma_t^{-1} - \Gamma_N^{-1}}{\mu}\right)\right] \quad (7)$$

where $Q(\cdot)$ is the Gaussian Q-function, $\mu = m_1^2/\sqrt{m_2}$, and $\sigma = \sqrt{\ln\left[m_2/m_1^2\right]}$. The variables m_1 and m_2 are the first and second moments of Γ_I^{-1}, respectively, and are a function of the cluster size C, the number of users per cell M, as well as the shadowing variance $\sigma^2_{w_i}$, the path loss coefficient γ_i, the PSK constellation size, and the rolloff parameter of the Nyquist pulse $q(t)$.

Fig. 3 shows the log-normal approximation of the ccdf of Γ_I^{-1} from which outage probability is calculated using (7) for a given cluster size C and number of users per cell M. Alternatively, these curves may be used to design a system in which capacity is maximized by minimizing the cluster size C with the constraint that the outage rate remains below a certain value, typically 1%.

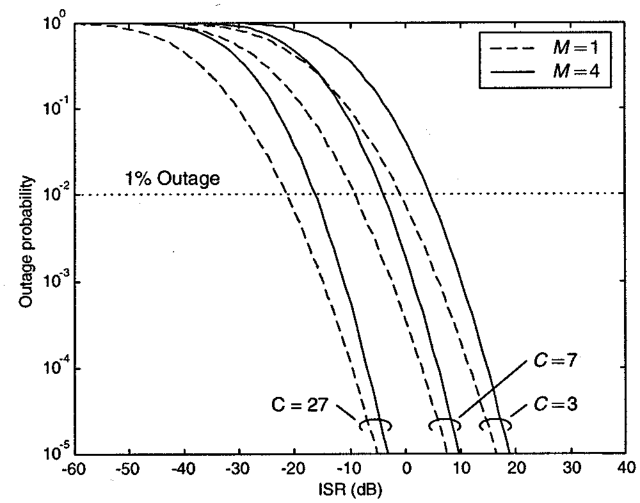

Fig. 3. Log-normal approximation to the complementary cumulative distribution function of ISR Γ_I^{-1} for $\sigma_{w_i} = \sqrt{2}\cdot 6$ dB $\forall i$, $\gamma_i = 4\ \forall i$, and BPSK modulation with pulse rolloff $\beta = 0.5$.

In this paper, the capacity of both a joint detection (JD) system and MMSE combining system is calculated for a given number of users M and antennas L as follows. First, for each detection technique, the threshold SINR Γ_t is determined corresponding to a bit error rate of 10^{-3}. For JD, the threshold SINR is calculated using the analytical expression for BER contained in [4], and for MMSE, the threshold SINR is found from graphs of BER vs. SNR generated using the method in [3].

Next, for a given cluster size C, μ and σ are calculated from m_1 and m_2. The allowed cluster sizes are given by $C = j^2 + jk + k^2$ where $j \geq 1$ and $k \leq j$ are the shift parameters of a standard hexagonal cell layout [8]. The first several allowed cluster sizes are $\{1, 3, 4, 7, 9, 12, 13, 16, 19, 21, 25, 27\}$. Next, using μ and σ, the equation $Q\left[\sigma^{-1}\ln\left(\Gamma_{I_r}^{-1}/\mu\right)\right] = 10^{-2}$ is solved numerically for Γ_{I_r}. This is the required SIR to achieve an outage rate of 1%.

By definition, the SIR Γ_I must be greater than the SINR Γ, thus if $\Gamma_{I_r} < \Gamma_t$ then it is not possible to achieve 1% outage using the given cluster size, and the next larger allowed cluster must be considered. This process is repeated until $\Gamma_{I_r} > \Gamma_t$. The resulting value of C is the minimum cluster size that satisfies $P_{out} \leq 1\%$ for the given number of users M and antennas L. The actual outage probability depends on the SNR Γ_N. For $P_{out} = 1\%$, the required SNR is $\Gamma_{N_r} = 1/\left(\Gamma_t^{-1} - \Gamma_{I_r}^{-1}\right)$. If the SNR is greater than Γ_{N_r}, then the outage rate is found using (7) and will be less than 1%.

Finally, capacity is defined as the product of the number of cochannel users per cell M and the number of channels available to each cell. The number of channels/cell is simply the total number of channels available to the entire system divided by the cluster size $C(M)$. Here the functional dependence of the cluster size on M is explicitly noted. Since the

TABLE I

MINIMUM CLUSTER SIZES FOR JD AND MMSE COMBINING FOR $\sigma_{w_i} = \sqrt{2} \cdot 6$ dB $\forall i$, $\gamma_i = 4\ \forall i$, AND BPSK MODULATION WITH 50% EXCESS BANDWIDTH.

M	L (JD) 2	3	4	L (MMSE) 2	3	4
1	9	7	7	9	7	7
2	13	9	7	36	12	7
3	16	9	7	-	36	12
4	19	12	9	-	-	28
5	25	12	9	-	-	-
6	27	13	9	-	-	-

number of channels/cell scales directly with the total number of channels, the total is set to unity, resulting in the simple capacity measure $M/C(M)$. For comparison with conventional TDMA systems in which only one intracell cochannel user is allowed, it is useful to define *relative* capacity as $M/C(M)$ divided by $1/C(1)$. Using this relative measure implies that the maximum achievable relative capacity is M since, for a fixed number of antennas L, $C(M) \geq C(1)$.

IV. RESULTS

Table I shows the minimum cluster sizes found using the method described in the previous section. For JD, the minimum cluster size increases gradually with each additional user, whereas for MMSE combining it increases quickly due to losses in diversity order. This has profound effects on relative capacity as shown in Fig. 4. With joint detection, as more and more intracell cochannel users are allowed, capacity continually increases with a rate proportional to the number of antennas. Furthermore, with 4 antennas, JD achieves the maximum relative capacity with up to 3 users. Experiments show that the capacity eventually saturates for a large number of users, indicating a soft limit. Note that capacity does not always increase monotonically towards the limit (e.g. the $L = 2$ curve), since the allowed cluster sizes are irregularly spaced.

In stark contrast to JD, the behaviour of MMSE combining exhibits a hard capacity limit: an optimum value of M exists, above which capacity actually decreases due to losses in diversity order. The corresponding peak capacity depends on the number of antennas, but with 4 antennas, the peak value is only two. In the case of $M = L$, the relative capacity can actually be less than unity, indicating a performance penalty compared to a standard TDMA system with a single user per cell ($M = 1$).

Fig. 5 shows the required SNR Γ_{N_r} to achieve an outage rate of 1%. As can be seen, for both JD and MMSE, the required SNR exhibits a general increase as each additional user is added, albeit non-monotonic due to the irregular spacing of the allowed cluster sizes. For JD, the increase is much slower, and the required SNR significantly less, than for MMSE com-

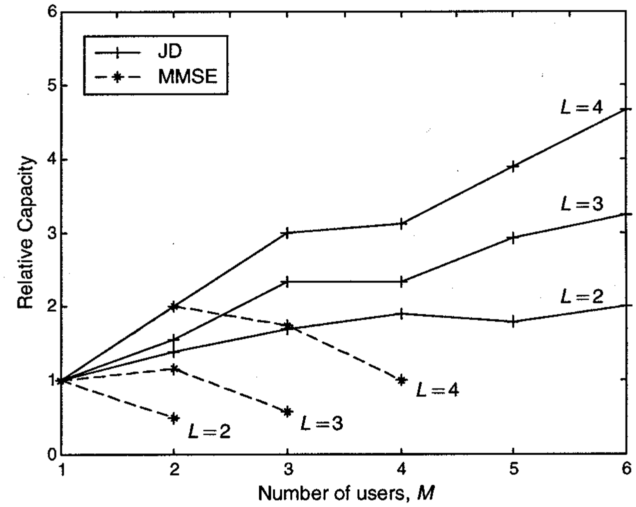

Fig. 4. Relative capacity of JD and MMSE combining systems for the minimum cluster sizes in Table I.

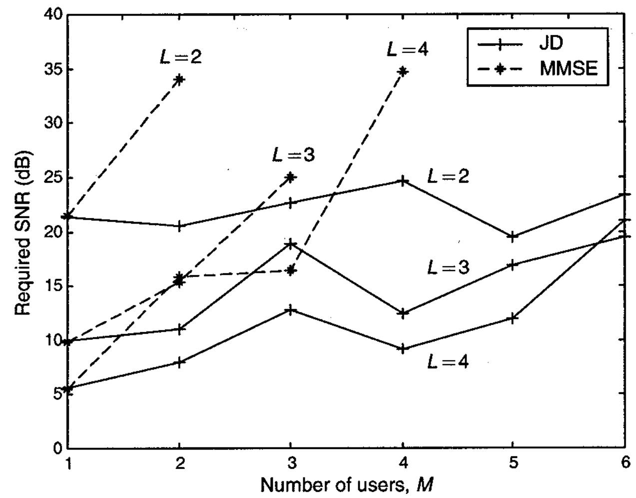

Fig. 5. Minimum required SNR for JD and MMSE combining systems for the minimum cluster sizes in Table I.

bining. For example, with four antennas, both JD and MMSE combining have the potential for doubling system capacity by allowing two intracell cochannel users, but for JD, the additional SNR required over a standard TDMA system with $M = 1$ is approximately 2 dB, whereas it is about 10 dB for MMSE.

The implications of a higher required SNR are that the received power P_o must be higher and/or the noise level N_o must be lower (recall that $\Gamma_N = P_oT/N_o$). The latter implies the use of a base station amplifier with a lower noise figure which increases cost. The former implies either an increase in transmit power, which increases mobile battery drain, and/or a reduction in cell size.

The effect on cell size may be seen by observing the power control law in (4) which assumes that the mobile units have infinite dynamic range. Taking into account the mobiles' limited transmit power places an upper limit on the cell radius R_0, so that power control may effectively compensate for shadowing at all locations within the cell. For a fixed maximum transmit power, a higher received power P_o implies the use of a smaller cell size. This implies increased infrastructure costs for a fixed coverage area due to the necessity of placing more base stations. A side benefit, though, is a system with reduced blocking probability in each cell.

V. Conclusions

In this paper, we have analyzed the uplink capacity of a TDMA cellular system employing joint detection (JD) with a diversity antenna array at the base station receiver in the presence of fading, shadowing, path loss, and cochannel interference. For reference, the analysis is compared to MMSE combining—a common technique for configuring diversity receivers. Both techniques allow several intracell users to share the same time/frequency slot in order to increase system capacity.

With JD and diversity arrays, TDMA capacity may be significantly extended. Even though the reuse distance must be increased as more and more intracell cochannel users are allowed, capacity continues to increase towards a soft limit, since, with L antennas, JD maintains L-fold diversity for all users. Moreover, for a system with a few users per cell, the relative improvement in capacity over a single-user TDMA system approaches M (the number of users per cell) as additional antennas are employed. For example, with 4 antennas, the capacity may be tripled with $M = 3$.

In contrast, with MMSE combining, the capacity increase is modest because of its fundamentally different behaviour. As each additional intracell cochannel user is allowed, the diversity order drops by one for all users, resulting in an optimum number of users beyond which capacity actually decreases. For example, with four antennas, the maximum capacity improvement over a single-user TDMA system is two.

For both JD and MMSE combining, the required receive SNR to achieve a fixed outage rate increases with M, although with JD the increase is much slower, and the required SNR much less, than for MMSE combining. Consequently, for a given capacity improvement, the use of joint detection allows for cheaper base station amplifiers, lower mobile battery drain, and lower infrastructure costs due to increased cell coverage.

Acknowledgment

This work was supported by a grant from Ericsson Inc., Research Triangle Park, North Carolina.

References

[1] J.H. Winters, J. Salz, and R.D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Transactions on Communications*, vol. 42, pp. 1740-1751, February/March/April 1994.

[2] J.H. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE Journal on Selected Areas of Communication*, vol. 2, pp. 528-539, July 1984.

[3] S.J. Grant and J.K. Cavers, "Performance enhancement through joint detection of cochannel signals using diversity arrays," *IEEE Transactions on Communications*, vol. 46., pp. 1038-1049, August 1998.

[4] S.J. Grant and J.K. Cavers, "Further results on the joint detection of cochannel signals using diversity arrays," *submitted to IEEE Transactions on Communications*, March 1999.

[5] S.J. Grant and J.K. Cavers, "Analytical calculation of outage probability for a general cellular mobile radio system," *in Proc. IEEE VTC'99-Fall*, Amsterdam, The Netherlands, Sept. 19-22, 1999.

[6] Y.Yeh and S.C. Schwartz, "Outage probability in mobile telephony due to multiple log-normal interferers," *IEEE Transactions on Communications*, vol. 32, pp. 380-388, April 1984.

[7] K.S. Gilhousen et al., "On the capacity of a cellular CDMA system," *IEEE Transactions on Vehicular Technology*, vol. 40, pp. 303-312, May 1991.

[8] V.H. MacDonald, "The Cellular Concept," *Bell Systems Technical Journal*, vol. 58, pp. 15-41, January 1979.