# A Configuration Space View of View Planning

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*Abstract*— **For sensor-based robot motion planning, view planning problem refers to planning the next sensing action to further facilitate the motion planning task. In [24] C-space entropy was introduced as a measure of knowledge of robot configuration space, or C-space. The robot plans the next sensing action to maximally reduce the expected C-space entropy, also called the Maximal expected Entropy Reduction, or MER criterion. It was shown that MER criterion resulted in much more efficient C-space exploration performance than physical space based view planning criteria, such as to maximize unknown physical volume in each view. From a C-space perspective, MER criterion consists of two important aspects: sensing actions are evaluated in C-space (geometric aspect); these effects are evaluated in an information theoretical sense (stochastic aspect). In this paper, we investigate how much of this better performance is attributable to the paradigmatic shift to evaluating the sensor action in C-space, i.e., the pure geometric component of MER, and how much is attributable to the stochastic aspect of MER. We propose C-space based pure geometric criteria (which are essentially geometric aspect of MER) for view planning and compare them with the MER criterion. We empirically show that a great deal of efficiency is attributable to the pure geometric aspect; however, we also show that the stochastic aspect, despite being based on simple assumptions, result in moderately more efficient C-space exploration over the pure geometric component of MER. We outline explanations for our findings.**

## I. Introduction

In this paper, we consider the sensor-based motion planning and exploration problem for general robot-sensor systems, where a range sensor is mounted on a robot with non-trivial geometry and kinematics [1], [10], [12], [19], [21], [23], [24]. Fig. 1 shows an eye-in-hand system, a two link robot equipped with a triangle field of view (FOV) range sensor on its end-effector. The white region in the figure is the free part of the physical space known to the robot; the light grey region is free but still unknown; the dark grey regions are unknown obstacles. The robot starts from its initial configuration, the vertical line in the middle of the figure, and its task is to explore its environment while avoiding collisions with the obstacles, known or unknown. A key sub-problem here is view planning [23], i.e. where the robot should sense next, and good view planning strategies can result in efficient exploration performance [19], [21], [24].

Unlike simple mobile robots, [2], [5], [7], [11], [17], [18], (often modelled as a point, hence trivial geometry and kinematics [7], [11]), in which case where the sensor senses (the physical space) and the natural space for motion planning (the configuration space) are the same, for general robot-sensor systems (where the robot has non-trivial geometry and kinematics, such as the eye-in-hand system considered here), where the robot can move (path planning) and what it should sense (view planning), has a much more complex relationship [23]. In this case, the robot must find additional physical space to manoeuvre itself, taking into account its own "size" and "shape" and not simply any additional physical space. This implies that the effect of the sensing action, which obviously senses physical space, must be (implicitly or explicitly) transformed to and viewed from the configuration space (C-space) of the robot.

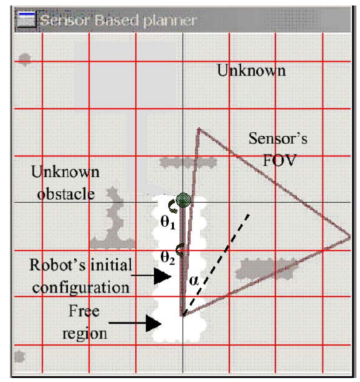



Fig. 1. Example of an eye-in-hand system: a planar 2-link robot with a triangle FOV range sensor.

Treating the unknown environment stochastically, the notion of C-space entropy was introduced as a measure of the robot's (lack of) knowledge of C-space [24]. The next best view is then the one that results in maximal (expected) entropy reduction (MER criterion) or, equivalently maximal (expected) information gain. In contrast, earlier approaches had simply used pure physical space based criteria, such as to maximize unknown physical space volume (MPV) in the sensor's field of view (FOV) [10]. A detailed presentation of other related work on view planning is given in [24]. For instance, entropy measures are used for physical space and object model respectively in [13], [16], [18], [22] for designing autonomous environment-exploration/object-model-construction strategies. See [14] for a survey on view planning. See also [3] on sensor-based path planning for a planar robot robot.

The MER based criteria have two important aspects: (1) the effects of sensing actions are evaluated in C-space (geometric aspect), (2) these effects are measured in an information theoretic sense, e.g., using entropy (stochastic aspect). The information theoretic perspective assumes a knowledge of the obstacles' distribution (in a stochastic sense) in the environment, e.g., in [19], [21], [24] a Poisson point process model of the obstacle distribution is assumed. The closed form expressions for maximal (expected) entropy reduction were derived for a planned view. These results, when implemented in a 2D simulator and on a six-dof eye-in-hand system, were shown to lead to more efficient C-space exploration (number of views needed to make a certain percentage of C-space known) than physical space based view planning criteria, such as MPV [10]. Please note that MPV and MER criteria do have complementary aspects and that the two can be combined to yield better exploration in both physical and configuration space [8]. Our main focus here is C-space exploration.

It is then valid to ask how much of this better performance is attributable to the paradigmatic shift to evaluating the sensor action in C-space, i.e., the pure geometric component of MER, and how much is attributable to the stochastic aspect of MER. In this paper, we formulate (indeed these criteria naturally suggest themselves, given the closed form expressions for MER with the Poisson point process assumption for obstacle distribution) C-space based pure geometric criteria (essentially aspect (1) of MER) for view planning and empirically (via 2-dof planar simulations) compare it with MPV and MER criterion for efficiency in C-space exploration. The MPV criterion performs the worst[1]; the proposed C-space based pure geometric criteria are significantly more efficient than MPV, showing that a great deal of efficiency is attributable to evaluating the sensor action in C-space, i.e., pure geometric component of MER. However, MER criterion, despite simplifying probabilistic assumptions, does result in moderately more efficient C-space exploration than pure C-space based geometric criteria. Intuitively, MER, notwithstanding the underlying simple stochastic models, such as Poisson point process, provides a means of addressing sensing actions' partial effects on a configuration, i.e., one configuration is "more (or less) known" than another configuration, while pure geometric criteria do not account for such partial effects, i.e., they are a "binary" version of MER criterion.

In the rest of this paper, first we will briefly recapitulate C-space entropy and MER criterion based on generic FOV sensor model; then new pure geometric criteria in C-space are formulated for view planning; the C-space exploration efficiency of these criteria and MER criterion are compared empirically.

## II. Background: C-space Entropy and MER Criterion

### A. Notation

Let $\mathcal{P}$ denote the physical space. The robot is denoted by $\mathcal{A}$ and its configuration space (C-space) by $\mathcal{C}$. $q \in \mathcal{C}$ denotes a robot configuration, and $\mathcal{A}(q) \in \mathcal{P}$ denotes the physical space occupied by the robot at $q$. Subscripts $free$ and $obs$ denote the free space and obstacles in both physical and C-space. For example, $\mathcal{P}_{free}$ denotes the entire physical free space and $\mathcal{P} = \mathcal{P}_{free} \cup \mathcal{P}_{obs}$ . Subscripts $u$ and $known$ denote the unknown and known quantities (in physical and C-space) and superscript $i$ denotes the iteration number, i.e., the number of scans (or views) that the view planning algorithm has already taken ($i = 0$, at the very start). Let $\mathcal{P}^i_{free}$, $\mathcal{P}^i_{obs}$, and $\mathcal{P}^i_u$ denote the known free physical space, known physical obstacles, and unknown physical space after iteration $i$. (There is a slight abuse of notation here for simplicity; we should really be using subscripts "known-free" and "known-obs", however, it makes them too long.) Hence $\mathcal{P}^i_{known} = \{\mathcal{P}^i_{free} \cup \mathcal{P}^i_{obs}\}$, and $\mathcal{P} = \{\mathcal{P}^i_{known} \cup \mathcal{P}^i_u\}$. Furthermore, $\mathcal{A}^i_u(q)$ denotes the part of the robot (at configuration $q$) lying in the unknown part of the environment at iteration $i$, i.e., $\mathcal{A}^i_u(q) = \mathcal{A}(q) \cap \mathcal{P}^i_u$. Similarly $\mathcal{A}^i_{known}(q) = \mathcal{A}(q) \cap \mathcal{P}^i_{known}$.

$\mathcal{C}^i_{free}$, $\mathcal{C}^i_{obs}$, and $\mathcal{C}^i_u$, respectively denote the known free C-space, known C-obstacles, and unknown C-space, after iteration $i$. A configuration $q$ is free, i.e., $q \in \mathcal{C}^i_{free}$ if $\mathcal{A}^i_u(q) = \emptyset \ \wedge \ \mathcal{A}(q) \cap \mathcal{P}^i_{obs} = \emptyset$. A configuration $q$ is obstacle, i.e., $q \in \mathcal{C}^i_{obs}$ if $\mathcal{A}(q) \cap \mathcal{P}^i_{obs} \neq \emptyset$. A configuration $q$ is unknown, i.e., $q \in \mathcal{C}^i_u$ if $\mathcal{A}^i_u(q) \neq \emptyset \ \wedge \ \mathcal{A}(q) \cap \mathcal{P}^i_{obs} = \emptyset$.

We attach a coordinate frame to the sensor's origin. Let $s$ denote the vector of parameters that completely determine the sensor's frame, i.e., sensor's configuration. For instance, assuming the sensor is attached to the end-effector of the robot, for planar case, $s = (x, y, \theta)$; for 3D case, $s = (x, y, z, \alpha, \beta, \gamma)$. Let $\mathcal{V}(s) \in \mathcal{P}$ denote the sensor's field of view (FOV) at configuration $s$. Finally, we use "\" to denote the set difference operation, i.e. for two sets $\mathcal{M}$ and $\mathcal{N}$, $\mathcal{M} \setminus \mathcal{N} = \{x | x \in \mathcal{M} \ \wedge \ x \notin \mathcal{N}\}$.

### B. C-space entropy and MER criterion

Assume a stochastic model of the obstacle distribution in the physical space. This assumption in turn induces a probability distribution on C-space, i.e., every configuration has a certain probability of being free according to the status of the physical region the robot occupies at this configuration. The notion of C-space entropy was introduced as an ignorance measure of C-space [24]. Mathematically, the C-space could be viewed as a collection of $n$ random variables (r.v.), $Q_j, j = 1, \ldots, n$, representing the status of each discretized (or randomly sampled for high dimensional cases) robot configuration $q_j$, being free ($Q_j = 0$) or in collision ($Q_j = 1$). The entropy of this joint distribution is called C-space entropy, $H(\mathcal{C})$. It should really be denoted by $H(\mathcal{C}|\mathcal{P}^i_{known})$, the entropy conditional on current known physical space. For notational brevity and since it is obvious that probabilities and hence entropy computations should be conditional on current state, in the following we will neglect this condition in the notations. We have [4],

$$H(\mathcal{C}) = - \sum_{Q_1=0,1} \ldots \sum_{Q_n=0,1} \Pr[Q_1 \ldots Q_n] \log \Pr[Q_1 \ldots Q_n] \quad (1)$$

Let $ER_{\mathcal{C}}(s)$ denote the *expected* C-space entropy reduction. MER criterion states that the next best sensing action is the one to maximize $ER_{\mathcal{C}}(s)$, i.e.,

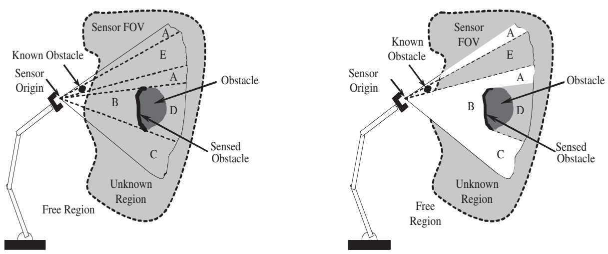
$$s^{i+1}_{max} = \arg\max_s ER_{\mathcal{C}}(s) = \arg\max_s E\{H(\mathcal{C}) - H(\mathcal{C}|\mathcal{V}(s))\} \quad (2)$$

The expectation computation above is carried over all possible sensing results.

[1] Our main aim is to compare the geometric C-space criteria with MER; we have included MPV as a baseline here. MPV criterion was already shown to be significantly worse than MER for C-space exploration [19], [21], [24].

### C. MER for generic FOV sensor model

In [19], [21], a closed form expression for MER criterion was given based on a generic non-zero volume FOV sensor model as shown in Fig. 2. Most commercially available range sensors that provides range images, e.g., the area scan laser ranger finder used in SFU Eye-in-Hand system [23], fall into this category. We recapitulate these results as follows.




(i) Before scan (ii) After scan.

Fig. 2. A generic range sensor's FOV $\mathcal{V}(s)$. After this sensing action, regions A, B and C are known free, the black contour is a sensed obstacle and region D, occluded by the sensed obstacle remains unknown. Region E also remains unknown due to occlusion by an already known obstacle.

A simple probabilistic model was used for the obstacle distribution in the physical space — the Poisson point process, which is essentially characterized as uniformly independently distributed point obstacles in space [15]. Indeed the Poisson point process model is a simplifying assumption because obstacles, in general, are not points in the real environments. Nevertheless, it matches well our rather reasonable intuition that the more the robot (at a given configuration) is in unknown region, the less is the chance that it would be collision free. It is an unbiased uniform distribution assuming no shape information of the obstacles, and hence reasonable when no *a priori* information is known about the environment. Above all, Poisson point model allows one to derive efficiently computed closed-form expression for expected C-space entropy reduction, which gives insights into the MER criterion. Please see Section II-D for further discussions on how Poisson point assumption combines geometric and stochastic aspects of MER in an elegant manner. The resulting algorithms, as shown via simulations, drastically improve the efficiency of C-space exploration when compared to pure physical space based criteria, such as MPV.

Other more complex models do not lend themselves to such closed form expressions and therefore would tremendously increase computational cost of entropy computations. For example, if we were to use occupancy grid maps [6] for the physical space, the expectation computation in Eq. (2) is to be carried out over all possible combinations of the grid statuses, thus having an exponential (in the number of unknown cells in the sensor FOV) computational complexity. Existing exploration approaches (for exploring physical space) for mobile robots ignore this complexity and do not compute the true expectation. For example, some assume that the unknown area to be sensed is completely free, i.e., only one sensing result is possible [5], [18]. This is clearly an oversimplifying assumption. Others use an ad-hocly defined information function [16].

Further ignoring mutual entropy terms for efficiency in computations [2], $ER_{\mathcal{C}}(s)$, can be approximated by the sum of the expected entropy reduction of each unknown configuration $q$, the marginal expected entropy reduction $er_q(s)$, i.e.,

$$ER_{\mathcal{C}}(s) \approx \widetilde{ER}(s) = \sum_{q \in \chi_u(s)} er_q(s) \quad (3)$$

In the above equation, $\chi_u(s)$, the unknown C-zone of $s$, is defined as the set of unknown configurations at which part of the robot can be sensed. Further by defining $\mathcal{V}_u(s)$ as the portion of the sensor FOV, $\mathcal{V}(s)$, that intersects $\mathcal{P}_u^i$ and is not occluded by known obstacles, we have $\chi_u(s) = \{q \in \mathcal{C}_u^i | \mathcal{A}_u^i(q) \cap \mathcal{V}_u(s) \neq \phi\}$.

The marginal (expected) entropy reduction of $q$, $er_q(s)$, is given by

$$\begin{aligned} er_q(s) &= E\{H(Q) - H(Q|\mathcal{V}(s))\} \\ &= H(Q) - e^{-\lambda|\mathcal{A}(q)\cap\mathcal{V}_u(s)|} \cdot H(Q \mid \mathcal{A}(q) \cap \mathcal{V}_u(s) \text{ free}) \end{aligned} \quad (4)$$

where $H(Q)$ is the entropy of $q$ before sensing, i.e., $H(Q) = -p(q)\log p(q) - (1-p(q))\log(1-p(q))$ in which $p(q)$, the void probability of $q$, is defined as the probability of $q$ being not in collision with obstacle. By Poisson point assumption, we have $p(q) = e^{-\lambda|\mathcal{A}_u^i(q)|}$, where $\lambda$ is the density parameter. In the implementation, one can simply assume a value for $\lambda$. (See [20] for discussions on $\lambda$'s effects on planning results.) The conditional in Eq. (4) refers to the event that the part of the robot at $q$ inside the unknown part of the sensor FOV is free of obstacle, and the void probability (the probability of being collision free) of $q$ conditional on this event is given by:

$$p(q \mid \mathcal{A}(q) \cap \mathcal{V}_u(s) \text{ free}) = e^{-\lambda \cdot |\mathcal{A}_u^i(q) \setminus \mathcal{V}_u(s)|}$$

$H(Q \mid \mathcal{A}(q) \cap \mathcal{V}_u(s) \text{ free})$ is then simply computed from $p(q \mid \mathcal{A}(q) \cap \mathcal{V}_u(s) \text{ free})$ in the above formulation.

MER criterion then gives the next sensing action $s$ to maximize $ER(s)$, i.e.,

$$s_{max} = \arg\max_s \widetilde{ER}(s) = \arg\max_s \sum_{q \in \chi_u(s)} er_q(s) \quad (5)$$

This is pictorially illustrated by Fig. 3. A sensing action $s$ reduces the expected entropy associated with each unknown configuration $q$, and the sum of these reductions (recall that we ignore mutual entropy terms) approximates the whole C-space entropy reduction. In view planning, we choose a sensing action $s$ that induces the maximal value. The corresponding view planning algorithm is as follows:

*Algorithm 1: MER Criterion*
*for every s* /* *according to a certain resolution* */
*determine* $\mathcal{V}_u(s)$
$\widetilde{ER}(s) = 0$ /* *initialize* */
*for every q*
*if* ($\mathcal{A}_{known}^i(q) \cap \mathcal{P}_{obs}^i = \phi \wedge \mathcal{A}_u^i(q) \cap \mathcal{V}_u(s) \neq \phi$)
*compute* $er_q(s)$ *using Eq. (4)*
$\widetilde{ER}(s) = \widetilde{ER}(s) + er_q(s)$
$s_{max} = \arg\max_s \widetilde{ER}(s)$

[2] Ignoring mutual entropy regards the statuses of two configurations independent of each other.

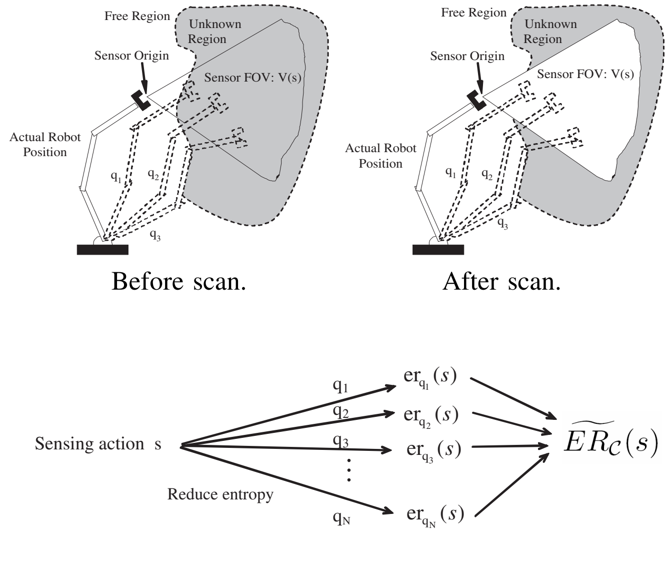



Fig. 3. A sensing action at configuration $s$. Assuming that the sensed region becomes free, the knowledge of the three different robot configurations shown are changed. $q_1$ is known free while $q_2$ and $q_2$ are still unknown. The entropy reduction (ignoring mutual entropy terms) provides a (expected) measure of this knowledge gained for each $q$. These marginal terms, $er_{q_j}(s)$, are totally determined by geometries of robot volume and sensor FOV, under Poisson point assumption. The expected C-space entropy reduction is simply a sum of these marginal terms.

Determining quantities such as $\mathcal{A}_u^i(q)$, $\mathcal{V}_u(s)$ involves straightforward geometrical computations. For instance, determining $\mathcal{V}_u(s)$ corresponds to determining the intersection of the sensor FOV with $\mathcal{P}_u$ while excluding portions of $\mathcal{P}_u$ occluded by already known obstacles (before sensing action), a relatively simple geometric computation. In case of complex shaped robot and environments, this region can be approximated by discretized representation of the environment, e.g., discretized grids used in the simulations described later. $er_q(s)$ for given $s$ and $q$ is therefore easily computed. The iteration over $q$, i.e., summation over C-space of the robot to determine $ER(s)$, may be prohibitive for robots with many degree of freedoms. In this case, the summation can be carried out over a large enough set of random samples [24]. The iteration over $s$, i.e., maximization over the sensor configuration space to determine $s_{max}$ will be directly proportional to the number of discretized sensing configurations.

### D. Discussion of MER results

Eqs. (3), (4) clearly bring out the geometric and stochastic aspects, mentioned earlier in the introduction, of MER criterion. The effect of each sensing action $s$ is essentially a weighted summation over a C-space region, the unknown C-zone of $s$, $\chi_u(s)$. The C-zone is totally determined by the geometry and kinematics of the robot and its environment (see $\chi_u(s)$ definition above). The weight for each configuration $q \in \chi_u(s)$ is governed by the stochastic aspect, in fact it is precisely the marginal entropy reduction. It essentially depends on (i) the current knowledge about the configuration, embodied in $H(Q)$, which in turn depends on $|\mathcal{A}_u(q)|$ under Poisson point assumption via $p(q)$, the void probability of configuration $q$, and (ii) the expected knowledge about the configuration after sensing, embodied in $H(Q|\mathcal{V}(s))$, which depends on $E\{\mathcal{A}_u(q)|\mathcal{V}(s)\}$.

One can show some qualitative correspondences to our intuition. For example, it can be shown [20] that if we fix the known volume $A_{known}^i(q)$, the higher is the portion of unknown region, occupied by the robot at configuration q, that can be potentially sensed, $\mathcal{A}(q) \cap \mathcal{V}_u(s)$, the higher is the weight for that configuration. On the other hand, if we fix the volume of $\mathcal{A}(q) \cap \mathcal{V}_u(s)$, those configurations at which the robot has smaller unknown region, $\mathcal{A}_u(q)$, and hence a better chance of becoming fully known, would have higher weights: these configurations would have a better chance of becoming fully known, thus expanding the $C_{known}$. For example, in the top left figure of Fig. 3 (before scan), assuming the volumes of $\mathcal{A}_u(q_j) \cap \mathcal{V}_u(s), j = 1, 2, 3$ are the same, $q_1$ would have the highest weight followed by $q_2$ and then $q_3$, since $q_1$ has the smallest unknown volume $\mathcal{A}_u(q_1)$, followed by $q_2$ and then $q_3$. This was shown in [24] for the simplified case of a point FOV sensor and was called "boundary property", i.e., configurations close to the boundary of $\mathcal{C}_{free}^i$ and $\mathcal{C}_u^i$ have a higher weight. We now show it for the general FOV case. Thus MER criterion can take into account the cumulative build up of knowledge through partial sensing actions, i.e., sensing actions that may not make a robot configuration $q$ completely known (free or obstacle), yet they do provide additional knowledge about $q$.

## III. Geometric Criteria in C-space for View Planning

### A. Maximal C-zone Volume criterion

Analogous to the MPV criterion which maximizes the volume of (unknown) physical space region within each view, we propose to maximize the volume of unknown C-zone of the physical space within each view, i.e., maximize $|\chi_u(s)|$, the volume of the C-space region affected by the view. We call this "Maximal C-zone Volume" criterion, or MCZV in short.

$$s_{max} = \arg\max_s |\chi_u(s)| = \arg\max_s \sum_{q \in \chi_u(s)} 1 \tag{6}$$

Compared with the MER criterion, Eq. (5), MCZV weighs the effect of the sensing action $s$ on all configurations $q \in \chi_u(s)$ equally. The current knowledge of the configuration $q$, i.e., how much is known about it, does not matter at all! Effectively the MCZV criterion is a special (unweighted) case of the MER criterion. The corresponding algorithm is as follows.

*Algorithm 2: MCZV Criterion*
*for every s* /* *according to a certain resolution* */
  *determine* $\mathcal{V}_u(s)$
  $CzoneV(s) = 0$ /* *initialize* */
  *for every q* /* *according to a certain resolution* */
    *if* ($\mathcal{A}_{known}^i(q) \cap \mathcal{P}_{obs} = \phi \wedge \mathcal{A}_u^i(q) \cap \mathcal{V}_u(s) \neq \phi$)
      $CzoneV(s) = CzoneV(s) + 1$
$s_{max} = \arg\max_s CzoneV(s)$

In the above algorithm, since the two iteration terms are the same as in Algorithm 1, the computational time of this algorithm is in the same order as the one based on MER criterion.

An interesting alternative explanation of MCZV criterion is as follows. If we assume that the region to be sensed is completely occupied by an obstacle (or a portion of an obstacle), then the entire unknown C-zone will become a C-obstacle! In other words, MCZV criterion could also be interpreted as the one that maximizes the C-obstacle, if the entire view were covered by an obstacle.

### *B. Maximal C-free Volume criterion*

Motivated by the above alternative interpretation, one could propose another complementary geometric criterion. Suppose we assume that the region to be sensed is completely free (of course, taking into account the visibility constraints w.r.t already known obstacles), i.e. $\mathcal{V}_u(s)$ is assumed free. Furthermore taking a greedy approach, we only consider the effect of a sensing action $s$ on those configurations, whose collision status becomes known free after sensing, one could choose the sensing action that results in the maximal volume of additional known free C-space. We call this "Maximal C-free Volume" criterion, or MCFV in short. Formally, MCFV is given by,

$$s_{max} = \arg\max_{s} \sum_{q \in \chi_u(s)} \delta(\mathcal{A}_u^i(q) \subseteq \mathcal{V}_u(s)) \qquad (7)$$

where $\delta(e)$ is a Boolean function defined on $e$: $\delta(e) = 1$, if $e$ is true; $\delta(e) = 0$, otherwise.

The corresponding algorithm is as follows,

*Algorithm 3: MCFV Criterion*
*for every s*    */* according to a certain resolution */*
    *determine* $\mathcal{V}_u(s)$
    $CFV(s) = 0$    */* initialize */*
    *for every q*    */* according to a certain resolution */*
        *if* $(\mathcal{A}_{known}^i(q) \cap \mathcal{P}_{obs}^i = \phi \wedge \mathcal{A}_u^i(q) \subseteq \mathcal{V}_u(s))$
            $CFV(s) = CFV(s) + 1$
$s_{max} = \arg\max_{s} CFV(s)$

Again the computational complexity of this algorithm is in the same order as MER and MCZV because of the same iteration terms.

Note that it could be thought of as a "binary" version of the MER criterion, i.e., only those unknown configurations that would become free as a result of the free sensing are counted. Thus it neglects the sensing actions' partial effect on still unknown configurations, i.e., more of the region occupied by a configuration will become known, thereby reducing its uncertainty. Intuitively, this is somewhat "greedy" in that it neglects the cumulative effect of sensing, and would likely be inefficient for cases where one single scan cannot adequately cover entire unknown parts of robot at relevant unknown configurations as we shall see in the simulation results.

One could also formulate more elaborate versions of MCFV and MCZV such as "Maximal Expected Cfree(Czone) Volume". These can be thought of weighted summation over unknown C-zone where weights are the probabilities of each configuration being sensed free/obsacle. Again, Poisson point process can be used for these probability computations.

## IV. Simulation Results for Exploration Efficiency Comparison

To compare the geometric criteria, MCZV and MCFV, with the MER criterion, we conducted a series of experiments on the simulated two-link eye-in-hand system shown in Figure 1. It consists of a 2 dof planar robot and a range sensor (triangle FOV) mounted on its end-effector. The sensor has an additional dof that rotates 360 degree around the wrist. The sensing angle (the angle between the two edge of the sensing triangle) is 60 degree. The task for the robot is to explore its environment, starting from pointing vertically downwards in its initial configuration.

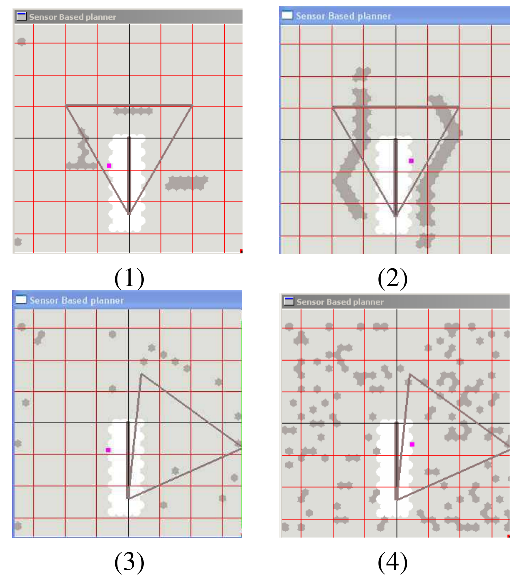



Fig. 4. The four different physical spaces used in the simulation: (1,2) structured environment, (3,4) random generated environment.

We use four different physical spaces as shown in Fig. 4 in the simulation: (1) and (2) are two structured environments; (3) and (4) are two unstructured environments where obstacles are randomly generated, with Poisson distribution with density parameters 0.02 and 0.2 respectively.

The sensor's configuration $s$ is the robot's configuration $q = (\alpha, \beta)$ plus an additional degree of freedom at the wrist, denoted by $\theta$, i.e., $s = (q, \theta) = (\alpha, \beta, \theta)$. ($\theta$ is the angle between the last link of the robot and the medial axis of the sensor FOV.) In the simulation, the data structures for the physical space and the robot is similar to [1].[3] The simulation program, written in C++, runs on a Pentium III 800.

The overall sensor-based MP planner used is SBIC-PRM (sensor-based incremental construction of probabilistic roadmap) reported in [23]. It consists of an incrementalized model-based PRM [9], that operates in the currently known environment; and a view planner that decides a reachable configuration within the currently known environment from which to take the next view. The two sub-planners operate in an interleaved manner, i.e., the robot uses the C-space roadmap to move to the planned view configuration, takes a scan from there, and updates the physical space and the C-space roadmap. The updated physical space and the roadmap

[3]Briefly speaking, the physical space is represented by well structured ($30 * 30$ grids in this simulation) overlapping cells of radius $r$ and the robot is represented by a collection of cells of radius $r/2$. While doing collision detection, all the physical space cells that have intersection with robot cells are checked to determine the status of this robot configuration.

are then used to plan the next view, and the iteration repeats.

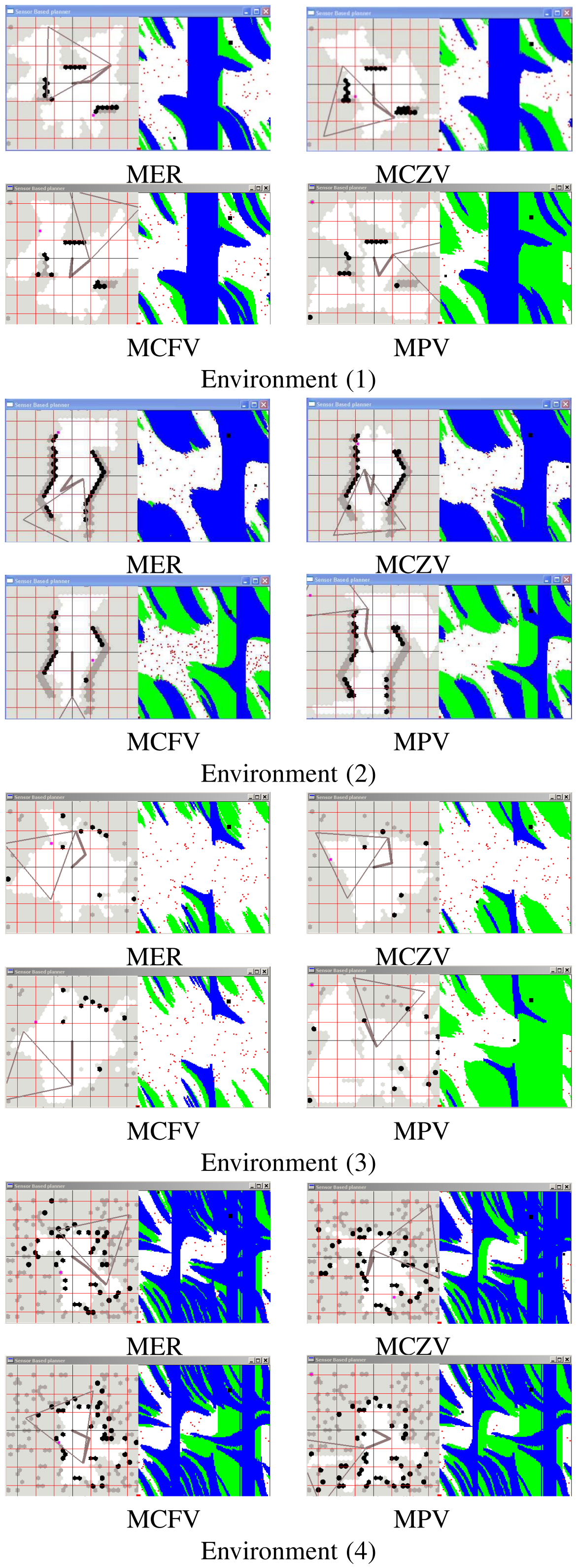



Fig. 5. Exploration in environments (1), (2), (3) and (4). The snapshots show the known physical space and C-space after 7 scans. The planned view for the 8th scan (the triangle) is also shown.

Fig. 5 shows snapshots of the simulation results after seven scans and the planned eighth scan for the MER criterion, MCZV criterion and MCFV criterion, respectively for environment (1), (2), (3) and (4). The view planning results for MPV criterion are also shown as a baseline comparison with the other three C-space based criteria. The left sub-image in each snapshot shows the physical space, and the right sub-image shows the C-space. In both physical and C-space, "grey" (or "green" in colored version), "white", and "dark" (or "blue" in colored version) regions denote unknown part $\mathcal{P}_u^i$ or $\mathcal{C}_u^i$, free part $\mathcal{P}_{free}^i$ or $\mathcal{C}_{free}^i$, and obstacles $\mathcal{P}_{obs}^i$ or $\mathcal{C}_{obs}^i$, respectively. (The small dots in right subimages are the nodes of the PRM used to plan paths in free C-space, $\mathcal{P}_{free}^i$.) For MER criterion, we assumed $\lambda$, the density parameter of the underlying Poisson point process, to be 0.5. Please note that, empirically, the view planning results are not sensitive to the choice of $\lambda$ value. See [20] for detailed discussions.

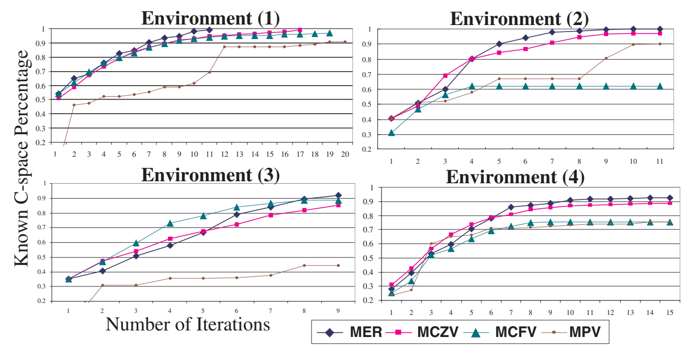



Fig. 6. Comparison of C-space exploration efficiency in environments (1), (2), (3) and (4) for the three view planning criteria: MER, MCFV and MCZV. The data shown are average performance for ten tests conducted.

Fig. 6 shows C-space exploration rates (average over ten runs[4]) for the same three criteria, with MPV criterion as the baseline. We can see that the C-space based geometric criteria (MCZV and MCFV) significantly outperform the physical space based MPV criterion. Of the two C-space based geometric criteria, MCZV is better. The MER criterion further moderately outperforms the C-space based geometric criteria. Specifically, for environment (1), MER criterion uses 10 scans to explore 98% of C-space while MCZV and MCFV perform comparably and use 16 scans and 20 scans respectively, MPV uses $> 30$ scans[5]; for environment (2), to expand 98% of C-space, the numbers of iterations by MER, MCZV, MCFV and MPV are 7, 13, $> 30$ and $> 30$; for environment (3), to expand 98% of C-space, the numbers of iterations by MER, MCZV, MCFV and MPV are 10, 14, 15 and $> 30$; and for environment (4), to explore 88% of C-space, the number of scans are 9, 13, $> 30$ and $> 30$ respectively.

Note that for environments (2) and (4), MCFV criterion did not perform well. For environment (2), MCFV performed even worse than MPV. Intuitively, these environments are cluttered with obstacles and each sensing action has small scannable region due to occlusions. This implies that many single scans may not (in general) cover the entire unknown volume of robot configurations. MCFV criterion ignores such scans and hence misses out on their potential effects.

[4]The standard deviations of these trials for different criteria are all sufficiently small (less than five percent of the average).

[5]In all the runs, MCFV failed to expand 88% of C-space within 30 iterations.

MER criterion evaluates the sensing actions' (expected) effects on both free and obstacle parts of $\mathcal{C}_u$ and thus provides a nice tradeoff between $\mathcal{C}_{free}$ and $\mathcal{C}_{obs}$ exploration. Moreover, MER also takes into consideration the sensing actions' "partial" effects on configurations that will not result in knowing the exact status of the configurations, but nevertheless reduce the uncertainties. MER criterion therefore is able to account for "cumulative" effects of sensor actions. This is intuitively why MER criterion moderately outperforms the C-space based geometric criteria. Recent preliminary experimental results (obtained as this paper goes to print and therefore not included here) on the SFU (six-dof) Eye-in-Hand system, the system hardware same as in [21], show that the MER criterion performs significantly better than pure geometric criteria. This suggests that the stochastic aspect of the MER criterion may play a significant role in more complex environment (3D) and for high dof robots.

The average view planning time and average running time per iteration are roughly the same for the above three algorithms: 51 and 54 seconds respectively. Note that the view planning time only counts for the view planner to plan the next sensing action, while the running time per iteration refers to the time for executing the whole sensor-based planner [23], which includes not only the view planning time but time for sensor scanning, known physical space and roadmap update, roadmap searching, and the robot movements (this additional time is roughly invariant with respect to the view planning criteria).

## V. Conclusion

The Maximal C-space Entropy Reduction (MER) criterion for view panning was shown to significantly outperform the pure physical space based view planning criteria in our earlier papers [19], [21], [23], [24]. The criterion consists of two important aspects, namely a geometric aspect in C-space, and a stochastic aspect based on stochastic assumption of the physical space. In this paper, we investigated how much of this better performance is attributable to the paradigmatic shift to evaluating the sensor action in C-space, i.e., the pure geometric component of MER, and how much is attributable to the stochastic aspect of MER. We formulated C-space based pure geometric criteria (essentially aspect (1) of MER criterion) for view planning and compared it with MPV and MER criterion. We showed that a great deal of efficiency is attributable to evaluating the sensor action in C-space, i.e., to the pure geometric component of MER criterion. In particular, MCZV performance is close to that of MER. MCFV, on the other hand, works well only in relatively uncluttered spaces. However, we also show that MER criterion, despite simplifying probabilistic assumptions, does result in moderately more efficient C-space exploration than the pure C-space based geometric criterion MCZV. Intuitively, MER provides a means of addressing sensing actions' partial effects on a robot configuration, i.e., one configuration is "more (or less) known" than another configuration, while pure geometric criteria do not account for such partial effects and are essentially an unweighted (in case of MCZV) or binary weighted (in case of MCFV) versions of the MER criterion. This suggests that using more sophisticated (and complex) stochastic models than the simple Poisson point process model of obstacle distribution would result in even better performance for MER criterion. A big problem, however, is that the resulting computational cost would also increase, since closed form expressions for such complex models are unlikely and brute force numerical computations of quantities such as expected entropy reduction would be prohibitively expensive.

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