Computing C-space Entropy for View Planning Based on Beam Sensor Model

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Abstract

The concept of C-space entropy was recently introduced in [1, 2, 3], as a measure of knowledge of C-space for sensor-based path planning and exploration for general robot-sensor systems. The robot plans the next sensing action to maximally reduce the expected C-space entropy, also called the maximal expected entropy reduction, or MER criterion. The expected C-space entropy computation, however, made two idealized assumptions. The first was that the sensor field of view (FOV) is a point; and the second was that no visibility (or occlusion) constraints are taken into account, i.e., as if the obstacles are transparent. We extend the expected C-space entropy formulation where the sensor FOV is a beam and furthermore, it is subject to visibility constraints, as is the case with real range sensors. Planar simulations show that this new formulation results in more efficient exploration.

1 Introduction

While most research in sensor-based path planning and exploration has concerned itself with mobile robots, our recent work has concentrated on general robot-sensor systems, where the sensor is mounted on a robot with non-trivial geometry and kinematics [1, 2, 3, 4]. See also [5, 6, 7, 8, 9]. This class of robots is broad and includes robots ranging from a simple polygonal robot to articulated arms, mobile-manipulator systems, and humanoid robots [10]. Figure 1 shows a simple example of such a robot-sensor system — an eye-in-hand system — an articulated arm with a wrist mounted range sensor. The robot must simultaneously plan paths and sense its environment for obstacles. A key problem in such sensor-based planning is therefore view-planning, i.e., where should the robot sense next? Efficient sensing strategies can drastically reduce the time used for a robot to achieve a desired task. In the general robot-sensor case, unlike for a simple mobile robot (modelled as a point), where the robot can move and what it should sense, has a much more complex relationship. "Where to move" is best posed and answered in configuration space, the natural space for path planning, whereas sensor senses in the physical space. In [4] we proposed an incremental framework, which consists of a model-based planner that plans paths within the currently known environment[1] and a view planner that plans the next sensing action (view) to explore. The two planners are interleaved. Subsequently, in [1, 2], we showed that for general robot-sensor systems, the view planning problem is appropriately posed in the configuration space of the robot — the next view should be planned to give maximum "knowledge" or "information" of the C-space of the robot. Treating the unknown environment stochastically, we introduced the notion of C-space entropy. The next best view is then the one that maximizes the expected entropy reduction (MER criterion) or, equivalently expected information gain. We derived

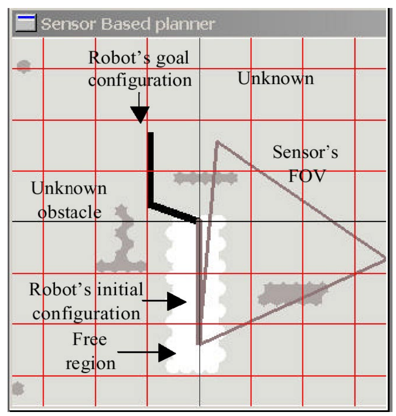

Figure 1: *An eye-in-hand system — a two-link robot with a wrist mounted range sensor (with triangle FOV) moving in an unknown environment. A key question (the view planning problem) is: where should the robot sense next?*

closed form expressions for expected C-space entropy reduction, or information gain under a Poisson model of the environment. However, two idealized assumptions were made in that paper: (i) the sensor has a point field of view (FOV), i.e., it senses a single point and (ii) no visibility constraints were taken into account, i.e., as if the sensor would "see" (get range measurement) through the obstacles. The next best view is planned using this formulation, i.e., the algorithm computes the point (say, x_{max}) which, if sensed, would yield maximum expected information gain and places the sensor so that the center of the actual FOV (a cone) coincides with x_{max}.

In this paper, we relax the above two assumptions and present the C-space entropy computation for a beam sensor while respecting visibility constraints, thereby modelling a real range sensor. This computation is valid for a Poisson model of the environment [11], admittedly a simplification, but the resulting closed form expressions give us insights and are useful at least as approximations. We present simulations that show clear improvement in the efficiency of exploration with the new formulation. Our initial

[1]This ensures that at each iteration the robot will always move within the currently known environment, and its motions are guaranteed to be collision-free.

simulations are planar for ease of visualization. We emphasize that our formulations and results are valid for 3D environments and are currently being implemented on a real six-dof eye-in-hand system consisting of a PUMA 560 with a wrist mounted area-scan laser range sensor that has been developed in our lab and was reported in [4].

2 Notation

Let $\mathcal{A}$ denote the robot and q denote a point in its configuration space, $\mathcal{C}$. $\mathcal{A}(q)$ then denotes the region in physical space, $\mathcal{P}$, occupied by the robot. Let $\mathcal{S}$ denote a sensor attached to the robot. We attach a coordinate frame to the sensor's origin. Let s denote the vector of parameters that completely determine the sensor frame, i.e., sensor's configuration. For instance, assuming the sensor is attached to the end-effector of the robot, for planar case, $s = (x, y, \theta)$; for 3D case, $s = (x, y, z, \alpha, \beta, \gamma)$. Let $\mathcal{V}(s) \in \mathcal{P}$ denote the region sensed (sensor FOV) by the sensor at configuration s. Subscripts $free$, obs, [2] and unk (or sometimes u) denote the known free, known obstacle and unknown regions, respectively in physical and configuration space. So, for example, $\mathcal{P}_{obs}$ denotes the known obstacles in physical space, $\mathcal{A}_{unk}(q)$ denotes the part of robot lying in unknown physical space at configuration q, and $\mathcal{C}_{free}$ denotes the known free configuration space.

3 Background:C-space Entropy and IGDF

We assume that the obstacles' distribution in the physical environment is modelled with an underlying stochastic process (e.g., the Poisson model used later). The kinematics and geometry of the robot, embodied by function $\mathcal{A}(q)$ map the probability distribution in physical space to a probability distribution over the C-space. Shannon's Entropy then provides a measure of the robot's ignorance of the status of C-space, [2]. For a point FOV sensor model, which only senses a point (or an infinitesimal ball) in physical space, one can then compute the expected entropy reduction (or, equivalently, expected information gain) per unit volume if a point $x \in \mathcal{P}$ was sensed (obstacle/free). The information gain density (IGD) function captures this notion and is defined as

$$IGD_{\mathcal{C}}(x) = \lim_{vol(\mathcal{B}(x))\to 0} \frac{-E\{\underset{\mathcal{B}(x)}{\triangle} H(\mathcal{C})\}}{vol(\mathcal{B}(x))}$$

where $H(\mathcal{C})$ denotes the current C-space entropy, $E\{\underset{\mathcal{B}(x)}{\triangle} H(\mathcal{C})\} = E\{H(\mathcal{C}|\mathcal{B}(x))\} - H(\mathcal{C})$ denotes the expected entropy change after $\mathcal{B}(x)$, a ball centered at point x, is sensed.

In order to get efficiency in computing, we neglect the mutual entropy terms, essentially treating each configuration as an independent random variable, i.e., $\widetilde{H}(\mathcal{C}) = \sum_{q_i \in \mathcal{C}} H(Q_i)$. In this equation, Q_i denotes the binary random variable (r.v.) corresponding to configuration q_i being free (=0) or in collision (=1); $H(Q_i)$ denotes the entropy of r.v. Q_i, i.e.,

$$H(Q_i) = p(q_i)\log(p(q_i)) + (1 - p(q_i))\log(1 - p(q_i)) \quad (1)$$

where $p(q_i) = \Pr[q_i = free]$ is the marginal probability that configuration q_i is collision-free, also called the void probability of q_i. With this simplification one can show that:

$$\widetilde{IGD}_{\mathcal{C}}(x) = \lim_{vol(\mathcal{B}(x))\to 0} \frac{-E\{\underset{\mathcal{B}(x)}{\triangle} \widetilde{H}(\mathcal{C})\}}{vol(\mathcal{B}(x))} = \sum_{q\in\mathcal{C}} igd_q(x)$$

where $igd_q(x)$ is given by:

$$igd_q(x) = \lim_{vol(\mathcal{B}(x))\to 0} \frac{-E\{\underset{\mathcal{B}(x)}{\triangle} H(Q)\}}{vol(\mathcal{B}(x))} \quad (2)$$

When $\mathcal{B}(x)$ is sensed, the sensed information affects the C-space entropy via each configuration q. $igd_q(x)$ is then the marginal contribution to information gain density via configuration q, if a point $x \in \mathcal{P}$ were to be sensed. Furthermore, $igd_q(x)$ equals 0 when $\mathcal{A}(q)$ does not contain x. So we need only compute the above summation over those q's such that $x \in \mathcal{A}_{unk}(q)$, also called the C-zone of x [2, 3], and denoted by $\chi(x)$.[3] Therefore one can write:

$$\widetilde{IGD}_{\mathcal{C}}(x) = \sum_{q\in\chi(x)} igd_q(x)$$

For the more general case, when a region $\mathcal{V}(s)$ is being sensed, the IGD function is now defined over the space of all sensor configurations. For each sensor configuration, s, it assigns a real value that corresponds to the expected information gain per unit volume of sensed region with the sensor placed at configuration s. Using the same simplifications as above (i.e., ignoring mutual entropy terms),

$$\widetilde{IGD}_{\mathcal{C}}(s) = \lim_{vol(\mathcal{V}(s))\to 0} \frac{E\{\underset{s}{\triangle}\widetilde{H}(\mathcal{C})\}}{vol(\mathcal{V}(s))} = \sum_{q\in\mathcal{C}} igd_q(s)$$

where one can write:

$$igd_q(s) = \lim_{vol(\mathcal{V}(s))\to 0} \frac{-E\{\underset{s}{\triangle} H(Q)}{vol(\mathcal{V}(s))} \quad (3)$$

As before, $igd_q(s)$ is the marginal contribution to IGD via configuration q when the sensor senses at configuration s. Similar to the point FOV sensor, the contribution of those configurations such that $\mathcal{V}(s) \cap \mathcal{A}(q) \neq \phi$ will contribute zero, hence the summation can be restricted to the C-zone of $\mathcal{V}(s)$, denoted by $\chi(s)$, and defined as the set of q's such that $\mathcal{A}(q) \cap \mathcal{V}(s) \neq \phi$. [4] Therefore, we can also write:

$$\widetilde{IGD}_{\mathcal{C}}(s) = \sum_{q\in\chi(s)} igd_q(s)$$

4 Beam Sensor Model

The beam sensor, as the name implies, senses along a beam (ray) of finite length, L, emanating from the sensor origin (See Figure 2). It returns the distance of the first hit point (obstacle) along the beam. Points along the beam that are in front of the hit point (i.e., closer to the origin than the hit point) are in free space. Points along the beam behind the hit point (i.e., farther from origin than the hit point) are deemed to be un-sensible (by occlusion constraints) in this particular sensing action and their status (obstacle/free/unknown) remains the same. Therefore, for a particular sensing action, a point along the beam

[2] Ideally, these subscripts should be *known-free* and *known-obs*. But we omit *known* for brevity.

[3] Intuitively, C-zone of x is the set of configurations such that the robot when placed in such a configuration contains point x. One could think of this as a generalization of inverse kinematics which applies only to the end-effector rather than the entire robot body.

[4] This extends the notion of C-zone to that of a set in physical space rather than a single point x.

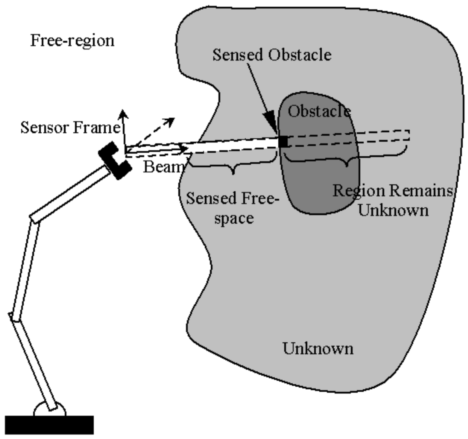

Figure 2: *The beam sensor model.*

may acquire one of three possible states: 0 (free), 1 (obstacle) or u (unsensible).

For mathematical formulation, we will assume that the sensor FOV, $\mathcal{V}(s)$, is a thin cylinder of infinitesimal radius (or equivalently, infinitesimal cross sectional area denoted by $\triangle a$) and length L. We discretize this cylinder into n "disks", each of length $\triangle l$, by planes orthogonal to its axis, as shown in Figure 3. As $\triangle a$ and $\triangle l$ approach zero, the cylinder becomes an ideal beam.

Let $\mathcal{V}_u(s)$ denote the portion of sensed region that lies inside $\mathcal{P}_{unk}$ and is *in front of the first known obstacle* along the sensing direction, i.e., $\mathcal{V}_u(s)$ denotes the largest possible sensing region the beam sensor can sense at s. $\mathcal{V}_u(s) \approx x_1 \cup x_2 \cup \ldots \cup x_m$ where $x_i, i = 1, \ldots, m$ are the disks lying inside $\mathcal{V}_u(s)$ (we are concerned with only the unknown portion of sensor FOV and label only those disks) in front of the first known obstacle. Note that $\mathcal{V}_u(s)$ may be a multiply connected set. If a disk (say, x_i) contains the hit point, its status would become 1 (obstacle). All disks x_j, $j < i$, would become 0 (free), and all disks x_k, $k > i$ would keep their status u (unknown) as shown in Figure 3. So we will get a "$0, 0, \ldots, 0, 1, u, u, \ldots, u$" sequence.

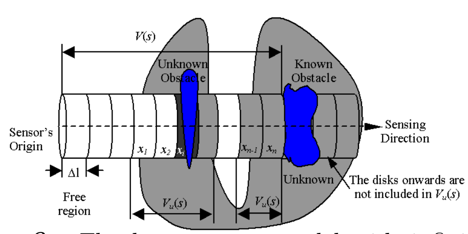

Figure 3: *The beam sensor model with infinitesimal width and discretized into "disks". The hit point lies in disk x_i with $i = 3$. Disks x_1 and x_2 become free and disks x_4 onwards remain unknown.*

4.1 Environment Model

We use a simple probabilistic model of physical space — the Poisson point process, essentially characterized by uniformly distributed points in space [11]. From the motion planning point of view, these points are obstacles in the physical space of the robot. Given the density parameter of this model, λ, the void probability of an arbitrary set $\mathcal{B} \in \mathcal{P}$ — the probability that there is no point (obstacle) in $\mathcal{B}$ — denoted by $p(\mathcal{B})$, is given by

$$p(\mathcal{B}) = \Pr[\mathcal{B} \subseteq \mathcal{P}_{free}] = e^{-\lambda \cdot vol(\mathcal{B})} \tag{4}$$

This implies that $p(q)$, the void probability of configuration q is given by

$$p(q) = \Pr[\mathcal{A}(q) \subseteq P_{free}] = e^{-\lambda \cdot vol(\mathcal{A}_{unk}(q))} \tag{5}$$

The location of the hit point is a random variable. The event that the i^{th} disk contains the hit point, denoted by $x_i = h$, corresponds to first $(i-1)$ disks, $x_1, x_2, \ldots, x_{i-1}$, being all in $\mathcal{P}_{free}$ and the i^{th} disk, x_i, containing an obstacle point, i.e., $\{x_j = 0, j = 1, \ldots, i-1 \wedge x_i = 1\}$ where $i \in \{1, \ldots, m\}$. The corresponding probability, denoted by $p(x_i = h)$ is then given by using Eq. (4):

$$p(x_i = h) = e^{-\lambda \cdot (i-1) \cdot \triangle a \cdot \triangle l} \cdot (1 - e^{-\lambda \cdot \triangle a \cdot \triangle l}) \tag{6}$$

with $1 \leq i \leq m$, and $\triangle a \cdot \triangle l$ being the volume of each disk.

4.2 Compute $igd_q(s)$

From the definition of $igd_q(s)$ in Eq. (3), we have:

$$igd_q(s) = \lim_{\substack{\triangle a \to 0 \\ \triangle l \to 0}} \frac{-\underset{s}{E}\{\triangle H(Q)\}}{L \cdot \triangle a} \quad \text{and}$$

$$\underset{s}{E}\triangle H(Q) = \{\sum_{i=1}^{m} p(x_i{=}h) \underset{x_i=h}{\triangle} H(Q)\} + \{p(x_1 \ldots x_m{=}0) \underset{x_1 \ldots x_m=0}{\triangle} H(Q)\} \tag{7}$$

The r.h.s. in the above expression consists of two terms: the first one (the summation) corresponds to any of the x_i's being the hit point; and the second one corresponds to there being no hit point in the sensed beam. As before, $\underset{x_i=h}{\triangle} H(Q)$ denotes the change in entropy if x_i was the hit point and it is defined as:

$$\underset{x_i=h}{\triangle} H(Q) = H(Q \mid x_i = h) - H(Q)$$

where

$$H(Q \mid x_i = h) = p(q \mid x_i = h) \cdot \log(p(q \mid x_i = h)) + (1 - p(q \mid x_i = h)) \cdot \log(1 - p(q \mid x_i = h))$$

and

$$p(q \mid x_i = h) = \Pr[q = free \mid x_i = h]$$

Let $\mathcal{V}_u(s \mid x_i = h)$ denote the status of sensed region assuming $x_i = h$. Now we will compute $\underset{s}{E}\{\triangle H(Q)\}$.

See Figure 4. Suppose for some q, $\mathcal{A}(q)$ intersects cylindrical $\mathcal{V}_u(s)$ and the intersection with $\mathcal{V}_u(s)$'s axis is denoted by $\mathcal{I}$. $\mathcal{I}$ is actually a set of intervals $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_t$. Let $\bar{\mathcal{I}}$ denote the complement of $\mathcal{I}$ in the unknown part of the cylinder's axis. Let $num(\mathcal{I}_j)$ represents the number of disks inside $\mathcal{I}_j$. So $num(\mathcal{I}) \approx \lfloor \frac{\mathcal{I}_i}{\triangle l} \rfloor$. As $\triangle l \to 0$, $num(\mathcal{I}_i) \cdot \triangle l \to len(\mathcal{I}_i)$.

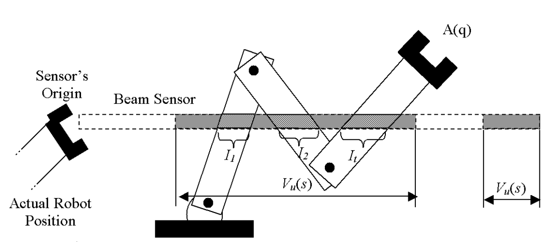

Figure 4: *Intersection of sensed unknown volume $\mathcal{V}_u(s)$ with the space $\mathcal{A}(q)$ occupied by robot, if it were at configuration q, is given by intervals $\mathcal{I}_j$, $1 \leq j \leq t$.*

We can further divide the summation term in the expression for $\underset{s}{E}\{\triangle H(Q)\}$ in Eq. (7) into two components. The first one is when the disk containing the hit-point, x_i, were to lie inside $\mathcal{A}(q)$, i.e., it were to lie within one of the intervals $\mathcal{I}_j$, $1 \leq j \leq t$, in set $\mathcal{I}$. We denote this component by $E\{\underset{x_i \in \mathcal{A}(q)}{\triangle} H\}$ (or simply

$E\{\triangle H_1\}$), and corresponding igd_q as $(igd_q)_1$. The second one is when x_i were not to lie within $\mathcal{A}(q)$, i.e., it were to lie in $\bar{\mathcal{I}}$. We denote this component by $E\{\underset{x_i \notin \mathcal{A}(q)}{\triangle} H\}$ (or simply $E\{\triangle H_2\}$), and the corresponding igd_q as $(igd_q)_2$. Let $E\{\triangle H_3\}$ denote the term (second term in r.h.s of Eq. (7)) if there were no sensed hit point, i.e., all the disks were free, corresponding igd_q being $(igd_q)_3$.

So we have:

$$igd_q = (igd_q)_1 + (igd_q)_2 + (igd_q)_3$$

or, equivalently,

$$\begin{aligned} & E\{\triangle H\} = E\{\triangle H_1\} + E\{\triangle H_2\} + E\{\triangle H_3\} \\ = \; & E\{\underset{x_i \in \mathcal{A}(q)}{\triangle} H\} + E\{\underset{x_i \notin \mathcal{A}(q)}{\triangle} H\} \\ + \; & p(x_1 = x_2 = \ldots x_m = 0) \cdot \underset{x_1 = x_2 = \ldots x_m = 0}{\triangle} H \end{aligned}$$

Computing the 1st Component ($E\{\triangle H_1\}$). When x_i (remember x_i is the hit-point) lies inside $\mathcal{A}(q)$, $\mathcal{A}(q)$ is in collision with obstacles. So $p(q \mid x_i = h) = 0$ and hence $H(Q \mid x_i = h) = 0$, thereby $\underset{x_i = h}{\triangle} H(Q) = -H(Q)$. Hence, $E\{\triangle H_1\} = -\sum_{x_i \in \mathcal{A}(q)} p(x_i = h) \cdot H(Q)$. Therefore,

$$(igd_q)_1 = \lim_{\substack{\triangle a \to 0 \\ \triangle l \to 0}} \frac{-E\{\triangle H_1(Q)\}}{L \cdot \triangle a} = \lim_{\triangle l \to 0} \frac{H(Q)}{L} \sum_{x_i \in \mathcal{I}} \lim_{\triangle a \to 0} \frac{p(x_i = h)}{\triangle a} \quad (8)$$

Let us take a look at $\lim_{\triangle a \to 0} \frac{p(x_i = h)}{\triangle a}$, the probability per unit volume that the hit point is x_i as the sensing cylinder approaches zero radius, i.e. the idealized beam. Using Eq. (6), and with simple algebra, we can show that

$$\lim_{\triangle a \to 0} \frac{p(x_i = h)}{\triangle a} = \lim_{\triangle a \to 0} \frac{e^{-\lambda(i-1)\triangle a \triangle l}(1 - e^{-\lambda \triangle a \triangle l})}{\triangle a} = \lambda \triangle l \quad (9)$$

Substituting in Eq. (8),

$$(igd_q)_1 = \lim_{\substack{\triangle a \to 0 \\ \triangle l \to 0}} \frac{-E\{\triangle H_1\}}{L \cdot \triangle a} = [num(\mathcal{I}_1) + \ldots + num(\mathcal{I}_t)] \cdot \frac{\lambda \cdot \triangle l}{L} \cdot H(Q)$$

As $\triangle l \to 0$, $num(\mathcal{I}_i) \cdot \triangle l \to len(\mathcal{I}_i)$. So we have,

$$(igd_q)_1 = \frac{len(\mathcal{I}_1) + \ldots + len(\mathcal{I}_t)}{L} \lambda H(Q) = \frac{\lambda}{L} len(\mathcal{A}(q) \cap \mathcal{V}_u(s)) H(Q) \quad (10)$$

The expression makes intuitive sense. Since it is the expected contribution from those cases where a sensed hit point lies inside $\mathcal{A}(q)$, we know that each such event would reduce the entropy to zero since the status of Q would become known (in collision) and hence the entropy reduction will be $-H(Q)$, the entropy before sensing. The multiplying factor $len(A(q) \cap \mathcal{V}_u(s)) \cdot \frac{\lambda}{L}$ simply represents the expectation (per unit length) of such an event happening under Poisson model.

Computing the 2nd Component ($E\{\triangle H_2\}$). When x_i lies outside $\mathcal{A}(q)$, $\mathcal{A}(q)$ is not in collision with sensed obstacle (the hit point). $(igd_q)_2$ is then

$$(igd_q)_2 = \lim_{\substack{\triangle a \to 0 \\ \triangle l \to 0}} \frac{-E\{\triangle H_2\}}{L \cdot \triangle a} = \lim_{\triangle l \to 0} \frac{-1}{L} \sum_{x_i \notin \mathcal{A}(q)} \lim_{\triangle a \to 0} p(x_i = h) \lim_{\triangle a \to 0} \frac{\underset{x_i = h}{\triangle} H}{\triangle a}$$

Let us look at $\lim_{\triangle a \to 0} \frac{\triangle H}{\triangle a}$. $H(Q)$ is a continuous function of $p(q)$. Now from Eq. (5), $p(q) = e^{-\lambda \cdot vol(\mathcal{A}_{unk}(q))}$. $vol(\mathcal{A}_{unk}(q))$ changes continuously with $\triangle a$ since hit point does not belong to $\mathcal{A}(q)$ and $vol(\mathcal{A}_{unk}(q))$ will increase by the additional free space sensed in $\mathcal{V}(s)$, a continuous function of $\triangle a$. Hence $p(q)$ is a continuous function of $\triangle a$ almost everywhere. Furthermore, as $\triangle a \to 0$, $vol(\mathcal{V}(s)) \to 0$, i.e., $vol(\mathcal{A}_{unk}(q))$ changes infinitesimally. Therefore, via the chain rule, we have

$$\lim_{\triangle a \to 0} \frac{\underset{x_i = h}{\triangle} H}{\triangle a} = \lim_{\triangle a \to 0} \frac{\triangle H}{\triangle p} \lim_{\triangle a \to 0} \frac{\triangle p}{\triangle vol(\mathcal{A}_{unk}(q))} \lim_{\triangle a \to 0} \frac{\triangle vol(\mathcal{A}_{unk}(q))}{\triangle a}$$

Now the first term $\lim_{\triangle a \to 0} \frac{\triangle H}{\triangle p}$ is simply the derivative of $H(Q)$ w.r.t. p and differentiating Eq. (1), we have $\frac{\triangle H}{\triangle p} = \log \frac{1 - p(q)}{p(q)}$. The second term is simply derivative of p w.r.t. $vol(\mathcal{A}_{unk}(q))$, and from Eq. (4) we have $\frac{\triangle p}{\triangle vol(\mathcal{A}_{unk}(q))} = -\lambda \cdot p$. Since $vol(\mathcal{A}_{unk}(q))$ decreases after the sensing, the negative of the third term is less than or equal to the length of the sensing cylinder, i.e., -$\frac{\Delta vol(\mathcal{A}_{unk}(q))}{\Delta a} \leq L$. Substituting these, we will have

$$\lim_{\triangle a \to 0} \frac{\underset{x_i = h}{\triangle} H}{\triangle a} \leq \lambda \cdot p(q) \cdot \log \frac{1 - p(q)}{p(q)} \cdot L$$

Therefore,

$$(igd_q)_2 \leq -\lambda p(q) \log \frac{1 - (q)}{p(q)} \lim_{\triangle l \to 0} \sum_{x_i \notin \mathcal{A}(q)} \lim_{\triangle a \to 0} p(x_i = h) \quad (11)$$

The set $\{x_i \notin \mathcal{A}(q)\}$ is a subset of $\{x_i \in \mathcal{V}(s)\}$. Therefore, for the summation in Eq. (11) we have,

$$\lim_{\triangle l \to 0} \sum_{x_i \notin \mathcal{A}(q)} \lim_{\triangle a \to 0} p(x_i = h) \leq \lim_{\triangle l \to 0} \sum_{x_i \in \mathcal{V}(s)} \lim_{\triangle a \to 0} p(x_i = h)$$

Using Eq. (9) with only first order expansion, we can easily show that

$$\lim_{\triangle l \to 0} \sum_{x_i \in \mathcal{V}(s)} \lim_{\triangle a \to 0} p(x_i = h) \leq \lim_{\triangle a \to 0} \lambda \cdot L \cdot \Delta a = 0$$

Hence,

$$(igd_q)_2 = 0 \quad (12)$$

This implies that those cases where the hit point is sensed but does not lie within a given $\mathcal{A}(q)$, the marginal information gain density (contribution due to configuration q) is zero. We can think of $(igd_q)_2$ as essentially a "derivative" of entropy w.r.t. volume for the case when the hit point would not lie inside the robot if the robot were at configuration q, weighted by the corresponding probability of such an event happening. The derivative is finite and the probability of the event happening (a hit point lying outside the robot but within the sensing beam) is very low and approaches zero under Poisson model as Δa approaches zero . The $(igd_q)_2$ being the product of the two, is therefore zero.

Computing the 3rd Component ($E\{\triangle H_3\}$). When there is no sensed hit point in the entire beam, we have:

$$\begin{aligned} (igd_q)_3 &= \lim_{\substack{\triangle a \to 0 \\ \triangle l \to 0}} \frac{-E\{\triangle H_3(Q)\}}{L \cdot \triangle a} \\ &= \lim_{\triangle l \to 0} \frac{-1}{L} \lim_{\triangle a \to 0} \frac{p(x_1 \ldots x_m = 0) \cdot \underset{x_1 \ldots x_m = 0}{\triangle} H}{\triangle a} \\ &= \lim_{\triangle l \to 0} \frac{-1}{L} \lim_{\triangle a \to 0} p(x_1 \ldots x_m = 0) \lim_{\triangle a \to 0} \frac{\Delta H}{\triangle a} \quad (13) \end{aligned}$$

Using Eq. (4) for the void probability, we have,

$$\lim_{\triangle a \to 0} p(x_1 = \ldots = x_m = 0) = \lim_{\triangle a \to 0} e^{-\lambda \cdot m \cdot \triangle a \cdot \triangle l} = 1$$

Using the same techniques as in computing $(igd_q)_2$,

$$\lim_{\triangle a \to 0} \frac{\underset{x_i = h}{\triangle} H}{\triangle a} = \lim_{\triangle a \to 0} \frac{\Delta H}{\Delta p} \lim_{\triangle a \to 0} \frac{\triangle p}{\triangle vol(\mathcal{A}_{unk}(q))} \lim_{\triangle a \to 0} \frac{\triangle vol(\mathcal{A}_{unk}(q))}{\triangle a}$$

The first two product terms are the same as in previous case. The change in volume, $\Delta vol(\mathcal{A}_{unk}(q)) = -(num(\mathcal{I}_1) + \ldots + num(\mathcal{I}_t)) \cdot \Delta l \cdot \Delta a$. Therefore,

$$\lim_{\triangle a \to 0} \frac{\underset{x_1 \ldots x_m = 0}{\triangle H}}{\triangle a} = \lambda p(q) \log \frac{1 - p(q)}{p(q)} (num(\mathcal{I}_1) + \ldots + num(\mathcal{I}_t)) \cdot \triangle l$$

Substituting these in Eq. (13), we get

$$(igd_q)_3 = - \lim_{\triangle l \to 0} \frac{\lambda}{L} (num(\mathcal{I}_1) + \ldots + num(\mathcal{I}_t)) \triangle l \cdot p(q) \log \frac{1 - p(q)}{p(q)}$$

$$= \quad -\frac{\lambda}{L} (len(\mathcal{I}_1) + \ldots + len(\mathcal{I}_t)) p(q) \frac{1 - p(q)}{p(q)}$$

An alternative form is

$$(igd_q)_3 = -\frac{\lambda}{L} \cdot len(\mathcal{A}(q) \cap \mathcal{V}_u(s)) \cdot p(q) \cdot \frac{dH(Q)}{dp} \qquad (14)$$

As for $(igd_q)_2$, $(igd_q)_3$ is essentially the "derivative" of entropy w.r.t. volume for the case when the sensor does not sense any hit point, weighted by the expectation of the corresponding event. The derivative is again finite, however, the probability of this event happening (no hit point) is very high and approaches unity as Δa approaches zero under Poisson model. The $(igd_q)_3$ being the product of the two, is therefore finite.

Now, we have computed all the three components of $E\{\Delta H\}$. We can easily get $igd_q(s)$ from Eq. (10), (12) and (14),

$$igd_q = \frac{\lambda}{L} \cdot len(\mathcal{A}(q) \cap \mathcal{V}_u(s)) \cdot (H(Q) - p \cdot \frac{dH(Q)}{dp})$$

$$= \quad -\frac{\lambda}{L} \cdot len(\mathcal{A}(q) \cap \mathcal{V}_u(s)) \cdot \log(1 - p)$$

And finally,

$$\widetilde{IGD}_c(s) = -\frac{\lambda}{L} \cdot \sum_{q \in \mathcal{X}(s)} len(\mathcal{A}(q) \cap \mathcal{V}_u(s)) \cdot \log(1 - p)$$

We can easily see that this result also applies to the point sensor model in the limit, i.e., as L, the length of the sensing beam, goes to zero. In this case, we have

$$\lim_{L \to 0} \frac{\mathcal{A}(q) \cap \mathcal{V}_u(s)}{L} = 1$$

Therefore, $\widetilde{IGD}_q(s) = \lambda \cdot (H(Q) - p \cdot \frac{dH(Q)}{dp})$, precisely the result we obtained in [1, 2, 3][5].

5 Algorithm for View Planning

Now that we have computed an expression for the IGD over sensor's configuration space, we can use the MER criterion to decide the next scan, which is essentially to take the next scan from that sensor configuration which maximizes the information gain, i.e., to choose the sensor configuration s_{max} such that

$$s_{max} = \max_s \{ - \sum_{q \in \mathcal{X}(s)} len(\mathcal{A}(q) \cap \mathcal{V}_u(s)) \cdot \log(1 - p) \}$$

The algorithm then is as follows:

for every s /* *according to a certain resolution* */
  determine $\mathcal{V}_u(s)$
  $\widetilde{IGD}(s) = 0$ /* *initialize* */
  for every q
    if ($\mathcal{A}(q)$ *overlaps with* $\mathcal{V}_u(s)$)
      compute $p(q)$
      compute $l(q) = len(\mathcal{V}_u(s) \cap \mathcal{A}(q))$
      compute $igd_q(s) = -\frac{\lambda}{L} \cdot l(q) \cdot \log(1 - p)$
    $\widetilde{IGD}(s) = \widetilde{IGD}(s) + igd_q(s)$
$s_{max} = \max(\widetilde{IGD}(s))$

[5] Please note that there is an algebraic error in earlier papers for the point FOV case. This is the corrected result.

Determining $\mathcal{V}_u(s)$ corresponds to determining the intersection of the beam with $\mathcal{P}_{unk}$ in front of known obstacles, a simple geometric computation. Note that iteration over q (C-space of the robot) may be prohibitive for robots with many degree of freedoms. In this case, the summation can be carried out over a large enough set of random samples.

6 Simulation Results

In order to test the effectiveness of our formulae, we conducted a series of experiments on the simulated two-link eye-in-hand preliminary system shown in Figure 1. The task for the robot is to move to the given goal configuration from the given start configuration. The overall planner used is SBIC-PRM (sensor-based incremental construction of probabilistic road map) reported in [3, 4]. Briefly, SBIC-PRM consists of an incrementalized model-based PRM, [12], that operates in the currently known environment; and a view planner that decides a reachable configuration within the currently known environment from which to take the next view. The two sub-planners operate in an interleaved manner.

We compare the results of four different view planning criteria for efficiency of exploration of the physical and configuration space. The first strategy, denoted by RV (random views), is to randomly choose a viewpoint as the next scan, and place the centre of the FOV (the cone) at that point. The second strategy, denoted by MPV (maximum unknown physical volume) is to choose the next viewpoint so as to maximize the unknown physical volume inside the scan [5]. The third strategy is to use point FOV based MER criterion for view-planning [1, 2], and place the centre of the actual FOV (the cone) at x_{max}, the viewpoint that results in maximum entropy reduction. The fourth is to use the beam FOV based MER criterion derived in this paper, and place the central axis of the actual FOV (the cone) at s_{max}, the sensor configuraion that results in maximum entropy reduction. In all these cases, the robot started off as in Figure 1.

As shown in the Figures 5, 6, 7, and 8, the first two strategies expand the known C-space much less than the last two MER criterion based strategy. Using RV gives us about 8% expansion of known C-space in 5 scans, and the robot reached its goal in 36 scans. MPV results in C-space expansion by about 54% in 5 scans, and the robot reached goal in 20 scans[6]. The point FOV based MER criterion gives us much better results, resulting in about 73% expansion in 5 scans and the robot reached the goal in 14 scans. The beam FOV based MER criterion was the best, better than point FOV based MER. It made the C-space expand by about 76 % in 5 scans and the robot reached the goal in 11 scans.

Figure 9 plots C-space v.s. number of iterations for above four view-planning algorithms. We can easily see that beam FOV based MER is the most efficient one, which expanded C-space to about 90% in 8 scans; point FOV based MER needed 11 scans; RV needed 33 scans; and MPV needed 19 scans respectively.

[6] See [3] for an explanation of unexpected high inefficiency of this strategy.

7 Conclusions

We presented closed form solutions for computing the expected C-space entropy reduction for a beam FOV range sensor. These results are extensions of our previous results that applied to a point FOV sensor and take into account the visibility constraints inherent in range sensors. Planar simulations show that our new results lead to more efficient exploration of the robot configuration space. Our next step is to implement these results for a real six-dof eye-in-hand system, a PUMA 560 with a wrist mounted area scan laser range finder. We will also extend these results further to that of a finite volume sensor (like an area scan range sensor), thus closely modelling a real range sensor providing range images.

The current formulation assumes a Poisson point process for obstacle distribution. It treats obstacles as points. Extending our formulation for a Boolean stochastic model [11] where geometric shape of obstacles is taken into account would be the next logical, but perhaps analytically challenging step.

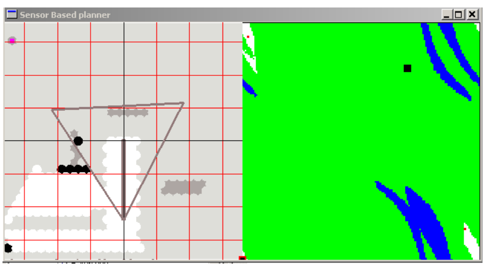

Figure 5: *C-space expansion after 5 scans: RV Strategy*

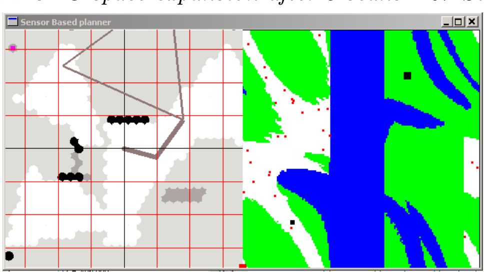

Figure 6: *C-space expansion after 5 scans: MPV Strategy*

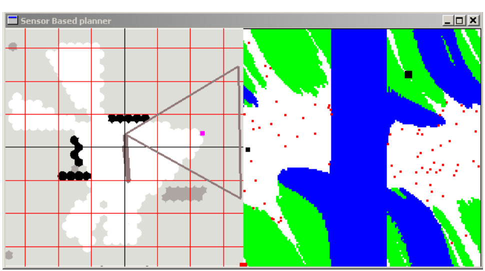

Figure 7: *C-space expansion after 5 scans: Point FOV Based MER Criterion*

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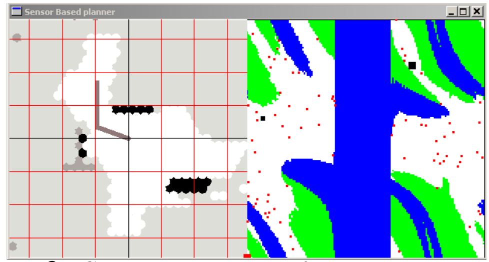

Figure 8: *C-space expansion after 5 scans: Beam FOV Based MER Criterion. The robot reached goal after 5 scans.*

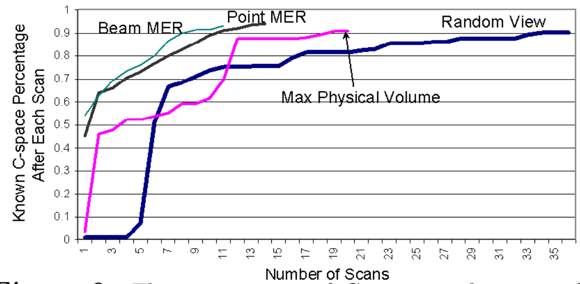

Figure 9: *The comparison of C-space exploration efficiency for the four view-planning algorithms: RV, MPV, Point FOV Based MER and Beam FOV Based MER*

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