Maximal Entropy Reduction Criterion for View Planning: Detailed Issues

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***Abstract*— In this report, we discuss several issues of Maximal expected Entropy Reduction (MER) criterion for generic range sensor based on a simple stochastic physical space model, the Poisson point process. First, we investigate using more realistic but complex stochastic models in the place of Poisson point process. We show that even with a simple extension, the computational cost for MER criterion is tremendously increased. This justifies the usage of Poisson point. It, albeit simple, matches our intuitions and improves tremendously the planning results over simple physical space criteria.**

We then show the monotonicity property of the marginal entropy reduction (of a single unknown robot configuration) under Poisson point model, i.e., the marginal entropy reduction is always monotone as the function of the volume of the known part of the robot. The monotonicity result also makes intuitive sense: the expected information gain is larger as the volume intersection, the volume of robot unknown part that lies in the sensor field of view (FOV), is bigger. We arrived the same results for MER criterion based on a point FOV sensor model.

We also investigate empirically the effects of (Poisson point) model parameter estimation on view planning qualities. The parameter estimation error is inevitable for unknown environment with imperfect statistical assumptions. We find that MER criterion based on generic range sensor models is relatively robust with respect to estimation errors. We also show, empirically, that the quality of estimations (of the density parameter of the Poisson point model) nevertheless improves the planning results marginally.

I. Background

In this section, we briefly recapitulate the notion of C-space entropy and MER criterion, and its application to view planning problems. See [4] for a detailed coverage.

A. C-space Entropy

Assume a stochastic geometric model for the physical space, i.e., a certain unknown region in the physical space takes a certain probability of being free, called void probability, according to this model. This in turn induces a stochastic model on the robot configuration space (C-space), i.e., each unknown robot configuration takes a certain void probability according to the void probability of the robot unknown part in the physical space at that configuration. The notion of C-space entropy was introduced as an ignorance measure of the C-space. Taking a discrete view, the C-space could be represented by a collection of n random variables (r.v.), $Q_1, \ldots, Q_i, \ldots, Q_n$, representing the status of each discretized (or randomly sampled for high dimensional cases) robot configuration q_i, being free ($Q_i = 0$) or in collision ($Q_i = 1$). The entropy of this joint distribution is called C-space entropy, $H(\mathcal{C})$[1], given by,

$$H(\mathcal{C}) = -\sum_{Q_1=0,1} \ldots \sum_{Q_n=0,1} \Pr[Q_1, \ldots, Q_n] \log \Pr[Q_1, \ldots, Q_n] \quad (1)$$

MER (Maximal expected Entropy Reduction) criterion states that the next best sensing action is the one to maximize the *expected* C-space entropy reduction, i.e.,

$$\begin{aligned} s_{max}^{i+1} &= \arg\max_s ER_c(s) \\ &= \arg\max_s E\{H(\mathcal{C}) - H(\mathcal{C}|\mathcal{V}(s))\} \end{aligned} \quad (2)$$

B. MER for generic FOV sensor FOV model

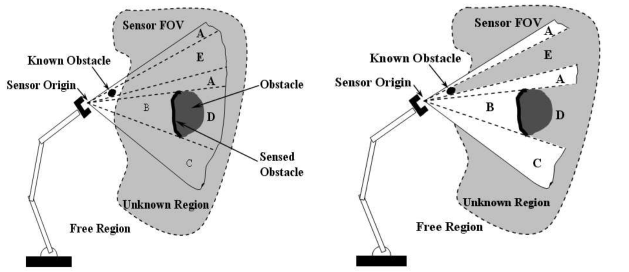
In [4], we derived closed form expressions for MER criterion based on generic non-zero volume FOV sensor model as shown in Fig. 1.

Ignoring mutual entropy terms for efficiency in computations [2], the expected C-space entropy reduction due to sensing action s, $ER_{\mathcal{C}}(s)$, can be approximated by the sum of the expected entropy reduction of each unknown configuration q, the marginal expected entropy reduction $er_q(s)$, i.e.,

$$ER_{\mathcal{C}}(s) \approx \widetilde{ER}(s) = \sum_{q \in \mathcal{C}_u^i} er_q(s) \quad (3)$$

[1] It should really be denoted by $H(\mathcal{C}|\mathcal{P}^i_{known})$, the entropy conditional on current known physical space. This however makes it too long. And since it is evident that probabilities and thence entropy computations should be conditional on current state, we will neglect this condition in the notations in the following discussion.

[2] Ignoring mutual entropy regards the statuses of two configurations independent of each other.



(i) Before scan (ii) After scan.

Fig. 1. Illustration of a generic range sensor's FOV $\mathcal{V}(s)$. After this sensing action, regions A, B and C are known free, the black contour is a sensed obstacle and region D, occluded by the sensed obstacle remains unknown. Region E also remains unknown, but it is occluded by an already known obstacle.

Further under Poisson point process assumption for obstacle distribution in physical space [3], in [4], we showed that for a given sensor configuration s, the marginal (expected) entropy reduction for an unknown configuration q is given by,

$$\begin{aligned} & er_q(s) = E\{H(Q) - H(Q|\mathcal{V}(s))\} \\ = \quad & H(Q) - e^{-\lambda \cdot |\mathcal{A}(q) \cap \mathcal{V}_u(s)|} \cdot H(Q \mid \text{event } 2') \end{aligned} \quad (4)$$

In the above equation, $H(Q)$ is the entropy of q before sensing, i.e., $H(Q) = -p(q) \log p(q) - (1 - p(q)) \log(1 - p(q))$ in which $p(q)$, the void probability of q, is defined as the probability of q being not in collision with obstacle. By Poisson point assumption, we have $p(q) = e^{-\lambda |\mathcal{A}_u^i(q)|}$. $\mathcal{V}_u(s)$ is defined as the portion of the sensor FOV, $\mathcal{V}(s)$, that intersects $\mathcal{P}_u^i$ and is not occluded by known obstacles. The conditional event $2'$ in Eq. (4) refers to the event that the part of the robot at q inside the unknown part of the sensor FOV is free of obstacle. The void probability (the probability of being collision free) of q conditional on this event is given by:

$$p(q \mid \mathcal{A}(q) \cap \mathcal{V}_u(s) \text{ free}) = e^{-\lambda \cdot |\mathcal{A}_u^i(q) \setminus \mathcal{V}_u(s)|}$$

$H(Q \mid \mathcal{A}(q) \cap \mathcal{V}_u(s) \text{ free})$ is then simply computed from $p(q \mid \mathcal{A}(q) \cap \mathcal{V}_u(s) \text{ free})$ in the above formulation.

In Eq. (4), if $\mathcal{A}(q)$ does not intersect $\mathcal{V}_u(s)$, (sensing action s does not cover any portion of the robot volume at q), marginal entropy reduction of q is zero. By defining the "unknown C-zone of s", $\chi_u(s)$, as the set of unknown configurations at which the robot *does* intersect $\mathcal{V}_u(s)$ [4], i.e., $\chi_u(s) = \{q \in \mathcal{C}_u^i | \mathcal{A}_u^i(q) \cap \mathcal{V}_u(s) \neq \phi\}$, we can rewrite Eq. (3) as,

$$\widetilde{ER}(s) = \sum_{q \in \chi_u(s)} er_q(s) \quad (5)$$

[3]Poisson point model is essentially characterized as uniformly distributed points in the physical space. Note that for sensor-based MP use, these points are regarded as obstacles.

[4]The unknown C-zone definition a physical space region can be thought of as a whole robot body inverse kinematics, i.e., a region in the physical space is related to regions in C-space. Thus, the physical space geometry is transformed to C-space geometry, which gives MER a C-space geometry flavor.

MER criterion then gives the next sensing action s to maximize the expected C-space entropy reduction, i.e.,

$$s_{max} = \arg\max_s \widetilde{ER}(s) = \arg\max_s \sum_{q \in \chi_u(s)} er_q(s) \quad (6)$$

II. Poisson Point Process Model

In this section, we investigate using more realistic but complex stochastic model for the physical space. We show that even with a simple extension to the Poisson point process, by adding a nonzero size to the cell, the computational cost can be increased to be in the order of the cell number exponential because of the lack of a closed-form solution. This justifies the Poisson point model in the MER computation.

A. Simple extension to Poisson point

As shown in a series of paper based on different sensor model [7], [8], [4], [5], albeit simple and unrealistic, Poisson point model, when applied to MER criterion, induces great improvements over simple physical space based view planning strategies, e.g., to maximize the unknown physical space volume inside the view (MPV). Here we investigate using some more realistic model and show the computational costs it introduces when being applied to MER criterion.

As a simple extension to the Poisson point process, here we consider the following stochastic model for the physical space[5]: the physical space is discretized into cells of a certain size, i.e., $\mathcal{P} = \{c_1, c_2, \ldots, c_j, \ldots, c_M\}$; and each cell c_j has a certain independent prior void probability, denoted by $p(c_j) = \Pr[c_j = 0]$. (Independence implies $p(c_{j_1}, c_{j_2}) = p(c_{j_1})p(c_{j_2})$.) For simplicity, we model the range sensor as an ideal one that returns, within its FOV, the distances (with respect to the sensor origin) of first sensed obstacles along the sensing direction, called hit points, and the space between senor origin and hit points is sensed free. Accordingly, the cell status, $p(c_j)$, changes after sensing: it becomes known-free ($p(c_j) = 1$), if its unknown part is sensed free; it becomes obstacle ($p(c_j) = 0$), if it has intersection with some sensed obstacles; its void probability is unchanged if neither of the above two cases happens.

In essence, this model is the same as occupancy grid map [2], with perfect (without uncertainties) sensing model for map update: the cell prior void is either unchanged, changed to 0 (free) or 1 (obstacle) after sensing.

The robot void probability at configuration q can be computed as the joint probability of its comprising cells, i.e.,

$$p(q) = \prod_{c_j \in \mathcal{A}(q)} p(c_j).$$

The product formula above is due to the independence of the cell probabilities.

[5]Another possibility would be to use Boolean model [3], which is characterized by associating a geometric shape (with some probability) with every point in the Poisson point model. However, due to the increased complexity of probability computations with this model, it is very unlikely that there exists a closed form analytical expressions for MER criterion.

B. MER based on simple extension

In the following, we show how to use this physical space model for MER criterion based on generic range sensors.

Using MER criterion, the next best view is such chosen that the expected entropy reduction (approximated by the sum of marginal entropies) is maximized, i.e.,

$$\begin{aligned} s_{max}^{i+1} &= \arg\max_s E\{H(\mathcal{C}) - H(\mathcal{C}|\mathcal{V}(s))\} \\ &\approx \arg\max_s \sum_{q\in\mathcal{C}_u} E\{H(Q) - H(Q|\mathcal{V}(s))\} \quad (7) \end{aligned}$$

In the above, the conditional $\mathcal{V}(s)$ denotes a possible sensing result if the sensor were to sense at sensor's configuration s. This result in turn induces, using the ideal sensor update model mentioned above, the posterior probabilities of the physical space cells and the posterior probability distribution of the C-space space changes accordingly. The entropy reduction expectation computation above is carried out over all such possible sensing results. This reduction expectation is used to evaluate each sensor configuration's potential effect on the C-space knowledge.

As before, we use $er_q(s)$ to denote the expected marginal entropy reduction for an unknown robot configuration $q \in \mathcal{C}_u$, if the sensor were to sense at s, i.e.,

$$er_q(s) = E\{H(Q) - H(Q|\mathcal{V}(s))\}.$$

For the lack of a closed form solution, the expected marginal entropy reduction has to be computed numerically. Suppose the physical space cells that comprises $\mathcal{V}_u(s)$, the unknown sensor FOV at s in front of known obstacles, are labeled $j = 1, 2, \ldots, k$. (For simplicity, we ignore those situations where the cells have only partial intersection with $\mathcal{V}_u(s)$. Thus this computation will be accurate up to a certain resolution.) Expanding the expectation expression, we have,

$$er_q(s) = \sum_{c_1=0,1} \cdots \sum_{c_k=0,1} \prod_j \Pr[c_j](H(Q) - H(Q|c_1, \ldots, c_k)).$$

Without visibility constraints, it is easy to see that the number of terms inside the summation is in the order of the exponential of unknown cell number. Thus, the complexity of evaluating the expected marginal entropy reduction is in the same order. Even with visibility constraints, the complexity is still roughly in the same order. This can be explained as follows. By decomposing the unknown sensor FOV into small sensing cones, Fig. 2, there can only exist one hit point (or hit obstacle cell) in each cone. Thus, for each cone, the number of possible sensing results will be the number of unknown cells in that cone plus 1 (corresponding to a free cone). So the number of terms in the above summation is the product of these numbers for all the cones. This induces a computational complexity of roughly the number of cells in each cone to the power of number of such discretized cones.

Note that by the above discussion, the resolution of the physical space stochastic model has a big effect on the entropy computation because of the lack of a closed form representation. If we make the resolution coarser to reduce the computation costs, the accuracy of the representation can not be guaranteed. On the other hand, for the sake of accuracy, the number of cells representing the physical space will increase and in turn induce a big overhead (growing exponentially) for MER criterion computation. This justifies the usage of Poisson point model: although simplistic, it results in a closed form solution for MER criterion which can be computed relatively fast, and in addition, it matches well our intuitions and MER based on it improves the view planning results significantly over simple physical space based criterion.

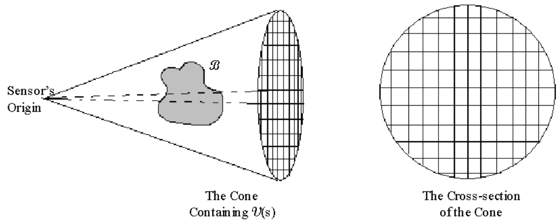


Fig. 2. Illustration of a possible way of discretizing $\mathcal{V}_u(s)$ into sensing cones.

III. Expected Entropy Reduction's Monotonicity

In the following, we show that although marginal entropy function (w.r.t. the volume of robot unknown part) under Poisson point model is not monotone, the expected marginal entropy reduction is always positive and monotone, i.e., as unknown part of the robot at q, $\mathcal{A}_u(q)$, intersects the sensor FOV at s in front of known obstacles, $\mathcal{V}_u(s)$, the expected marginal entropy value of q is always decreasing and the decreasing amount is bigger, larger this intersection is.

The expected marginal entropy reduction for an unknown robot configuration q is given by [4]:

$$er_q(s) = H(Q) - e^{-\lambda\cdot|\mathcal{A}(q)\cap\mathcal{V}_u(s)|} \cdot H(Q \mid \text{event } 2'). \quad (8)$$

In the above formula, the conditional event $2'$ denotes the event that the intersection $\mathcal{A}(q) \cap \mathcal{V}_u(s)$ is free of obstacles. So conditionally on event $2'$, the volume of unknown part of the robot is reduced from $|\mathcal{A}_u(q)|$ to $|\mathcal{A}_u(q)| - |\mathcal{A}(q) \cap \mathcal{V}_u(s)|$. Under Poisson point model, the posterior void probability $p(q|\text{event } 2')$ is given by:

$$p(q|\text{event } 2') = e^{-\lambda(|\mathcal{A}_u(q)| - |\mathcal{A}(q)\cap\mathcal{V}_u(s)|)} \quad (9)$$

Note that before sensing, the value $H(Q)$ is fixed. Thus, by Eqs. (8) and (9), we have $er_q(s)$ as a function of $|\mathcal{A}(q)\cap\mathcal{V}_u(s)|$. Instead of directly analyzing the positiveness and monotonicity of this function, which turns out to be very complicated, we are going to analyze this indirectly using its derivative, given by:

$$\begin{aligned} &\frac{d\, er_q(s)}{d|\mathcal{A}(q)\cap\mathcal{V}_u(s)|} \\ &= -\lambda e^{-\lambda|\mathcal{A}(q)\cap\mathcal{V}_u(s)|} \log(1 - p(q|\text{event } 2')) \\ &= -\lambda e^{-\lambda|\mathcal{A}(q)\cap\mathcal{V}_u(s)|} \log(1 - e^{-\lambda(|\mathcal{A}_u(q)| - |\mathcal{A}(q)\cap\mathcal{V}_u(s)|)}) \end{aligned} \quad (10)$$

Since the term inside the logarithm function above is always less than 1, the above derivative is always positive. This guarantees that marginal entropy reduction as a function of $|\mathcal{A}(q) \cap \mathcal{V}_u(s)|$ is monotonically increasing and the increase is always positive. In other words, if the additional unknown part of the robot inside sensor FOV is larger, so is the expected entropy reduction. This is shown in Fig. 3 for different $|\mathcal{A}_u(q)|$ values. This matches our intuition very well: more information of an unknown robot configuration is expected if the sensor can potentially see a larger portion of the robot volume.

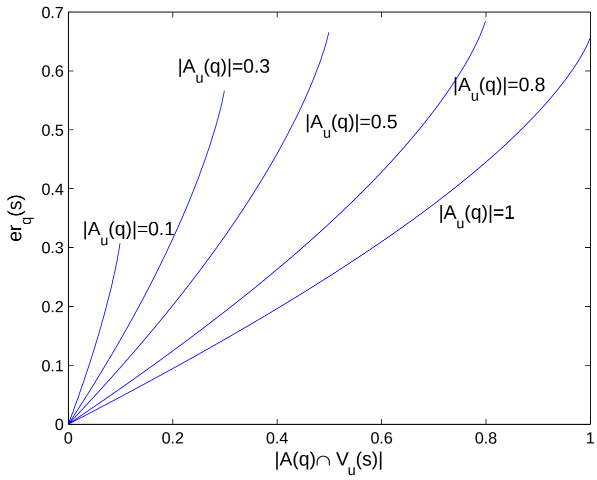


Fig. 3. The marginal entropy reduction value, $er_q(s)$, as a function of expected sensed volume, $|\mathcal{A}(q) \cap \mathcal{V}_u(s)|$. The plots shown are for different unknown volume before sensing, $|\mathcal{A}_u(q)|$. λ is set to be 1 for all the plots.

The formula Eq. (10) also revels the "boundary" property of the $er_q(s)$ function: the expected marginal entropy reduction is larger for those unknown robot configurations at which the robot has less unknown volume. This can be shown from Eq. (10 directly using monotonicity of the basic functions involved: for a fixed $|\mathcal{A}(q) \cap \mathcal{V}_u(s)|$, when $|\mathcal{A}_u(q)|$ increases, the value of $e^{-\lambda(|\mathcal{A}_u(q)|-|\mathcal{A}(q)\cap\mathcal{V}_u(s)|)}$ decreases, which in turn causes increases in both $-e^{-\lambda(|\mathcal{A}_u(q)|-|\mathcal{A}(q)\cap\mathcal{V}_u(s)|)}$ and $\log(1-e^{-\lambda(|\mathcal{A}_u(q)|-|\mathcal{A}(q)\cap\mathcal{V}_u(s)|)})$, and this results in a decrease in $er_q(s)$. As noted in [8], this boundary property implies that the MER criterion will "favor" those boundary robot configurations, unknown configurations lying near the boundary of $\mathcal{C}^i_{free}$ and $\mathcal{C}^i_u$, when summing up a sensing action's potential effects on the marginal entropy reductions. (This is why the property was named "boundary" property.) The "boundary" property of MER criterion based on generic range sensors comforts to that for MER criterion based on point FOV sensors as shown in [8], [7]. This will have impacts on the sampling strategies to approximate the C-space entropy reduction. For example, [7] adopts an importance sampling strategy and use a sampling distribution that favors more near the $\mathcal{C}^i_{free}$ and $\mathcal{C}^i_u$ boundaries.

IV. Parameter Estimation Effect on MER Criterion

In this section, we investigate empirically the effect of parameter estimation error on view planning qualities.

A. *The weighting effect of λ*

The density parameter λ is an important parameter of Poisson point process. Intuitively, it tells how clustered the physical space is. However, for sensor-based MP, this knowledge may not be available a priori. For these cases, an estimated λ has to be used. Here we try to answer the question: how does this estimation affect the qualities of the view planning results using MER criterion?

Note that the λ value shape the marginal entropy reduction function, Eq. (4). This implies that while doing view planning and summing up the potential effects of a sensing action on different configurations, these configurations are weighted differently according to their marginal entropy reductions. As shown in Fig. 4, the choices of smaller λ will tend to weight different configurations more evenly, for example, the $\lambda = 0.1$ plot; those of larger λ favors more those configurations at which the robot has less unknown volume, for example, $\lambda = 10$ plot.

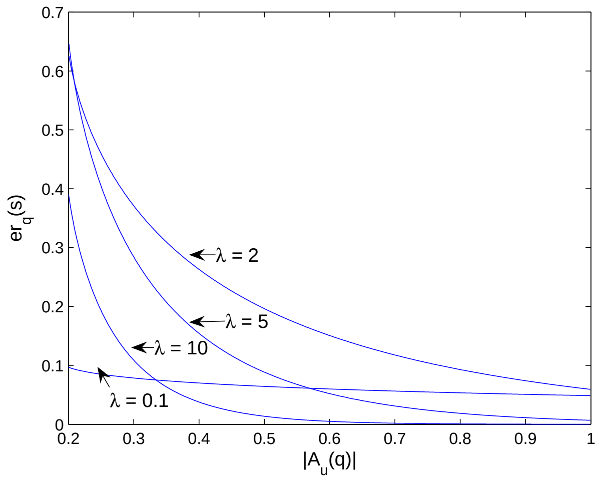


Fig. 4. The marginal entropy reduction value, $er_q(s)$, as a function of the unknown robot volume before sensing, $|\mathcal{A}_u(q)|$, assuming a fixed intersection volume, $|\mathcal{A}(q) \cap \mathcal{V}_u(s)|$. The plots shown are for different λ values. The volume values are all normalized to be the ratio to the volume of the whole robot.

However, it is hard to tell the effects of the weighting ability of the λ values on the view planning qualities based on MER criterion. This is due to the fact that MER includes both the stochastic (the weighting effects by a stochastic model) component and the geometric component, and much of the view planning mechanism is attributed to the geometric one [6].

In the following, we empirically evaluate the effect of different λ values on the view planning results.

B. *Empirical results of the effect of λ*

In the following, we use conducted a series experiments on the 2D simulated eye-in-hand system shown in Figure 5. It consists of a 2 dof planar robot and a range sensor (triangle FOV) mounted on its end-effector. The sensor has an additional dof that rotates 360 degree around the wrist. The sensing angle (the angle between the two edge of the sensing triangle) is 60 degree. The task for the robot is to explore its environment, starting from pointing vertically downwards in its initial configuration.

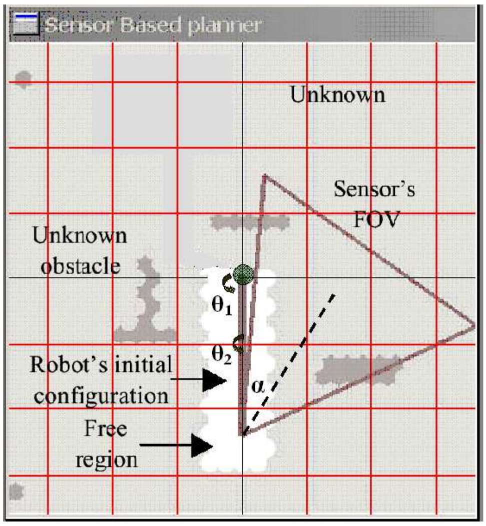


Fig. 5. Examples of Eye-in-Hand system: a planar 2-link robot with a triangle FOV range sensor.

We then simulated a series of different physical spaces, both structured and randomly generated, for the robot to explore. Four of them are shown in Fig. 6: (1) and (2) are two structured environments; (3) and (4) are two unstructured environments where obstacles are randomly generated, with Poisson distribution with density parameters 0.02 and 0.2 respectively.

We tested MER based view planner using different λ values (from 0.01 to 20) on these environments and recorded the planning results (the exploration rate of the C-space). For all these environments, the planning results are really close to each other. For example, for environment (1), the average standard deviation of the known C-space percentages after each view is less than 0.4%; for environment (2), this number is less than 0.6%. Our conjecture is that for structured environments, the physical space regions are distinct from each other and geometries play a bigger role than the stochastic effects. We also observed that for larger λ values, bigger than 10, the results deteriorate marginally. This is maybe due to the fact that larger λ tends to less weight those configurations at which the robot has lots of unknown volume and thus ignores their effects, while their cumulative effects may play a big role in the C-space exploration. (This may be more so for the beginning of the exploration when a lot of configurations are largely unknown.) For environment (3) and (4), the average standard deviations are 3.3% and 3% respectively. It is obvious that stochastic component plays a bigger role in these two randomly generated environments. We also found that for environment (2), the planning results are marginally (about 2% percent) better around for $\lambda = 0.01 \sim 0.04$, the range around the true $\lambda = 0.02$ used to generate the environment. For environment (3), the better (about 1% percent) performance range is $\lambda = 0.1 \sim 1$, still around the true $\lambda = 0.2$.

By these empirical results, we can conclude that MER criterion for view planning is fairly robust with respect the choices of the stochastic model parameter λ. Although we have seen some improvements when λ value used is closed to the environment characteristics, these improvements are fairly marginal.

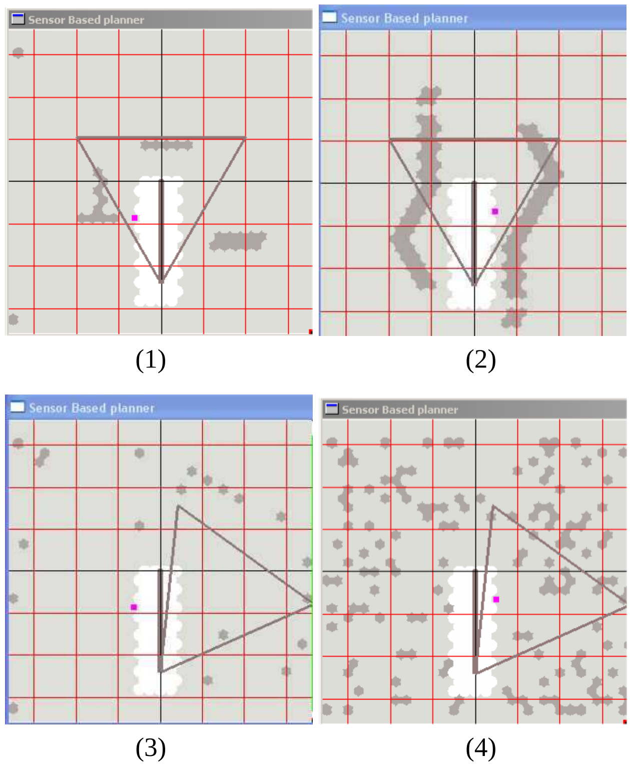


Fig. 6. The three different physical spaces used in the simulation: (1, 2) structured environment, (3,4) random generated environment.

V. Conclusion

In this report, we investigated several issues of MER criterion based on generic range sensor models. First to justify the usage of Poisson point process as our physical space model, we showed that even with a simple extension, to add a size to the probabilistic physical space cells, the MER computational costs increase exponentially, due to a lack of closed form solution. Thus from a computational efficiency consideration, Poisson point is the right choice for MER. Second, we investigate the monotonicity of C-space entropy reduction function based Poisson point process. We showed that marginal entropy reduction is a monotone function: as robot (at the corresponding configuration) intersects more with the sensor FOV, the expected information gain is larger. This matches our intuitions well. We also showed that for configurations at which the robot has less unknown volume, the expected entropy reduction is larger. This is called "boundary" property, reported previous for MER based on point sensors. Last, to evaluate the stochastic modeling effects on view planning qualities based MER criterion, we conducted extensive simulations on both structured and randomly generated environment. We showed empirically that the view planning results are very robust to choices of the density parameter of the Poisson point process model. We also found some improvements in the view planning results when the density parameter is close to the true value used to generate the environment. However, this improvement is really marginal.

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