# On Usage of Wave Variables in Force-Reflecting Teleoperation

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## Abstract

*This paper reviews the notion of 'Wave variables', which have been used to stabilize force-feedback teleoperations when large time-delays exist. Different system configurations suggested by previous researchers are simulated and compared. The effect of critical parameters such as the wave impedance of the communication line and the bandwidth of a low-pass filter , used to eliminate higher frequency harmonics and noise are studied. The results may provide a basis for future development in this growing field. The effects of wave impedance, communication line filter bandwidth and position feedback are studied and discussed.*

## 1 Introduction

Since their introduction by Niemeyer and Slotine [4], the wave-variables have been the subject of numerous papers, and found their way to areas as distant as neurology [5]. Anderson [2] pointed out that the main cause of instability in force-feedback teleoperations with time delays is the non-passive nature of the communication lines. Using some ideas from the scattering theory, he suggested to modify the control law to make the system transfer function appear like that of a passive transmission line([13],[14].

Based on Anderson's ideas, Niemeyer and others put together the concept of 'Wave variables' by redefining the system power flow. Usually, the power flow is defined as the product of an effort and flow pair. Let $\mathbf{P}$ be the power introduced to the system and let $\mathbf{F}$ and $\dot{\mathbf{x}}$ be the total effort and flow signals of the system. It should be noted that although the most usual effort and flow signals are force and velocity, it is not necessary to assume that $\mathbf{F}$ and $\dot{\mathbf{x}}$ really represent force and velocity parameters and they can be any arbitrary effort and flow signals.

Instead of looking at the power flow as a whole, we divide it to two components, one flowing along the positive direction from the input port to the output port ($\frac{1}{2}\mathbf{u}^T\mathbf{u}$), and the other one flowing opposite the positive direction from the output port to the input port($\frac{1}{2}\mathbf{v}^T\mathbf{v}$)(Figure 1).

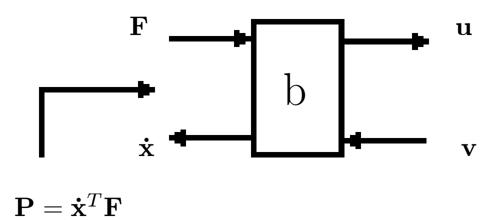


Figure 1: The wave variable transmission block diagram.

To introduce wave variables, we redefine the power flow as:

$$\mathbf{P} = \dot{\mathbf{x}}^T\mathbf{F} = \frac{1}{2}\mathbf{u}^T\mathbf{u} - \frac{1}{2}\mathbf{v}^T\mathbf{v} \tag{1}$$

This means we have assumed a physical wave traveling along the positive direction($\mathbf{u}$) and another physical wave traveling opposite the positive direction($\mathbf{v}$). These new variables are referred to as 'Wave variables' and they do not correspond to any physical quantity by themselves. Yet they can be dealt with like physical traveling waves, carrying energy over the communication line.Therefore the wave variables can be calculated in terms of $\dot{\mathbf{x}}$ and $\mathbf{F}$ as:

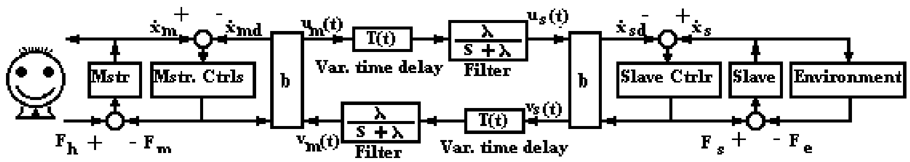


Figure 2: The overall system block diagram as used in all sections except for section 4. The filter blocks are added in section 3.

$$\mathbf{u}(t) = \frac{b\dot{\mathbf{x}}(t) + \mathbf{F}(t)}{\sqrt{2b}} \qquad \mathbf{v}(t) = \frac{b\dot{\mathbf{x}}(t) - \mathbf{F}(t)}{\sqrt{2b}} \quad (2)$$

In which $b$ is a tuning parameter which is called the 'Wave impedance' in analogy with network theory concepts. It should be noted that these equations can be written in different terms, depending on how we define our tuning parameter. But the current shape is preferred because it resembles similar equations in the scattering theory. Now that the wave variables are defined, for a given system, any pair of the above variables $(\mathbf{u}, \mathbf{v}, \dot{\mathbf{x}}, \mathbf{F})$, can be selected as input or output variables. It is possible to show that the communication line is passive if the energy stored in the outgoing wave of $\mathbf{v}$ is limited to the energy of incoming wave of $\mathbf{u}$. Now, if these wave variables are transmitted instead of the actual force or velocity signals, the overall system would be passive and no instability will happen.

Wave variables have no direct physical interpretation. Their unit of $\sqrt{Watt}$(with a physical dimension of $M^{\frac{1}{2}}LT^{-\frac{3}{2}}$) makes it hard to obtain any physical meaning for them. However, the symmetry in the behavior of wave variables of opposite directions results in interesting and useful characteristics. For example, given any pair of the four variables, the other two can be calculated by using a duality relation.

When a signal is sent through a communication line with a variable time delay, parts of the signal face longer delays and so arrive later than the parts originally following them. The output is a distorted signal and if the variance of the delay is not too much, the output signal resembles a thicker version of the original(Figure.3)

More recent research on wave variables includes works by Yokokohji et al. , in which they have tried to rebuild the wave signals distorted after passing a variable time-delay communication link [10].In these works, it is suggested to add a compensator block at each side of the original wave-variable-based teleoperation system. When the delay is constant, the adjustment by the compensator becomes zero and their proposed system becomes equivalent to the original wave-variable-based one.

Although alternative strategies to stabilize force-reflecting teleoperations have been suggested (e.g. [12]), using wave-variable notation is an approach based on physically based modeling and circuit theory.

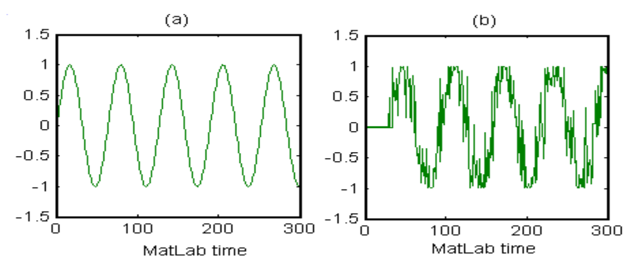


Figure 3: The wave variable (a) before and (b) after passing through a transmission line with a variable time delay.

## 1.1 System Configuration

In this paper, we have performed simulations to study the characteristics of wave-variable-based teleoperations. In the system we have used, a force pulse $(\mathbf{F}_h)$is applied(supposedly by a human operator) to a unit mass master manipulator, which is locally controlled by a PD controller. The force on this manipulator $(\mathbf{F}_m)$ forms one input for the master side wave transformer, which is a computational block using equations 2 to determine the motion velocity of the master manipulator($\dot{\mathbf{x}}_\mathbf{m}$). The wave transformer uses $\mathbf{F}_m$ as well as feedback wave variable $\mathbf{v}_m$ as its input. $\mathbf{v}_m$ is the feedback wave variable which is sent from the slave side to the master side.

The slave side manipulator also has a unit mass. It uses a wave transformer and a PD controller similar to those of the master manipulator. The force applied to the slave manipulator and the linear velocity of this manipulator are referred to as $\mathbf{F}_s$ and $\dot{\mathbf{x}}_\mathbf{s}$, respectively. The slave manipulator is assumed to be in contact with a spring, while it is subject to a viscous friction damping force at the same time. Thus the environment is modeled by a spring and a damper.

A delay block is used as a model for the communication line. This means we assume that the effect of crosstalks and other distortions are negligible in the communication link. Different with what is usually assumed, we have used a variable time delay. The length of the delay is a Gaussian random variable, whose mean and variance can be changed to give us a faster or slower communication link as well as more deterministic or less deterministic values. The average delay was set to 4 seconds and the variance was 1 second.

In one section, it is needed to transmit wave integrals as well as original wave signals. Therefore the communication line is changed. In one experiment the integral of each wave variable is superimposed on the original wave-variable and transmitted in the same communication channel. A low-pass filter is used at the other end of the channel to reconstruct each signal.

The integrals of force and velocity represent the momentum and position of each manipulator. Therefore, just as wave variables encode force and velocity of manipulators, their integrals encode the momentum and positions of the same manipulator. This matter will be discussed in section 4.

Knowing the magnitude of the force, the position and the velocity of manipulators, we have all the necessary parameters to assure that the respective parameters at the master and slave sides track each other during teleoperation process.

In another case, the integral of the wave variable is accompanied in transmission by the integral of the squared wave variable, which means the energy stored in the traveling waves. This way:

$$\mathbf{U} = \int_0^t \mathbf{u} d\tau \quad \textbf{and} \quad \mathbf{E} = \int_0^t \mathbf{u^2} d\tau$$

are both transmitted.

### 1.2 Paper overview

In section 2 we study the effect of changing the wave impedance $b$ on system behavior, by plotting the forces and velocities at the master and slave sides.

In section 3 we add a low-pass filter to reduce the noisy behavior generated by the variable time delay. The bandwidth of this filter($BW$) is changed and the resulting force and velocity curves are plotted against time.

In section 4, two different ways are suggested to reduce position error. The first way is to transmit the wave integrals, which contain position and momentum information, along with the wave variables themselves. The second way is to transmit wave integral and wave energy. A special filter is then used to reconstruct the original wave signal based on the information obtained from wave integral and wave energy. Advantages and disadvantages of this methods are briefly stated.Section 5 gives some conclusions and final remarks.

## 2 Wave Impedance

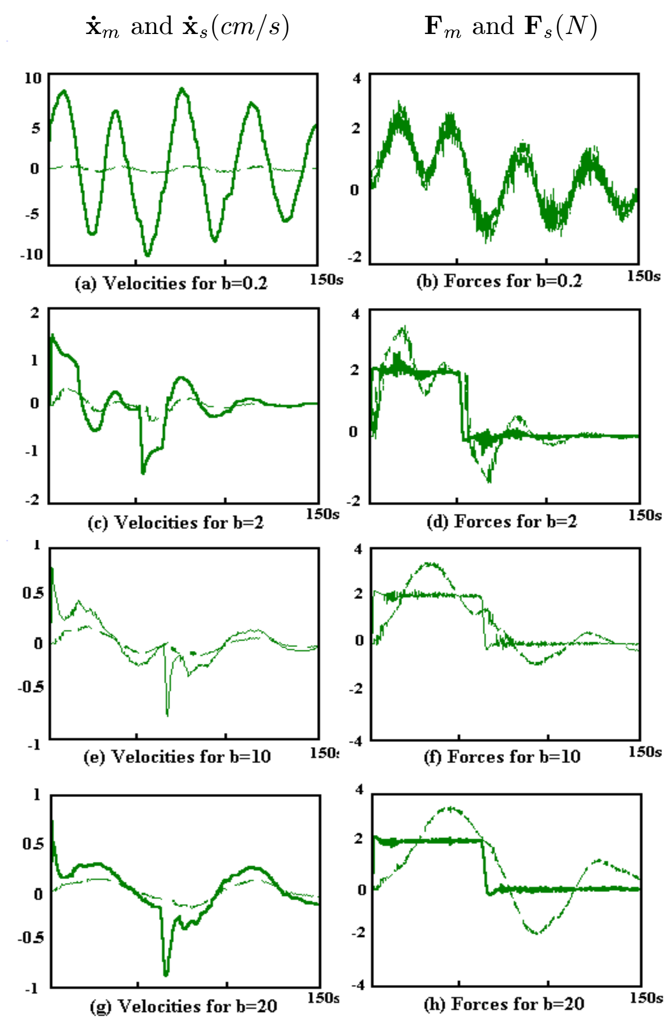


Figure 4: $\dot{x}_m, \dot{x}_s, F_m, F_s$ for different values of the wave impedance $b$.

The wave impedance $b$ is a tuning parameter which can trade off the speed of motion and levels of force [3]. By changing the value of $b$, we change the relative weight of $\dot{\mathbf{x}}$ compared to $\mathbf{F}$ in equation 2. Therefore, it is shown that an increase in impedance will raise force levels while reducing the mobility. The system will appear more damped, or in other words can be considered as a heavy tool. This situation is ideal for applications that involve applying a force to an object in workspace. The opposite happens when the wave impedance $b$ is decreased. Now a higher velocity can be achieved while keeping the same wave values. So the system behaves much lighter compared to the previous case and can be considered as a light tool that can be moved around easily.

In the present set of simulations, a force input like what is plotted in figure 5, is given as input to $F_h$. $\dot{\mathbf{x}}_\mathbf{m}$, $\dot{\mathbf{x}}_\mathbf{s}$, $\mathbf{F_m}$, $\mathbf{F_s}$ are plotted vs. time for a few different values of the wave impedance $b$, from b=0.2 to b=20. Based on Eq.2, at smaller values of $b$, the contribution to $\dot{\mathbf{x}}$ is smaller. So the wave variable $\mathbf{u}(t)$ and $\mathbf{v}(t)$ carry stronger information about $\mathbf{F}$(t) signals. Thus at $b$=0.2, we can see that while $\mathbf{F_m}$ and $\mathbf{F_s}$ track each other closely, there is a great difference of magnitude between $\dot{\mathbf{x}}_\mathbf{m}$ and $\dot{\mathbf{x}}_\mathbf{s}$. Moreover, $F_m$ and $F_s$ hardly resemble the desired force signal of $F_h$.

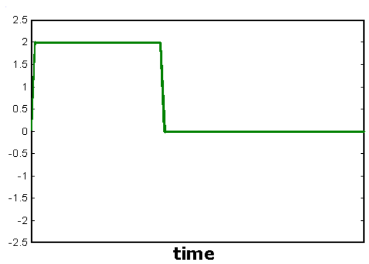


Figure 5: The force input given as $F_h$ used in all simulations.

As $b$ is increased to 2, $\mathbf{F_m}$ and $\mathbf{F_s}$ start to resemble $F_h$. At the same time the magnitude of $\dot{\mathbf{x}}_\mathbf{s}$ is increased and the ratio between $\dot{\mathbf{x}}_\mathbf{s}$ and $\dot{\mathbf{x}}_\mathbf{m}$ is closer to unity than before.

When the value of $b$ is increased to 10, $\dot{\mathbf{x}}_\mathbf{s}$ and $\dot{\mathbf{x}}_\mathbf{m}$ start to track each other, while the resemblance between $\mathbf{F_s}$ and $\mathbf{F_m}$ is still visible. It was observed that with the values that we have used in modeling, $b$=10 gives the most acceptable results in terms of tracking between the force and velocity signals comparing to other values of $b$ tested.

When $b$ is increased further(to 20), the effects of the time delay show up and the resemblance and tracking between $F_s$ and $F_m$ decays. This shows that a higher value of $b$ puts more emphasis on $\dot{\mathbf{x}}$ and therefore the tracking of $\mathbf{F}$s is no longer guaranteed.

The wave transformation also acts like a feedback path, and introduces a damping element with the damping of $b$. However, unlike what happens in real damping elements, the energy is not dissipated in this case, but rather stored in wave variables traveling in the system.

The importance of the wave impedance is in the fact that it can be changed during system operation. Therefore, even after fixing all the built-in parameters, we can still change the value of $b$ to emphasize on forces or velocities. This means that while in contact with an object, we can increase the wave impedance and make the system appear as a heavy tool, applying a large amount of force, but showing little mobility. On the other hand, when the goal is to move the tool, we can just decrease the impedance and make the system appear as a light object to give it a higher mobility.

## 3 Wave filtering

The necessity of wave filtering is mostly for eliminating the high-frequency noises and higher-order harmonics of the wave signals. Since all the information of the robots are included in $\mathbf{u}(t)$ and $\mathbf{v}(t)$, by filtering these signals smoother responses are achieved for both force and velocity on both sides. A variable time delay block is a nonlinear element by nature, therefore it introduces nonlinearities, noise and undesired harmonics in the circuit. These unwanted harmonics and noises can cause huge spikes in the magnitude of force at some instants, and these spikes can be dangerous in many applications that need preciseness, e.g. telesurgery. However, at the same time filters introduce a phase lag to the system that can potentially result in instability.

By introducing a first-order low-pass filter into the transmission path of the wave, the noisy behavior of the force signal can be reduced. But if the bandwidth of the filter is reduced below a certain value, the resulting velocity signals at the output will start to show a noisy behavior similar to what had appeared before in the force signals. This effect is more visible when the slave robot is interacting with a stiff object in the environment. Uncontrolled noise spikes in the force (the value of which may be critically high) may result in damage to objects in the manipulator workspace, or even fatal problems in applications such as telesurgery. Meanwhile in usual applications, there is little interest in noisy behavior of the velocity, as long as the overall movement of the manipulator is not affected.

Figure 6 shows the behavior of the force and velocity signals for two cases. In the first case, the bandwidth of the filter($\lambda$ in Figure 2), is set to 5 $rad/s$.

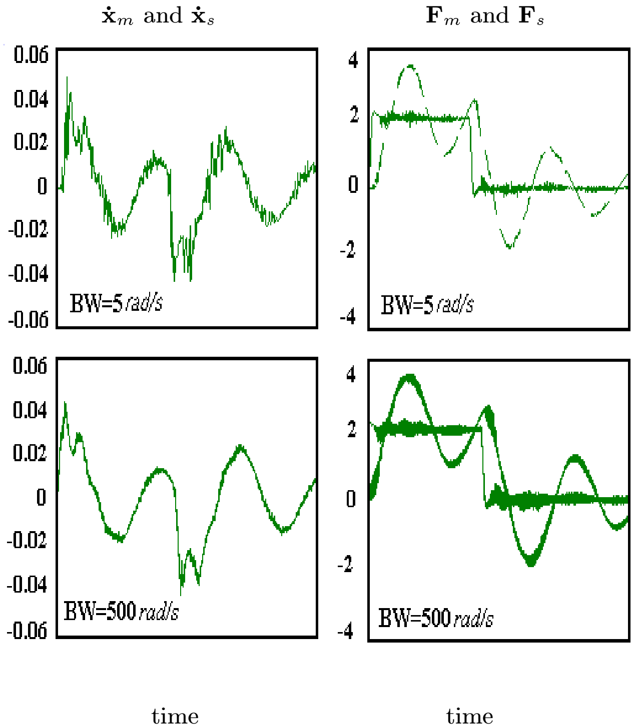


Figure 6: $\dot{x}_m, \dot{x}_s, F_m, F_s$ for different values of the filter bandwidth.(a) BW=5 $rad/s$ (b) BW=500 $rad/s$

# 4 Wave Integrals

The wave variables as defined in equation 2, contain information about only velocity and force. Therefore, position tracking is not guaranteed and due to data losses or other reasons, the master and slave manipulators can gradually drift apart. An immediate solution to this problem is transmission of wave variable integrals along with original wave variables [3]. Just as the wave signals encode force and velocity, their integrals encode momentum and position($\mathbf{p}$ and $\mathbf{x}$.

$$\mathbf{U} = \int_0^t \mathbf{u} d\tau = \frac{b\mathbf{x} + \mathbf{p}}{\sqrt{2b}} \qquad \mathbf{V} = \int_0^t \mathbf{v} d\tau = \frac{b\mathbf{x} - \mathbf{p}}{\sqrt{2b}} \tag{3}$$

On the master side, the wave integral, $\mathbf{U}_m$, can be calculated as:

$$\mathbf{U}_m = \sqrt{2b}\mathbf{x}_m - \mathbf{V}_m \tag{4}$$

and on the slave side, the velocity and the wave integral can be obtained from:

$$\mathbf{x}_{sd} = \frac{\sqrt{2b}\mathbf{U}_s - \mathbf{p}_s}{b} \qquad \mathbf{V}_s = \frac{b\mathbf{x_{sd}} - \mathbf{p}_s}{\sqrt{2b}} \tag{5}$$

where $\mathbf{p}_s$ is the slave robot momentum.

Although this method provides the necessary data for the slave manipulator, it might show certain drawbacks of its own. If $\mathbf{x}$ and $\dot{\mathbf{x}}$ go through different distortion or data losses during communication, there might be some mismatch between $\mathbf{x}$ and $\int \dot{\mathbf{x}} \mathbf{d}\tau$. It is not known which one should determine the motion of the manipulator. Probably the best way to control is to compute their average and use that instead of $\mathbf{x}$ to find the desired position of the slave manipulator.

## 4.1 Wave energy transmission

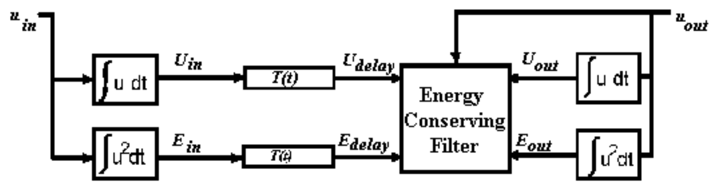


Figure 7:System schematic for wave energy transmission [8]

It has been suggested in [8] that instead of sending the wave signal alone, or transmitting the wave signal with its integral, it is useful to transmit both the wave integral and wave energy and to extract the wave signal on the slave side using a filter.

The wave integral is

$$\mathbf{U} = \int_0^t \mathbf{u} d\tau$$

and the wave energy is

$$\mathbf{E}_{in} = \int_0^t \mathbf{u^2} d\tau$$

So, assuming the $T(t)$ is the variable time delay of our communication block, we will have

$$U_{delay}(t) = U_{in}(t - T(t)) = \int_0^{t-T(t)} \mathbf{u}_{in} d\tau \tag{6}$$

and

$$E_{delay}(t) = E_{in}(t - T(t)) = \int_0^{t-T(t)} \mathbf{u}_{in}^2 d\tau \tag{7}$$

.

The *distance to go* is defined as

$$U(t) = U_{delay}(t) - U_{out}(t) \tag{8}$$

$$E(t) = E_{delay}(t) - E_{out}(t) \geq 0 \tag{9}$$

and the filter is designed such that

$$u_{out}(t) = \begin{cases} \alpha \frac{E(t)}{U(t)} & \text{if} \quad U(t) \neq 0 \\ 0 & \text{if} \quad U(t) = 0 \end{cases} \tag{10}$$

in which $\alpha$ is a tuning parameter.

As it has been discussed before, the nonlinearities in the communication line with a variable time delay may cause the system to behave as an actively, so producing energy inside the system. Therefore, the assumption in 9 that

$$E_{delay}(t) \geq E_{out}(t)$$

may not always be correct. In fact our simulations showed that at some points it was necessary to set this energy to zero to stabilize the system. This manual change may not be possible in real case. Once the energy limit is set, the results match perfectly with what is predicted in [8].

It should also be noted that equation 11 is an analytical definition rather than a practical one. While in mathematics any nonzero number has an inverse, in physical world the inverse of a very small number can be very large, which is to be considered as infinity. Because $U(t)$ is a physical parameter, it is unlikely to be exactly equal to zero. Thus the condition in the second case should be changed to something like

$$u_{out}(t) = \begin{cases} \alpha\frac{E(t)}{U(t)} & \text{if} \quad \|U(t)\| > 10^{-n} \\ 0 & \text{if} \quad \|U(t)\| < 10^{-n} \end{cases} \qquad (11)$$

where n is a number to be taken large enough to keep the filter working properly, while it should be small enough to prevent the output to increase to unacceptably large values.

## 5 Summary and Conclusions

With the tendency to use the Internet as the communication means, it is necessary to adapt every teleoperation method to deal with variable time delays. We have taken some steps to evaluate the works of previous researchers and interpret their ideas with our own simulation results, combined with critical analysis. The results can be a guideline for further research in this area.

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