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On the Effect of Compliance in Robotic Contact Tasks Problem

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Abstract

Generally, in most dexterous manipulation tasks the manipulator undergoes a transition from free motion to contact configuration with its environment. An example can be the exploratory unconstrained motions of the force guided manipulator for establishing contact with an environment. This transition usually involves the impact stage. The impact usually results in an unstable performance of the closed-loop controller of the manipulator. One of the main remedies to achieve a stable closed-loop control of the manipulator is to introduce some compliance property into the closed-loop system (i.e. through hardware or through feedback control laws) [3], [8] , [4]. Based on the Second Method of Lyapunov and the theory of guaranteed stability of uncertain systems, this paper presents results on showing the effect of compliance in increasing the stability bounds of the manipulator during the impact phase. The paper also presents discussions on how the effect of compliance in the model of mechanical manipulator can be studied in the closed-loop stability. Experimental results are also presented to demonstrate the effect of compliance in the stable response of the manipulator during the impact phase.

1 Introduction

Various methodologies have been proposed in the literature for stable control of the robotic manipulator during its phase transition from free to constrained motions. One of the main conclusions for stable control of such transition has been suggested to be the presence of some compliant properties[3], [8] , [4]. These properties can be introduced either in the construction of the robotic manipulator or can be created through the feedback control laws. This paper discusses the motivation of having compliant properties for stable control of robotic mechanisms during the impact with the environment.

There have been various remedies for designing of a stable impact controller. For example, Parker and Paul (87) [1] discussed that the compressibility property of air in pneumatic actuation and incorporation of velocity feedback can reduce the effect of impact and results in an stable performance. Youcef-Toumi and Gutz (89) [2] modelled the impact dynamics and suggested that for stable control of impact, velocity feedback can be used to create and active damping during the impact phase with the added integral force control. Using the notion of impedance control, Kazerooni, Waibel and Kim (90) [3] created an active impedance in their closed-loop control system. They also verified analytically that for stable control, there must exist some compliant properties either in the manipulator or in the environment. Volpe and Khosla (91) [4] experimentally verified that during the impact phase, proportional gain explicit force controller can be used to suppress oscillation (i.e. active compliance property). Recently, Hyde and Cutkosky (94) [5] experimentally compared various impact control strategies on a manipulator with compliant finger-tip and suggested an approach based on the input-shaping to increase the stability bounds. A compliant model of the manipulator in contact with the rigid environment has been proposed in [6]. Results concerning the effect of compliance in reducing the loop-gain of a robust controller are given in [7]. Also, experimental observation regarding the importance of compliance in stability of the controller in robotic impact task is presented in [8]. This paper presents results on effect of compliance in the closed-loop stability of the robotic impact task controller. The results are obtained based on the Second Method of Lyapunov and the theory of guaranteed stability of uncertain systems developed by [11].

The paper is organized as follows: section (2) presents some preliminaries in regard to the closed-loop modelling of the manipulator; section (3) presents the stability results on the effect of compliance with some experimental investigation and finally section (4) presents concluding remarks.

2 Closed-loop Modelling

This section presents review of a robust force controller of a model of manipulator in contact with a rigid environment. The contact between the manipulator and the environment is modeled as a compliant contact. The source of compliance is assumed to arise from a passive material (or device) attached to the end-point of the manipulator.

Let the nonlinear model of the manipulator in contact with the environment be given as:

$$\mathbf{M}_\theta \ddot{\theta} + h(\theta, \dot{\theta}) + g(\theta, \dot{\theta}, \ddot{\theta}) = \tau(t) - \mathbf{J}^T R \tag{1}$$

where $\mathbf{M}_\theta$ is the inertia matrix, $h(\theta, \dot{\theta})$ is a vector of centrifugal, Coriolis and gravity forces, $g(\theta, \dot{\theta}, \ddot{\theta})$ is a vector of uncertainties and disturbances, and R is the contact force exerted on the environment.

Expressing the above equation in the end-point reference frame (e.g. a frame where the normal to the contacting surface is co-linear with the $z-axis$) using the following identities:

$$\dot{\bar{x}} = \mathbf{J}\dot{\theta}$$
$$\tau = \mathbf{J}^T F$$
$$\ddot{\theta} = \mathbf{J}^{-1}\ddot{\bar{x}} - \mathbf{J}^{-1}\dot{\mathbf{J}}\dot{\theta}$$

where $\mathbf{J}$ is a mapping from the joint coordinates of the manipulator to the end-point coordinates $\bar{x}$. Substituting the above identities into equation (1), we have:

$$\mathbf{M}_{\bar{x}}\ddot{\bar{x}} + h_{\bar{x}} + g_{\bar{x}} = F - R \tag{2}$$

where:

$$\mathbf{M}_{\bar{x}} = \mathbf{J}^T\mathbf{M}_\theta\mathbf{J}^{-1}$$
$$h_{\bar{x}} = \mathbf{J}^{-T}h(\theta,\dot{\theta}) - \mathbf{M}_{\bar{x}}\dot{\mathbf{J}}\dot{\theta}$$
$$g_{\bar{x}} = \mathbf{J}^{-T}g(\theta,\dot{\theta},\ddot{\theta})$$

Let a non-linear model-based controller be given as:

$$F = \mathbf{M}_{\bar{x}}F' + h_{\bar{x}} + g_{\bar{x}} + R \tag{3}$$

(here we are assuming that the exact knowledge of the model and uncertainties are know). Defining F' as:

$$F' = \mathcal{F} - \dot{R} - R \tag{4}$$

The contact force as a function of the compliance of the end-point of the manipulator can be written as:[7]

$$R = \mathbf{K}_c(\bar{x} - \bar{x}_w) \tag{5}$$

where $\mathbf{K}_c$ is a matrix defining the compliance property of the end-point. $\bar{x}_w$ is the distance from the end-point of the manipulator at the initial contact with the environment.

Substituting (3) and (4) into (2) and noting $\ddot{\bar{x}} = \mathbf{K}_c^{-1}R$, we have:

$$\ddot{R} + \mathbf{K}_c\dot{R} + \mathbf{K}_c R = \mathbf{K}_c\mathcal{F} \tag{6}$$

In the case where the exact model parameters of the manipulator are not available and there are uncertainties and disturbances acting on the manipulator, the model-based controller of equation (3) can be written as:

$$F = \tilde{\mathbf{M}}_{\bar{x}}F' + \tilde{h}_{\bar{x}} + \tilde{g}_{\bar{x}} + \tilde{R} \tag{7}$$

where $\tilde{\mathbf{M}}_{\bar{x}}$ is an estimate of the inertia matrix, $\tilde{h}_{\bar{x}}$ is an estimate of the Coriolis and gravity force vector and $\tilde{g}_{\bar{x}}$ is an estimate of the disturbances and uncertainties. $\tilde{R}$ is a measure of the actual contact force vector. In the above equation F' is defined as:

$$F' = \left[(\ddot{R}^r - \dot{R}^e - R^e)/\mathbf{K}_c + \mathcal{F}\right] \tag{8}$$

where $R^e = R - R^r$. Here $(.)^r$ stands for reference parameters. $\mathcal{F}$ is the robust force controller which will be defined later.

Substituting equations (8) and (7) and then into the open-loop dynamics, we obtained the following,

$$-\left[(\ddot{R}^r - \dot{R}^e - R^e)/\mathbf{K}_c + \mathcal{F}\right] = -\tilde{\mathbf{M}}_{\bar{x}}^{-1}\mathbf{M}_{\bar{x}}\ddot{\bar{x}} + \tilde{\mathbf{M}}_{\bar{x}}^{-1}\left[(\tilde{h}_{\bar{x}} - h_{\bar{x}}) + (\tilde{g}_{\bar{x}} - g_{\bar{x}}) + (\tilde{R} - R)\right] \quad (9)$$

adding $\ddot{\bar{x}}$ to both side and incorporating the relationship of the end-point compliance of the manipulator (equation 5), we obtain:

$$\ddot{R}^e + \dot{R}^e + R^e - \mathbf{K}_c\mathcal{F} =$$
$$\mathbf{K}_c\tilde{\mathbf{M}}_{\bar{x}}^{-1}(\mathbf{M}_{\bar{x}} - \tilde{\mathbf{M}}_{\bar{x}})\ddot{\bar{x}} + \mathbf{K}_c\tilde{\mathbf{M}}_{\bar{x}}^{-1}\left[(\tilde{h}_{\bar{x}} - h_{\bar{x}}) + (\tilde{g}_{\bar{x}} - g_{\bar{x}}) + (\tilde{R} - R)\right]$$

or,

$$\ddot{R}^e + \dot{R}^e + R^e = \mathbf{K}_c\mathcal{F} + \mathbf{K}_c W \quad (10)$$

where the uncertainty W is defined as:

$$W = \tilde{\mathbf{M}}_{\bar{x}}^{-1}(\mathbf{M}_{\bar{x}} - \tilde{\mathbf{M}}_{\bar{x}})\ddot{\bar{x}} + \tilde{\mathbf{M}}_{\bar{x}}^{-1}\left[(\tilde{h}_{\bar{x}} - h_{\bar{x}}) + (\tilde{g}_{\bar{x}} - g_{\bar{x}}) + (\tilde{R} - R)\right]$$

The above equation can be put in the following state-space form:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u + \mathbf{B}W \qquad ; \qquad y = \mathbf{C}x \quad (11)$$

where $x = (R^e; \dot{R}^e)^T$ and $\mathbf{B}$ contains the complaint model of the end-point.

Combining the above with a model of the auxiliary dynamics, of the form:[10]

$$\dot{\xi} = \mathbf{\Lambda}\xi + \beta R^e \quad (12)$$

(the above equation is used to model the exogenous inputs which can act on the system. the poles of these models are all located on the right hand side of the imaginary axis [9]). We then obtain the following combined dynamics:

$$\left\{\begin{array}{c}\dot{x}\\ \dot{\xi}\end{array}\right\} = \left[\begin{array}{cc}\mathbf{A} & 0\\ \beta\mathbf{C} & \mathbf{\Lambda}\end{array}\right]\left\{\begin{array}{c}x\\ \xi\end{array}\right\} + \left[\begin{array}{c}\mathbf{B}\\ 0\end{array}\right]u + \left[\begin{array}{c}\mathbf{B}\\ 0\end{array}\right]W$$

where the output y is defined as:

$$y = \left[\begin{array}{cc}\mathbf{C} & 0\end{array}\right]\left\{\begin{array}{c}x\\ \xi\end{array}\right\}$$

or we can write:

$$\dot{z} = \bar{\mathbf{A}}z + \bar{\mathbf{B}}u + \bar{\mathbf{B}}W \quad (13)$$

In the following, the effect of impact force , which is an impulsive force having a large(bounded) amplitude and with very short duration is assumed to be modelled as an additive component to the contact force R or $R = R + ,$. Following similar derivation as above, the dynamics of the system by including the additive impact term can be written as:

$$\dot{z} = \bar{\mathbf{A}}z + \bar{\mathbf{B}}u + \bar{\mathbf{B}}\Phi + \bar{\mathbf{B}},^{-} \quad (14)$$

where $\bar{\Gamma}$ is refer to as the first stage reduction of the effect of impact force through the inverse of the mass matrix or $\bar{\Gamma} = \tilde{\mathbf{M}}_{\bar{x}}^{-1}\Gamma$.

The objective is now to design an additional control input u which can result in the closed-loop system to be stable in the presence of bounded uncertainties defined by W (i.e. $\|W\| \leq \rho$)[12] and the presence of impact force Γ.

Let us now define a controller of the form:

$$u = \mathbf{K}z + p \tag{15}$$

here the gain matrix $\mathbf{K}$ are chosen such that it stabilizes the unstable poles of the systems defined in equation (11) and the unstable poles of the auxiliary dynamics defined in equation (12). the controller input p is defined as :[11]

$$p = \begin{cases} -\frac{\bar{\mathbf{B}}^T\mathbf{P}z}{\|\bar{\mathbf{B}}^T\mathbf{P}z\|}\rho & if \quad \|\bar{\mathbf{B}}^T\mathbf{P}z\| > \epsilon \\ \\ -\frac{\bar{\mathbf{B}}^T\mathbf{P}z}{\epsilon}\rho & if \quad \|\bar{\mathbf{B}}^T\mathbf{P}z\| \leq \epsilon \end{cases} \tag{16}$$

where ϵ is a small positive number determined by the designer. $\mathbf{P}$ is a positive definite symmetric matrix representing the solution to the following Lyapunov equation for some $\mathbf{Q} > 0$,

$$\mathbf{P}\bar{\mathbf{A}} + \bar{\mathbf{A}}^T\mathbf{P} = -\mathbf{Q} \tag{17}$$

3 Stability Analysis

In the previous section a model of the manipulator combined with the model auxiliary dynamics was presented in the context of uncertain dynamical system. In this section, the stability of the closed-loop system is presented. First, the stability of the controller in the absence of impulsive force which arises from the collision between the manipulator and the environment is presented. Second, it is shown how the complaint property of the manipulator can affect the sensitivity of the closed-loop to the input impulse force and the closed-loop stability.

Let us consider the Lyapunov function candidate of the form:

$$V = z^T\mathbf{P}z \tag{18}$$

implementing the controller defined in (15) into equation (13), the derivative of the Lyapunov function candidate along the solution trajectory can be written as:[12]

$$\begin{aligned} \dot{V} &= \dot{z}^T\mathbf{P}z + z^T\mathbf{P}\dot{z} \\ &= -z^T\mathbf{Q}z + 2\alpha^T(u + W) \\ &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + 2\alpha^T(u + W) \end{aligned} \tag{19}$$

where we defined $\alpha = (\bar{\mathbf{B}}^T\mathbf{P}z)$. By Rayleigh's principle, and noting that $\mathbf{Q}$ is chosen as a positive definite matrix, we have $\lambda_{min}(\mathbf{Q}) \leq z^T\mathbf{Q}z$ and $\lambda_{min} > 0$.

Let us now consider the controller of equation (16), for $\|\alpha\| > \epsilon$. In equation (19) we can write:

$$\begin{aligned} \alpha^T(u+W) &= \alpha^T(\tfrac{-\alpha\rho}{\|\alpha\|} + W) \\ &= \alpha^T(\tfrac{-\alpha\rho}{\|\alpha\|}) + \alpha^T W \\ &\leq \|\alpha^T(\tfrac{-\alpha\rho}{\|\alpha\|})\| + \|\alpha^T W\| \\ &= -\|\alpha\|\rho + \|\alpha\|\rho = 0 \end{aligned} \tag{20}$$

for $\|\alpha\| \leq \epsilon$ we have:

$$\begin{aligned} \alpha^T(u+W) &= \alpha^T(\tfrac{-\alpha\rho}{\epsilon} + W) \\ &\leq \alpha^T(-\tfrac{\alpha\rho}{\epsilon}) + \|\alpha\|\rho \\ &= -\|\alpha\|^2\rho/\epsilon + \|\alpha\|\rho \\ &= (-\|\alpha\|^2/\epsilon + \|\alpha\|)\rho \end{aligned} \tag{21}$$

the maximum value of the above is when $\|\alpha\| = \epsilon/2$. Therefore,

$$\dot{V} \leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + \epsilon\rho/2 \tag{22}$$

as a result we can have $\dot{V} < 0$.

During the impact phase we assumed that the effect of impact force can be represented as an additive component to the contact force. Using the model given in equation (14), the controller given in equation (15) and the Lyapunov function candidate, the rate of change of this function candidate along the solution trajectory can be written as:

$$\begin{aligned} \dot{V} &= -z^T\mathbf{Q}z + 2\alpha^T(u + W + \bar{\Gamma}) \\ &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + \|\alpha\|\|(u + W + \bar{\Gamma})\| \end{aligned} \tag{23}$$

From equation (23) it can be seen that the effect of impulsive force due to the impact of the manipulator with the environment enters the rate of change of Lyapunov function. However, its effect is factored by the magnitude $\|\bar{\mathbf{B}}^T\mathbf{P}z\|$ which includes the model of the end-point compliance of the manipulator given in the definition of $\bar{\mathbf{B}}$. Given $\mathbf{P}$, one can reduce the effect of impact force on the closed-loop stability of the controller by introducing more compliant structure or material (low magnitude of $\mathbf{K}_c$). For example one design methodology can be the introduction of the compliance end-point to the design of manipulator. As a result, the magnitude of $\|\bar{\mathbf{B}}^T\mathbf{P}z\|$ can be reduced by mechanical design of the manipulator.

Let us define a constant $\eta > 0$ such that a given $\mathbf{P}$, $\mathbf{K}_c$ and bounded error vector z we can have $\|\bar{\mathbf{B}}^T\mathbf{P}z\| < \eta \leq \epsilon$. Then from the definition of the robust controller given in equation (16), the rate of change of Lyapunov function can be written as:

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + 2\alpha^T(u + \phi + \Gamma) \\ &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + \|\alpha\|(-\|\alpha\|\rho/\epsilon + W + \bar{\Gamma}) \end{aligned} \tag{24}$$

Let us further assume that the magnitude of impact force is bounded. This is a valid assumption since the impact force is also a function of the approach velocity of the manipulator

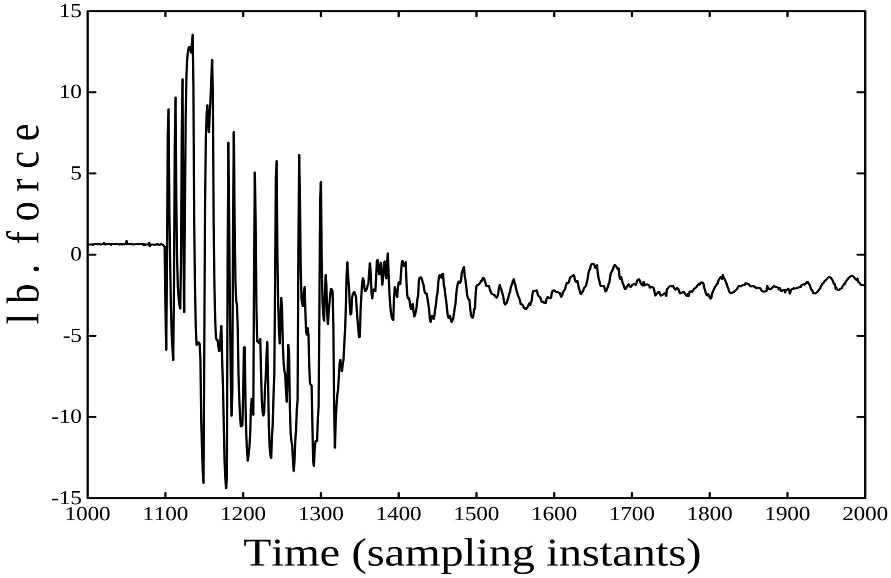

Figure 1: Force response of the manipulator during the impact phase in the case where no compliance is introduced.

to the environment and also $\bar{,}$ is the result of first stage reduction of the impact force, i.e. $\|\bar{,}\| \leq \rho$. As a result of the above assumptions and equation (24) we have:

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + \|\alpha\|(-\|\alpha\|\rho/\epsilon + \rho + \nu) \\ &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + 3\epsilon\rho \end{aligned} \tag{25}$$

and consequently, $\dot{V} \leq 0$.

To further examine the effect of compliance in the stable response of the controller during the impact phase of the manipulator with the environment, we carried series of experiments. In these experiments, the manipulator approaches a rigid environment with a given velocity. When the contact is detected, the manipulator switches from the position control to force control. Figure (1) shows the response of the manipulator when there is no effective compliance neither in the manipulator nor in the contacting environment. Clearly, the response is an unstable one since the manipulator bounces from the environment (positive values of force indicate lost of contact). Figure (2) shows the response of the manipulator with the same approach velocity for the case compliance material has been introduced at the point of contact. This response is an stable one.

4 Conclusions

There are many factors effecting the stable phase transition from the free motion to the constrained motion of the manipulator. Among these factors are the presence of compliance

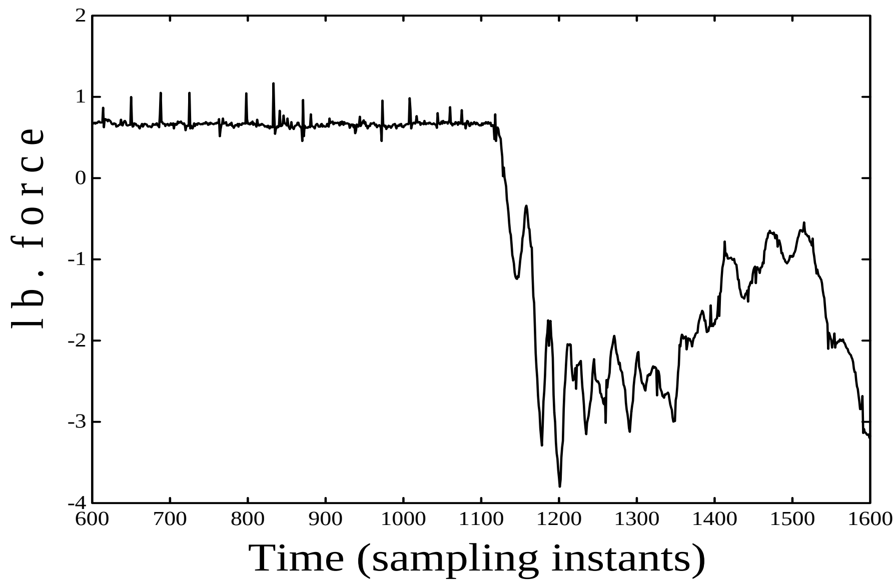

Figure 2: Force response of the manipulator during the impact phase having the same approach velocity of that of figure (1) with the introduction of compliance.

in the interacting bodies, the approach velocity, the sampling time for both the manipulator and the force sensor and the effect of switching index for changing from the position control mode to the force control mode during the impact and during post-impact phase are among the few.

This paper presented a discussion based on the control methodology for uncertain dynamical system for the effect of compliance during the impact phase of the manipulator. It was shown that one of the agent which can decrease the sensitivity of the closed-loop system to the effect of impulsive force which arises during the impact is the magnitude of the compliant property of the manipulator. Experimental results are also presented to demonstrate the effect of the compliance on the stability of the manipulator during the impact phase.

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