A Natural Framework for Designing Bounce-less Controller for Robotic Contact Tasks

Shahram Payandeh
Experimental Robotics Laboratory (ERL)
School of Engineering Science
Simon Fraser University
Burnaby, British Columbia
Canada V5A 1S6
shahram@cs.sfu.ca

Abstract. In robotic tasks where the manipulator has to make transition from free space motion to constrained one, there always exists a inevitable phase transition. A number of controllers have been proposed in the literature with various discussions on their practical implications. In this paper for the first time a novel framework for studying design of controllers for robotic contact tasks problem is proposed. This framework presents a natural set-up to study the performance of controller. The method of this paper combines the embedded passive compliant properties of the interacting system (i.e. manipulator and the contacting environment) with the basic controller to achieve the hybrid bounce-less property. Here the interacting environment is used as a natural switching surface for the discontinuous controller. The application of the proposed framework is demonstrated through experimental results.

Key Words. Robotic, contact tasks, discontinuous controller, differential inclusions, stability, experimental study.

1. Introduction

In general, the contact task is defined as the ability of the *computer controlled machine* such as dexterous robotic devices to follow a free space trajectory and then make and maintains contact with the environment for exerting a desired contact force on it. In general, they make up a large proportion of the tasks to which the robot can be applied. One application can be the force-guided autonomous motion of the manipulating system for detecting obstacles in its environment and then maintaining the contact.

Recently, various initiatives focused on the development of control laws which can result in an stable controller for the manipulator during the transition from free motion to constrained motion. Kazerooni (1,) proposed a controller based on the notion of impedance control where the closed-loop dynamics of the manipulator is *matched* with a target dynamics with compliance properties. The stability of the controller is then investigated based using the small gain theorem. Mills (2,) proposed a discontinuous controller and discussions on its performance based on the notion of the generalized dynamical systems. In this scheme, the environment is modeled as spring and damper (a compliant model of the environment). A practical approach for controlling the contact transition was proposed by Payandeh (3,). In this method, the gains of the controller are switched during different phases of motion to achieve a stable contact force regulation. Several methods for controlling the transition from free motion to constrained motion with the objective on minimizing fingertip load oscillation during transition was investigated by Hyde and Cutkosky(4,). A suitable controller based on the input command shaping was proposed. Using one degree of freedom manipulator, experiments were performed to demonstrate the stable performance of the controller. Marth, Tarn and Bejczy (5,) proposed a model-based algorithm combined with explicit force control to regulating the phase transition. The force controller is switched on when the contact is established. The stability results were presented for the impact phase of the manipulator with the rigid environment. The non-collocation effects between sensors and actuators have been studied in (7,), (8,). A thorough study of force robotic force controllers has been provided in (9,), (10,).

As correctly pointed-out by Kazerooni, Mills and Payandeh, in contact transition, the dynamics of the closed-loop system changes and the external contact force measurements which was omitted as a part of feedback control system in the free motion is now a part of the controller. However, the methodology of using the force signal as a natural indication of the contact phenomenon has been incorporated in different ways. Mills has use the force signal to switch the controller from the position control PD mode to the force control PD mode. While Kazerooni used the force signals as a part of the feedback loop while maintaining the structure of the controller unchanged (i.e. impedance control). In this way, the position controller can also be used as a force control through the impedance parameterization. Payandeh has used the measure of the instant of contact to change the gains of the controller while maintaining the structure of the robust controller fixed during the phases of free motion, impact and constrained motion.

In this paper, the force signal is also used as a switch for the controller from the position control mode to the force control mode. However, the frame work which is proposed to further study the performance of the closed-loop system is new. It is proposed that general free to contact model of the manipulator is best presented using the notion of the *differential inclusions* which is a class of dynamical systems with the discontinuous right-hand side. Here the physical contact surface can be used as a natural *switching surface* for the controller.

The paper is organized as follows: section (2) presents an overview of modelling of physical systems which results in an implementation of a bounce-less controller; in section (3) a discontinuous model of the manipulator and the analysis of the switching for bounce-less performance is presented, section (4) presents discussions on some experimental results and finally section (5) presents discussions and future work.

2. Toward Design of a bounce-less Controller

In general in has been realized that to accomplish a practical bounce-less controller in robotic contact tasks, there has to exist some compliance either in the structure of the manipulator (e.g. structural flexibility), in the contacting environment (e.g. compliant environment) or in the closed-loop controller (e.g. stiffness controller). This presence of compliance either in the contacting environment or in the manipulator structure can reduce the effective impulsive forces which arises due to the collision between two solid bodies(12,). The advantages of the presence of compliance has been shown both analytically and experimentally, see for example, (,), (,), (6,),(11,), (,) and (,). This in effect reduces the contact dynamic from the infinite mode to a finite mode and can reduce (eliminate) the number of

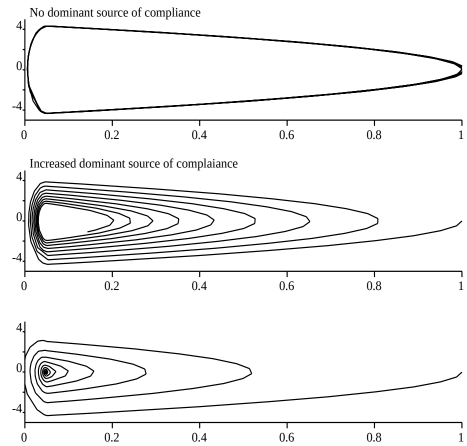

Fig. 1. Simulation results of an ideal manipulator when interacting with a rigid environment with various degrees of compliance.

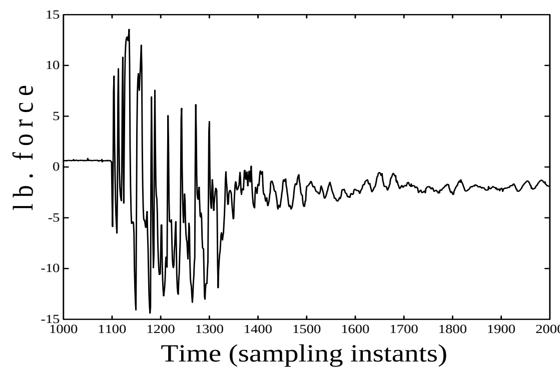

Fig. 2. Contact force response of the manipulator during the impact phase in the case where no dominant compliance is introduced.

bounces. For example, Figure (1) shows a simulated position response of an ideal manipulator when approaching a stiff environment.

In general, compliance in the contacting bodies (either in the environment or manipulator) allows the increase in the closed-loop bandwidth and hence increasing the bounds of the desired controller gains (,). For example, Figure (2) and (3) shows the actual response of the manipulator when approaching a rigid environment with an identical controllers. It can be seen for the case where there is no dominant compliance presence, the manipulator bounces from the contacting environment (positive value of approximately 1 lbf represents the non-contact phase, any value less than this represents the contact phase).

In the following this effective passive approach for introducing compliance in the model of the manipulator has been used in conjunction with a robust controller to achieve a stable phase transition and force regulation in robotic contact tasks.

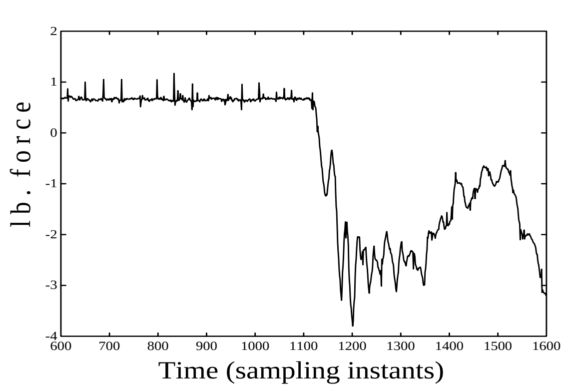

Fig. 3. Contact force response of the manipulator during the impact phase having the same approach velocity of that of figure (2) with the introduction of dominant source of compliance.

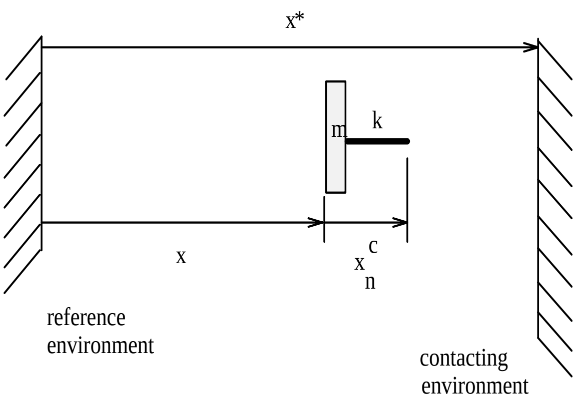

Fig. 4. Definition of the variables associated with the one-dimensional example

2.1. *A Natural Discontinuous Model of the Closed-Loop System*

Let us consider a one dimensional compliant mechanism with the actuation force. The dynamic equation of the system in an unconstrained phase can be written as:

$$m\ddot{x}(t) = -r + u(t)^1 \tag{1}$$

where for this case the reaction force from the environment during the contact phase is zero, or $r = 0$. The dynamic equation of this system in contact with a rigid environment can be written as:

$$m\ddot{x}(t) = -r + u(t)^2 \tag{2}$$

where $r = k_c(x^\star - x)$ for $x^\star - x < x_n^c$ is the reaction force acting on the mass due to interaction with the rigid environment and the k is a model of compliant. Figure (4) shows all the position variables involved.

Let us now define a controller for both contact and non-contact phases of the mechanisms of the

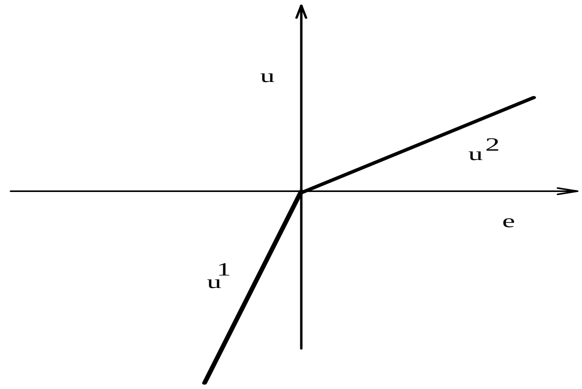

Fig. 5. A definition of the discontinuous controller

form:

$$u(t) = \begin{cases} u(t)^1 = k_p(x^d - x) - k_d\dot{x} & \text{for } r < 0 \\ u(t)^2 = k_p(r - r^d) - k_d\dot{r} & \text{for } r > 0 \end{cases} \tag{3}$$

where $x^d, \dot{x}^d$ is the desired trajectory of the end-point and r^d The desired contact force to be exerted on the environment (i.e. x^d is generated through an exploratory planning strategy to located the unknown environment). At the instant when the contact is detected, the controller regulates the desire contact force specified by r^d. By taking into account the relationship between the contact force and the state variables of the mechanisms, the controller $u(t)^2$ can be written as:

$$u(t)^2 = k_p(k_c x^d - k_c x) - k_d k \dot{x} \tag{4}$$

Now for example defining $e = (x^\star - x) - x_c$, a representation of the control inputs which was defined in equations 3 and 4 can be defined in Figure 5. It can be seen in this representation that the controller is discontinuous about the origin where the initial contact between the manipulator and the environment is detected.

In general, the two dynamic model of the object in both regions can be written as:

$$m\ddot{x}(t) = -r + u(t) \tag{5}$$

The right-hand side of the system of the above differential equation is discontinuous. the discontinuity both arises from the presence of the reaction force from the environment and the the nature of the controller at $x + x_n^c = x^\star$. The set of points in the state space where the right-hand side of (5) is discontinuous is called a surface of discontinuity similar to what is found in the theory of variable structure (13,). For this example, the surface of discontinuity S, can be represented by:

$$S = \left\{(x, v) \in \mathbf{R}^2 \mid s(x, v) = 0\right\} \tag{6}$$

This surface divides the phase space into two open regions, $R^+ = \left\{(x, v) \in \mathbf{R}^2 \mid s > 0\right\}$ and $R^- =$

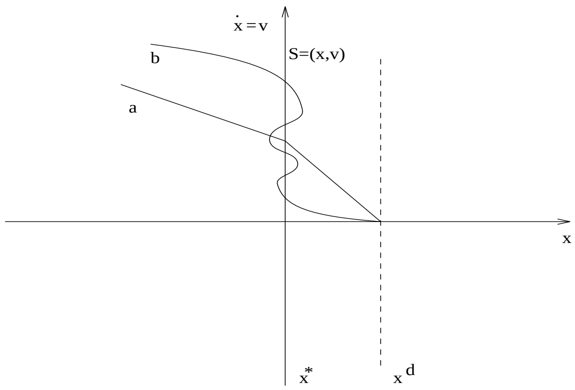

Fig. 6. Definition of the discontinuity in the state variable which arises naturally from the impact with the environment

$\{(x,v) \in \mathbf{R}^2 \mid s < 0\}$(14,). Inside these regions, the right-hand side of (5) satisfies a local Lipschitz condition, therefore we are guaranteed existence, uniqueness, and continuous dependence on initial conditions for solution. Trajectories are well defined an continuous until they hit the surface of discontinuity S where state derivatives are discontinuous. The notion of a solution of (5) must be generalized for trajectories which have discontinuous derivatives.

Figure (6) shows two types of trajectories. The trajectory b is for the case when the manipulator bounces at the discontinuous surface $s = (x, v) = 0$ and the controller switches from the free motion to constrained motion. Trajectory represents the motion where there trajectory does not have any bouncing at the natural surface of discontinuity.

2.2. *Existence of the Solution*

Let $\dot{x} = f(x,t)$ represent the dynamic of the system (i.e. the state-space description of the dynamics of a physical system (e.g. equations (1-2)))where $x \in \mathbf{R}^n$. $f(x,t)$ where $f : \mathbf{R}^n \times \mathbf{R} \rightarrow \mathbf{R}^n$ represents the vector-valued function which is piece-wise continuous in a finite domain G which itself consist of finite number of domains G_i. Let M be a set which consist of boundary points of these domains. For each point (t,x) of a domain G, a set $F(t,x)$ is an n-dimensional space is specified. If at the point (t,x) the function f is continuous, the set $F(t,x)$ consists of one point which coincides with the value of the function f at this point. If (t,x) is a point of discontinuity of the function f, the set $F(t,x)$ should be defined. The solution to $\dot{x} = f(t,x)$ is called a solution to the differential inclusion:

$$\dot{x} \in F(t,x) \tag{7}$$

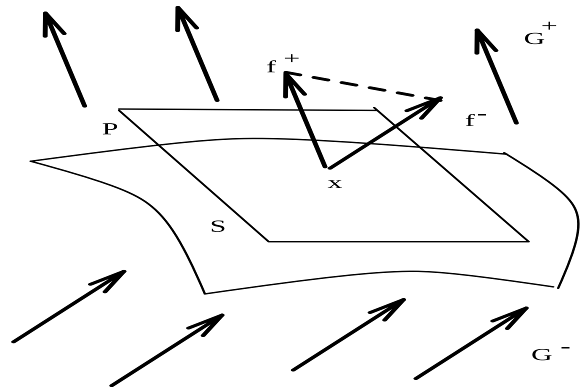

Fig. 7. A schematic of a solution to the differential inclusions

The objective is to construct $f(t,x)$ at the point of discontinuity of the function f, under which the differential inclusion (7) can be applied to approximate the description of the process.

Consider the case of the example, $f(t,x)$ is discontinuous on a smooth surface S given by $s(x,v) = 0$. The surface S separates its neighborhood in the x space into domains G^- and G^+ (i.e. before and after contact with the environment). For $t = constant$ and for the point $x^\star$ approaching the point $x \in S$ from the domain G^- and G^+. Let the function $f(t,x^\star)$ have the limit values:(15,)(Filippov's solution)

$$lim_{x^\star \in G^-, x^\star \rightarrow x} f^-(t,x^\star), \qquad lim_{x^\star \in G^+, x^\star \rightarrow x} f^+(t,x^\star)$$

Then the set $F(t,x)$ is a linear segment joining the endpoints of the vectors $f^-(t,x)$ and $f^+(t,x)$ (Figure 7). Or, in other words it can be stated that a vector function $x(.)$ is called a solution on $[t_0,t_1]$ if it is absolutely continuous on the interval and for almost all $t \in [t_0,t_1]$,

$$\dot{x} \in K[f](x) \tag{8}$$

where:

$$K[f](x) = \bar{co}\left\{limf(x^\star) \mid x^\star \rightarrow x, x^\star \notin N_f \bigcup N\right\} \tag{9}$$

where $N_f \subset \mathbf{R}^m$, $\mu N_f = 0$(18,).

The content of Filippov's solution is that the tangent vector to a solution, where it exists, must lie in the convex closure of the limiting values of the vector field in progressively smaller neighborhoods around the solution point ($B(x,\delta)$, or:

$$K[f](x) \equiv \bigcap_{\delta>0} \bigcap_{\mu N=0} \bar{co} f(B(x,\delta) - N, t)$$

Filippov presented a calculus for computing the differential inclusion which was then further stated by (,).

3. Analysis of a bounce-less controller

Let the model of a manipulator in non-contact phase expressed in its end-point coordinate frame be given as:

$$\mathbf{M}_{\bar{x}}\ddot{\bar{x}} + h_{\bar{x}} + g_{\bar{x}} = F \tag{10}$$

where: $\mathbf{M}_{\bar{x}} = \mathbf{J}^T\mathbf{M}_\theta\mathbf{J}^{-1}$ is the mass matrix, $h_{\bar{x}} = \mathbf{J}^{-T}h(\theta,\dot{\theta}) - \mathbf{M}_{\bar{x}}\dot{\mathbf{J}}\dot{\theta}$ is the Coriolis and gravity, $g_{\bar{x}} = \mathbf{J}^{-T}g(\theta)$ is a vector of disturbances and uncertainties and $\mathbf{J}$ is the Jacobian of the manipulator.

Let a non-linear controller be given as:

$$F = \tilde{\mathbf{M}}_{\bar{x}}F_1^{'} + \tilde{h}_{\bar{x}} + \tilde{g}_{\bar{x}} \tag{11}$$

where $F^{'} = (\ddot{x}^r - \dot{e}_1 - e_1 + \mathcal{F}_1)$ and $\mathcal{F}$ will be defined later. $\tilde{(.)}$ represents an estimate of the actual parameters.

Substituting the control law of equation (11) into the open-loop dynamics and expanding and simplifying term we have:

$$\begin{aligned}\ddot{e}_1 + \dot{e}_1 + e_1 \quad &= \quad \mathcal{F}_1 + \tilde{\mathbf{M}}_{\bar{x}}^{-1}(\tilde{\mathbf{M}}_{\bar{x}} - \mathbf{M}_{\bar{x}})\ddot{\bar{x}} + \\ &\quad\; \tilde{\mathbf{M}}_{\bar{x}}^{-1}((\tilde{h}_{\bar{x}} - h_{\bar{x}}) + (\tilde{g}_{\bar{x}} - g_{\bar{x}}))\end{aligned} \tag{12}$$

In the contact phase, dynamic equation of the system can be written as:

$$\mathbf{M}_{\bar{x}}\ddot{\bar{x}} + h_{\bar{x}} + g_{\bar{x}} = F - R \tag{13}$$

where $R = \mathbf{K}_c x$ is the contact force expressed as a change in displacement of the complaint end-point from its nominal dimension (e.g. x_n^c). The nonlinear controller of equation (11) can then be written as:

$$F = \tilde{\mathbf{M}}_{\bar{x}}F_2^{'} + \tilde{h}_{\bar{x}} + \tilde{g}_{\bar{x}} + \tilde{R} \tag{14}$$

where $\tilde{\mathbf{M}}_{\bar{x}}$ is an estimate of the inertia matrix, $\tilde{h}_{\bar{x}}$ is an estimate of the Coriolis and gravity force vector and $\tilde{g}_{\bar{x}}$ is an estimate of the disturbances and uncertainties. $\tilde{R}$ is a measure of the actual contact force vector. In the above equation $F_2^{'}$ is defined as:

$$F_2^{'} = \left[(\ddot{R}^r - \dot{R}^e - R^e)/\mathbf{K}_c + \mathcal{F}_2\right] \tag{15}$$

where $R^e = R - R^r$. Here $(.)^r$ stands for reference parameters. $\mathcal{F}_2$ is the robust force controller which will be defined later.

Substituting equations (15) and (13) and then into the open-loop dynamics, we obtained the following,

$$\begin{aligned}-\left[(\ddot{R}^r - \dot{R}^e - R^e)/\mathbf{K}_c + \mathcal{F}_2\right] &= \\ -\tilde{\mathbf{M}}_{\bar{x}}^{-1}\mathbf{M}_{\bar{x}}\ddot{\bar{x}} &+ \\ \tilde{\mathbf{M}}_{\bar{x}}^{-1}\Big[(\tilde{h}_{\bar{x}} - h_{\bar{x}}) &+ \\ (\tilde{g}_{\bar{x}} - g_{\bar{x}}) + (\tilde{R} - R)&\Big]\end{aligned}$$

adding $\ddot{\bar{x}}$ to both side and incorporating the relationship of the end-point compliance of the manipulator, we obtain:

$$\begin{aligned}\ddot{R}^e + \dot{R}^e + R^e - \mathbf{K}_c\mathcal{F}_2 &= \\ \mathbf{K}_c\tilde{\mathbf{M}}_{\bar{x}}^{-1}(\mathbf{M}_{\bar{x}} - \tilde{\mathbf{M}}_{\bar{x}})\ddot{\bar{x}} &+ \\ \mathbf{K}_c\tilde{\mathbf{M}}_{\bar{x}}^{-1}\left[(\tilde{h}_{\bar{x}} - h_{\bar{x}}) + (\tilde{g}_{\bar{x}} - g_{\bar{x}}) + (\tilde{R} - R)\right]&\end{aligned}$$

or,

$$\ddot{R}^e + \dot{R}^e + R^e = \mathbf{K}_c\mathcal{F}_2 + \mathbf{K}_c W \tag{16}$$

where W is the vector of uncertainty defined above.

The above equation can be put in the following state-space form:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u + \mathbf{B}W \quad ; \quad y = \mathbf{C}x \tag{17}$$

where $x = (R^e; \dot{R}^e)^T$ and $\mathbf{B}$ contains the complaint model of the end-point.

Combining the above with a model of the auxiliary dynamics, of the form:(?)

$$\dot{\xi} = \mathbf{\Lambda}\xi + \beta R^e \tag{18}$$

(the above equation is used to model the exogenous inputs which can act on the system. the poles of these models are all located on the right hand side of the imaginary axis (?)). We then obtain the following combined dynamics:

$$\left\{\begin{array}{c}\dot{x}\\ \dot{\xi}\end{array}\right\} = \left[\begin{array}{cc}\mathbf{A} & 0\\ \beta\mathbf{C} & \mathbf{\Lambda}\end{array}\right]\left\{\begin{array}{c}x\\ \xi\end{array}\right\} + \left[\begin{array}{c}\mathbf{B}\\ 0\end{array}\right]u + \left[\begin{array}{c}\mathbf{B}\\ 0\end{array}\right]W$$

where the output y is defined as:

$$y = \left[\begin{array}{cc}\mathbf{C} & 0\end{array}\right]\left\{\begin{array}{c}x\\ \xi\end{array}\right\}$$

or we can write:

$$\dot{z} = \bar{\mathbf{A}}z + \bar{\mathbf{B}}u + \bar{\mathbf{B}}W \tag{19}$$

The effect of impact force representation , which has a bounded magnitude with a short duration is assumed to be modeled as an additive component to the contact force R (?). Following similar derivation as above, the dynamics of the system by including the additive impact term can be written as:

$$\dot{z} = \bar{\mathbf{A}}z + \bar{\mathbf{B}}u + \bar{\mathbf{B}}W + \bar{\mathbf{B}}\bar{,} \tag{20}$$

where $\bar{,}$ is refer to as the first stage reduction of the effect of impact force through the inverse of the mass matrix or $\bar{,} = \tilde{\mathbf{M}}_{\bar{x}}^{-1},$.

The objective is now to design an additional control input u which can result in the closed-loop system to be stable in the presence of bounded uncertainties defined by W (i.e. $||W|| \leq \rho$)(**?**) and then in combination with the presence of bounded impact force , .

The objective now is to design a controller which can results in a stable and robust performance of the manipulator during various stages of motion. Then based on the performance of the controller in various discontinuous regions, we can deduct the performance at the surface of discontinuity.

Let us now define a controller of the form:

$$u = \mathbf{K}z + p \tag{21}$$

In general, the gain matrix $\mathbf{K}$ can be chosen such that it stabilizes the unstable poles of the systems defined in equation (11) and the unstable poles of the auxiliary dynamics defined in equation (12). Let the new closed-loop system be defined as:

$$\begin{aligned} \dot{z} &= \left[\bar{\mathbf{A}} + \bar{\mathbf{B}}\mathbf{K}\right] z + \bar{\mathbf{B}}p + \bar{\mathbf{B}}W \\ &= \tilde{\bar{\mathbf{A}}}z + \bar{\mathbf{B}}p + \bar{\mathbf{B}}W \end{aligned} \tag{22}$$

For the case of additive bounded impact force we can define the closed-loop system as:

$$\dot{z} = \tilde{\bar{\mathbf{A}}}z + \bar{\mathbf{B}}p + \bar{\mathbf{B}}W + \bar{\mathbf{B}}\bar{,} \tag{23}$$

the controller input p is defined as :(**?**)

$$p = \begin{cases} -\frac{\bar{\mathbf{B}}^T\mathbf{P}z}{||\bar{\mathbf{B}}^T\mathbf{P}z||}\rho & if \quad ||\bar{\mathbf{B}}^T\mathbf{P}z|| > \epsilon \\ -\frac{\bar{\mathbf{B}}^T\mathbf{P}z}{\epsilon}\rho & if \quad ||\bar{\mathbf{B}}^T\mathbf{P}z|| \leq \epsilon \end{cases} \tag{24}$$

where ρ is the bound on the uncertainty and ϵ is a small positive number determined by the designer. $\mathbf{P}$ is a positive definite symmetric matrix representing the solution to the following Lyapunov equation for some $\mathbf{Q} > 0$,

$$\mathbf{P}\tilde{\bar{\mathbf{A}}} + \tilde{\bar{\mathbf{A}}}^T\mathbf{P} = -\mathbf{Q} \tag{25}$$

It should be noted that gains $\mathbf{K}$ are chosen such that $\tilde{\bar{\mathbf{A}}}$ is asymptotically stable, i.e. $\lambda(\tilde{\bar{\mathbf{A}}}) \subset C_{-}$ where C_{-} corresponds to the left hand side of the complex plane. The following will show that the controller form for p defined in (18) renders the system (16) and then (17) *globally practically stable*. This implies that the system solution is *uniformly bounded* and *uniformly ultimately bounded*(**?**).

Theorem 1: The system defined in equation (16) with $||W|| \leq \rho$ with the controller defined in (16) results in a global practical stable system where the solution is uniformly ultimately bounded.

Proof: Let us consider the Lyapunov function candidate of the form:

$$V = z^T\mathbf{P}z \tag{26}$$

implementing the controller defined in (18) into equation (16), the derivative of the Lyapunov function candidate along the solution trajectory can be written as:(**?**)

$$\begin{aligned} \dot{V} &= \dot{z}^T\mathbf{P}z + z^T\mathbf{P}\dot{z} \\ &= -z^T\mathbf{Q}z + 2\alpha^T(u + W) \\ &\leq -\lambda_{min}(\mathbf{Q})||z||^2 + 2\alpha^T(p + W) \end{aligned} \tag{27}$$

where we defined $\alpha = (\bar{\mathbf{B}}^T\mathbf{P}z)$. By Rayleigh-Ritz inequality, and noting that $\mathbf{Q}$ is chosen as a positive definite matrix, we have $\lambda_{min}(\mathbf{Q})z^Tz \leq z^T\mathbf{Q}z$ and $\lambda_{min} > 0$.

Let us now consider the controller of equation (18), for $||\alpha|| > \epsilon$. In equation (21) we can write:

$$\begin{aligned} \alpha^T(u + W) &= \alpha^T(\tfrac{-\alpha\rho}{||\alpha||} + W) \\ &= \alpha^T(\tfrac{-\alpha\rho}{||\alpha||}) + \alpha^T W \\ &\leq ||\alpha^T(\tfrac{-\alpha\rho}{||\alpha||})|| + ||\alpha^T W|| \\ &= -||\alpha||\rho + ||\alpha||\rho = 0 \end{aligned} \tag{28}$$

and for the case when $||\alpha|| \leq \epsilon$ we have:

$$\begin{aligned} \alpha^T(u + W) &= \alpha^T(\tfrac{-\alpha\rho}{\epsilon} + W) \\ &\leq \alpha^T(-\tfrac{\alpha\rho}{\epsilon}) + ||\alpha||\rho \\ &= -||\alpha||^2\rho/\epsilon + ||\alpha||\rho \\ &= (-||\alpha||^2/\epsilon + ||\alpha||)\rho \end{aligned} \tag{29}$$

the maximum value of the above is when $||\alpha|| = \epsilon/2$. Therefore,

$$\dot{V} \leq -\lambda_{min}(\mathbf{Q})||z||^2 + \epsilon\rho/2 \tag{30}$$

consequently,

$$\dot{V} < 0 \tag{31}$$

where $\lambda_{min} > 0$ since $\mathbf{Q}$ is positive definite. Thus condition (25) is met for all time and all z such that:

$$\lambda_{min}(\mathbf{Q})||z||^2 - \epsilon\rho/2 > 0 \tag{32}$$

or the bounds on the error can be obtained to be:

$$\|z\| \geq \left(\frac{\epsilon\rho}{2\lambda_{min}(\mathbf{Q})}\right)^{\frac{1}{2}} \tag{33}$$

□

Theorem 2: Given the model in equation (17) with the bounds on the uncertainty vector W and $,$, the controller defined as: $p = -\frac{\alpha}{\|\alpha\|}\bar{\rho}$ if $\|\alpha\| > \bar{\epsilon}$ and $p = -\frac{\alpha}{\bar{\epsilon}}\bar{\rho}$ if $\|\alpha\| \leq \bar{\epsilon}$ where $\bar{\rho}$ is the new bound in the vector of uncertainties and the impact force and $\bar{\epsilon}$ is the new designer choice scalar for the case of impact task results in a closed-loop system to be global practical stable and the solution (i.e. the error vector z) to be uniformly ultimately bounded.

Proof: Let us consider the Lyapunov function candidate of the form defined in equation (20). The rate of change of this function candidate along the solution trajectory can be written as:

$$\dot{V} = -z^T\mathbf{Q}z + 2\alpha^T(p + W + \bar{,}) \tag{34}$$

Or, from Rayleigh-Ritz inequality, the above can be written as:

$$\dot{V} \leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + 2\alpha^T(p + W + \bar{,}) \tag{35}$$

From equation (29) it can be seen that the effect of impulsive force due to the impact of the manipulator with the environment enters the rate of change of Lyapunov function. However, its effect is factored by the magnitude $\|\bar{\mathbf{B}}^T\mathbf{P}z\|$ which includes the model of the end-point compliance of the manipulator given in the definition of $\bar{\mathbf{B}}$. Given $\mathbf{P}$, one can reduce the effect of impact force on the closed-loop stability of the controller by introducing more compliant structure or material (low magnitude of $\mathbf{K}_c$). For example one design methodology can be the introduction of the compliance end-point to the design of manipulator. As a result, the magnitude of $\|\bar{\mathbf{B}}^T\mathbf{P}z\|$ can be reduced by mechanical design of the manipulator.

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + 2\alpha^T(u + \phi + ,) \\ &\leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + \|\alpha\|(-\|\alpha\|\rho\epsilon + W + \bar{,}) \end{aligned} \tag{36}$$

Following similar argument as previous theorem for $\|\alpha\| > \bar{\epsilon}$ we have from equation (30):

$$\begin{aligned} \alpha^T(p + W + \bar{,}) &= \alpha^T(\tfrac{-\alpha\bar{\rho}}{\|\alpha\|} + W + \bar{,} \\ &\leq \|\alpha^T\left(\tfrac{-\alpha\bar{\rho}}{\|\alpha\|}\right)\| + \|\alpha^T(W + \bar{,})\| \\ &= \|\alpha\left(-\bar{\rho} + \|W + \bar{,}\|\right) = 0 \end{aligned}$$

Since $\lambda_{min} > 0$ the above equation and equation (29) implies $\dot{V} < 0$.

Similar argument can be stated for proving the the stability of the closed-loop system for the case when $\|\alpha\| \leq \bar{\epsilon}$. In equation (30) we can write:

$$\begin{aligned} \alpha^T(p + W + \bar{,}) &= \alpha^T(\tfrac{-\alpha\bar{\rho}}{\|\alpha\|} + W + \bar{,} \\ &\leq \|\alpha^T\left(\tfrac{-\alpha\bar{\rho}}{\|\alpha\|}\right)\| + \|\alpha^T\|\bar{\rho} \\ &= (-\|\alpha\|^2/\bar{\epsilon} + \|\alpha\|)\bar{\rho} \end{aligned}$$

the maximum value of the above is when $\|\alpha\| = \bar{\epsilon}/2$, Therefore $\dot{V} \leq -\lambda_{min}(\mathbf{Q})\|z\|^2 + \bar{\epsilon}\bar{\rho}/2$. Consequently we have $\dot{V} < 0$ where $\lambda_{min} > 0$ since $\mathbf{Q}$ is positive definite. Thus the above stability condition hold for all time and z such that:

$$\lambda_{min}(\mathbf{Q})\|z\|^2 - \bar{\epsilon}\bar{\rho}/2 > 0$$

where we can write the bound on the error to be:

$$\|z\| \geq \left(\frac{\bar{\epsilon}\bar{\rho}}{2\lambda_{min}(\mathbf{Q})}\right)^{\frac{1}{2}} \tag{37}$$

□.

Similar controller can be applied to the case of the free motion before the manipulator makes contact with the environment. For this case, it can be shown that there also exist a lyapunov function candidate of the form $V_1 = z_1^T\mathbf{P}_1 z_1$ such that $\dot{V}_1 < 0$. As a result, in this region one can also have a *globally practically stable* closed-loop system.

At the point of discontinuity, the global asymptotic stability theorem of Lyapunov for continuous system is modified by replacing the derivative $\dot{V}(.)$ with the Dini-derivative $D^*V(.)$, where the $*$ represents any four possible Dini-derivatives (16,). At any point where the derivative $\dot{V}(.)$ exists, all four Dini-derivatives will have a common value equal to the derivative at that point.

For the points in S, we must look at Dini-derivatives. All the discontinuities are simple, therefore both left and right limiting values of the derivatives of $V(.)$ exist outside of S. The Dini-derivatives are simply these limiting values. Since $\dot{V}$ is negative semi-definite for all points outside S, the Dini-derivatives are negative semi-definite for point in S (17,).

3.1. *Experimental Study of the Discontinuous System*

To demonstrated the feasibility of the proposed discontinuous controller, a series of experiments were conducted.

The computational system used was a PC 386 as the host and the spectrum 320C30 DSP board as the servocontroller. The user writes the control code on the DOS development system in C, com-

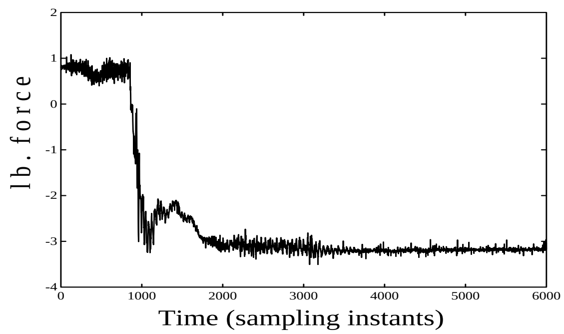

Fig. 8. The response of the discontinuous controller for manipulator approaching a compliant environment and then exerting -3 lbf on the environment. In this figure each sampling instances corresponds to $0.4ms$.

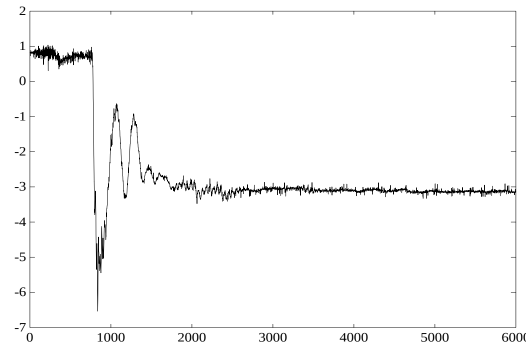

Fig. 9. The response of discontinuous controller for manipulator approaching a stiff environment and then exerting -3 lbf on the environment. In this figure each sampling instances corresponds to $0.4ms$.

piles and links the program and down-loads it to the DSP for execution. During runtime, the DSP executes without interference from the host. The DSP has dual-ported memory, which means that the PC can access this memory for reading or writing without interrupting the DSP, and vice-versa. The trajectory planning of the manipulator was done on the host computer, which calculated the reference set points for the controller to follow. These references were then passed on to the DSP controller. Two NSK Megatorque direct drive motors were used as actuators for the two links. The motors are capable of delivering high torque (maximum torque=249 N.m), have low friction, and come equipped with accurate angular position sensor (153,600 counts/rev) A six axis (ATI 15/50) force/torque sensor was used for measuring the contact forces. This six axis sensor can read forces upto 15 pounds and has a maximum sampling frequency of 2500 Hz (this is the rate that is used in the experiments). The links of the manipulator are made of aluminum with a semi-I cross sections which can reduce the flexibility of the links to zero (very stiff link construction).

The objectives of the experiment were to investigate the performance of the discontinuous model of the system defined in equations (15) and (16) where the controller is switched from the none-contact phase to contact phase upon the detection of contact. For these experiments, the gains and switching instances are selected a priori. In all of the experiments the motion of the manipulator is along a single axis of the end-point frame.

Figure (6) shows the response of a discontinuous controller for the case when approaching a compliant environment (i.e. the environment is constructed to be wooden cantilever beam with flexibility) and Figure (8) shows the response of the controller for the case of approached a very stiff environment (i.e. a rigid wall).

4. Discussions and Future Work

Application of the robotic system to various tasks involves the inevitable phase transition from the unconstrained motion to the constrained one. Controllers proposed in the literature in general switch the control action from the position control mode to the force control mode upon detection of the instance of contact and hence resulting in a dynamical system with a discontinuous forcing function.

This paper presented a natural frame-work for studying the performance of such robotic contact tasks where the action of the controller is discontinuous. The frame work is based on the Filippov's notion of the differential inclusions and Clarke's notion of the generalized gradient. Some preliminary modelling of the closed-loop system was presented. It is remain to analytically show the stability bounds of the closed-loop discontinuous system equation (17).

Some experimental results have been presented to demonstrate the feasibility of such controller. For these experiments, the gains of the controller have been selected for each phases of the contact task. The result shows the practicality of such controller.

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5. REFERENCES

[1]Kazerooni, H., Waibel, B. and Kim, S.,*On the stability of Robot Compliant Motion Control: Theory and Experiments*, Transaction of

ASME, Journal of Dynamics Systems Measurements and Control, Vol. 112, pp. 417-425, 1990

[2]Mills, J. and Lokhorst, D., *Control of Robotic Manipulators During General Task Execution: A Discontinuous Control Approach*, International Journal of Robotics Research, Vol. 12, No. 2, April, 1993, pp. 146-163

[3]Payandeh, S., *A Method for Controlling Robotic Contact Tasks*, Robotica, Vol. 14, 1996, pp. 281-288

[4]Hyde, J. and Cutkosky, M., *Contact Transition Control: An Experimental Study*, Proceedings of International Conference on Robotics and Automation, 1993, pp. 363-368

[5]Marth, G., Tarn, T. J., and Bejczy. A.K.,*Stable Phase Transition Control for Robot Arm Motion*, Proceedings of International Conference on Robotics and Automation, 1993, pp. 355-362

[6]Payandeh, S., *On the Effect of Compliance in Robotic Contact Tasks Problem*, Proceedings of 1995 American Control Conference, pp. 387-391

[7]Colgate, E., Hogan, N, *An Analysis of Contact Instability in terms of Passive Physical Equivalents*, Proceedings of 1993 IEEE Robotics and Automation Conference, pp. 404-409

[8]Eppinger, S. and Seering, W. ,*Three Dynamic Problems in Robotic Force Control*, IEEE Transaction on Robotics and Automation, Vol. 6, pp. 751-758, 1992

[9]Volpe, R. and Khosla, P., *A theoretical and Experimental Investigation of Impact control for Manipulators*, International Journal of Robotics Research, Vol. 12, pp. 351-365, 1993

[10]Mandal, N. and Payandeh, S., *Control Strategies for Robotic Contact Tasks: An Experimental Study*, Journal of Robotic Systems, Vol. 12, No. 1, pp. 67-92, 1995

[11]Payandeh, S., *Loop gain definition in a Robust Force Control Problem*, Proceedings of 1993 American Control Conference, pp. 2499-2502

[12]Payandeh, S., *Causality and Robotic Contact tasks Problem*, IEEE Transaction on Systems, Man and Cybernetics, Vol. 22, No. 5, Sept. 1992, pp. 1210-1214

[13]Utkin, V., I., *Variable Structure with Sliding Modes*, IEEE Transaction on Automatic Control, Vol. AC-22, No. 2, pp. 212-222, 1977

[14]Siljak, D., *Nonlinear Systems, The Parameter Analysis and Design*, Wiley, New York, pp. 539-544

[15]Filippov,A.F., *Differential equations with Discontinuous right hand side*, American Mathematical Society Translations, Vol. 42, ser 2, pp. 199-231, 1964

[16]Rouche, N., Habets, P. and Laloy, M.,Stability Theory by Liapunov's Direct Method, Springer-Verlag, 1977

[17]Yoshizawa, T., *Stability Theory by Lyapunov's second Method*, The mathematical Society of Japan, Gakujutsutosho Printing Co., 1966.

[18]Sheviz, D. and Paden, B.,*Lyapunov Stability Theory of Nonsmooth Systems*, IEEE Transaction on Automatic Control, Vol. 39, No. 9, 1994, pp. 1910-1994

[19]Clarke, F. H., *Optimization and Nonsmooth Analysis*, A Wiley-Interscience Publication, 1983

[20]Payandeh, S. and Goldenberg, A., *A Robust Force Controller: Theory and Experiments*, Proceedings of IEEE International Conference on Robotics and Automation, pp. 36-41