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On Fitted Stratified and Semi-Stratified Geometric Manipulation Planning with Fingertip Relocations

Abstract

This paper presents two object manipulation planning methods based on fitted stratified and semi-stratified approaches using finger relocations. The problem is discussed in the framework of a motion planning problem. The goal of the methods is to steer an object from an initial configuration to a final configuration while it is possible to reposition the fingertips on the surface in a predefined way. We assume there is no rolling and sliding but finger relocations are allowed. The first technique follows a pure stratified approach, however unlike the previously published method, the exact kinematic model of the manipulation system is matched with a virtual model masking the behavior of the original system. This provides a simpler model than the earlier stratified method by reducing the generally hard symbolic computation problem to a simple (almost pure numerical) one. The paper also introduces a semi-stratified manipulation planning based on the newly defined fitted system. This second method enhances the stratified motion planning with a definition of systematic finger relocation sequence. The proposed decomposition is based on the selection of suitable reference contact points. As the main benefit, the method enables a greater freedom in defining the desired fingertip trajectories. The methods are illustrated through an example of object reorientation.

KEY WORDS—stratified motion planning, dexterous manipulation, fingertip relocations, differential geometry

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1. Introduction

The coordinated object manipulation problem is composed of a *manipulation planning* and a *controlled implementation*. This work is toward the manipulation planning based on stratified approaches whose eventual aim is to produce finger joint trajectories that achieve a desired grasp via driving the manipulated object to a desired configuration.

There exist three categories of manipulation task for multi-fingered hand systems: (a) *object manipulation* tries to reach the desired object configuration without considering the contact configuration; (b) *grasp adjustment* attains the desired contact configuration by disregarding the object configuration; and (c) *dextrous manipulation* leads the robot hand to its final state, taking into account the desired object and contacts configuration, respectively.

This paper is concerned with object and dextrous manipulation, that is, it intends to lead the object into a desired configuration while the fingertips reach the desired contact points on the surface, if it is possible. (It will be seen that the methods are restricted in a way to reach arbitrary contact points.)

All the three strategies mentioned above need a contact model to establish an unambiguous description between the object and the robotic hand. Several contact models are discussed for example in Murray et al. (1994). Most planning algorithms are concerned with either *point contact without friction*, *point contact with friction* or *soft finger* contact models. The planners also incorporate *fixed*, *sliding* and *rolling* contact points based on the friction model. A good overview of the manipulation philosophies can be found in Bicchi and Kumar (2000).

Fixed contact points offer a simple object manipulation philosophy where the location of the contact points does not change relative to the object frame (Chevallier and Payandeh 1995). To accomplish a grasp adjustment Hong et al. (1990) suggested a finger gaiting technique where the contacts are temporarily broken during the finger relocations. Restrictions arise when the fingers reach the limit of their workspaces. This situation then requires a regrasping algorithm (Han and Trinkle 1998).

The rolling contact is able to complete the object manipulation, grasp adjustment and dextrous manipulation task, respectively (Bicchi et al. 1998). Although this approach does not need extra regrasping strategies, it may need a relatively complex kinematic description. The literature on rolling contact manipulation is very extensive. Some recently proposed algorithms have been reported in Marigo and Bicchi (2000) and Bicchi et al. (1997). In general, the manipulation algorithm based on rolling contact expects fewer fingers than the dextrous manipulation combining the fixed point and the finger gaiting. However, if the fingers reach the limit of their workspace, rolling contact manipulation will also need a finger gaiting technique (Han and Trinkle 1998).

The sliding contact points can be adopted mainly for the grasp adjustment tasks. Planning controlled slippage for improving robotic dexterity has been studied in Cole et al. (1992) and Payandeh (1997).

We assume throughout the paper that the contact points are fixed, i.e., that sliding and rolling are not permitted. At the same time, we allow finger relocations from a point of the surface onto another one.

Dextrous manipulation involves many geometric, mechanical and control aspects which can offer different levels of coordination of a robotic hand. At the higher planning level, the algorithm attempts to handle globally the manipulation issue decomposing the problem into primitive, high level functions. One may consider this strategy for convex polyhedra and smooth objects (Rus 1999; Cherif and Gupta 1997; Gupta 2001) where contact points between the object and fingers are always maintained. Our proposed solution to the manipulation problem and other similar approaches (Omata and Farooqi 1996; Leveroni and Salisbury 1995) allow the repositioning of fingertips. Our paper approaches the manipulation problem from the point of view of open loop control which leads to a motion planning problem (MPP) with constraints. Our paper may be placed between higher planning manipulation planning and lower level manipulation strategies such as Yoshikawa (2000) and Zefran et al. (1996), where dynamic models are also regarded. In other words, lower level strategies require a kinematic or dynamic model of the system.

Most techniques have studied a simpler, kinematic model. The easier treatment comes from the fact that the kinematic model has no drift in the equations. It means that the equations of motion can be written in the form $\dot{x} = F(x)u$, where x is the state vector, and u is the input vector of the system. It occurs also in this paper. As mentioned above, the manipulation task can be transformed into a general MPP with constraints. Numerous motion planning algorithms (MPAs) have been studied for smooth systems. They mostly employ some machineries from differential geometry (Isidori 1996). The main challenge appears by nonholonomic systems since the control vector fields do not span the configuration space in this case, although the system may be controllable. Several approaches exist to solve the smooth MPP from different aspects (see, for example, Murray and Sastry (1993) and Kiss et al. (2000)). From the viewpoint of this paper, the method that pieces together a trajectory as a sequence of flows plays a key important role because it leads the system (for example a fingertip) along a specified trajectory. The sequence of the flows is defined by the vector fields of the system (Lafferriere and Sussman 1991; Sussmann 1992). The method implies a precise solution for MPP of nilpotent systems.

In most dextrous manipulation (Han and Trinkle 1998; Hong et al. 1990), object manipulation and grasp readjustment are accomplished separately. It is especially valid for dextrous manipulation planning using fixed contact points and finger relocation. The reason is that different constraints appear cyclically in the nonlinear system representing different equations of motion. Although all the vector fields are smooth, the compound system may not be smooth because discontinuous inputs can change the equations of motion of the system. *Stratified motion planning* (Goodwine 1998) offers a general approach for the system whose (smooth) equations may change in the configuration space. This method combines the object manipulation and the grasp adjustment tasks into a unified dextrous manipulation problem (Goodwine 1999). The key element of the method is to divide the configuration space into smooth submanifolds (strata) where, in each stratum, a different smooth nonlinear system is valid. The method compounds these systems into a smooth common system later called the *bottom stratified extended system* where, with some restrictions, smooth motion planning (MP) can be applied. Stratified MPA delivers a unified dextrous manipulation concept to solve this manipulation task. However, the approach has some drawbacks, where (sometimes very hard) symbolic computational difficulties are the most noteworthy (Goodwine 1999). In addition, it is also hard to interpret the resulting trajectory.

This paper is concerned with object and dextrous manipulation, where there is no rolling and sliding, but where finger relocation is allowed. We propose two new manipulation algorithms adopting stratified MPA but their foundation is a simple fictitious (fitted) system that reduces the complexity of the computations to almost pure numerical procedures. They also let one interpret the state trajectories easily from the results in the configuration space.

The first proposed method based on the philosophy of stratified motion planning (Harmati et al. 1999, 2000b) uses a special fictitious system called the fitted system. The special parameterization of the fitted system yields simple vector fields where one also can easily check any system's property (e.g. stratified controllability) This method is able to carry out a dexterous manipulation in a restricted workspace of fingertips. However, the method does not ensure automatically force closure stability and finger collisions.

The second proposed method is a semi-stratified motion planning on a fitted system using task decomposition (Harmati et al. 2000c). Beside stratified MP, this semi-stratified motion planning includes also a strategy for systematic finger relocations. The finger relocations are based on a subsegment generation procedure, which provides suitable chosen reference fingertip positions to a desired object motion. Hence, the method restricts the fingertip positions (i.e., arbitrary fingertip positions on the object cannot be achieved, causing dexterity in manipulation to fail), hence it aims primarily at object manipulation. In return, it is able to guarantee force closure stability and collision avoidance. Additionally, it provides a greater degree of freedom in finger relocation than fitted stratified manipulation because it allows any trajectory in the free space for the fingertips.

To summarize, we assume fixed contact points without sliding and rolling, however they can be relocated on the surfaces. Additionally, we assume a kinematic model of the manipulation system and force closure stability of the grasping. (The last one may be guaranteed by a sufficient number of fingers.) Beside these conditions, fitted stratified and semi-stratified manipulation is concerned in the frame of MPP using finger relocations.

The paper is organized as follows. An example of dextrous manipulation is defined in Section 2. Here, the robotic hand is equipped with four fingers and the goal is to manipulate an object of smooth surface. Section 3 gives an overview of stratified MP. Stratified MP is based on smooth MP and operates on sequences of flows. The solution of the smooth and stratified MPP consists of the sequence of flows along the vector fields of the system. The manipulation system will be investigated as a stratified system in Section 4. Some issues appearing in this section give motivation to develop new methods. We propose a new manipulation method based on the fitted model in Section 5. Its modified version, the semi-stratified manipulation, is presented in Section 6. Finally, simulation results are presented in Section 7, highlighting the different features between the proposed and the earlier stratified approach.

2. Manipulation Example

The results of this paper will be demonstrated using an example. This section introduces this manipulation system and presents some notations used in the remainder of the paper. The algorithms suggested in the next section are introduced via an egg-shaped object manipulation. The x, y and z coordinates of the surface points (Goodwine 1999) shown in

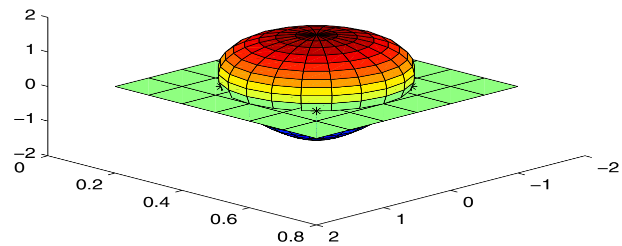

Fig. 1. The manipulated object with the 4 initial contact points (denoted by $*$) in the plane $z = 0$. The task is a rotation around $[1\ 0\ 1]^T$. (z axis is directed upwards.)

Figure 1 are parameterized by the equation

$$c(u, v) = \begin{pmatrix} (1 + \frac{u}{\pi}) \cos u \cos v \\ (1 + \frac{u}{\pi}) \cos u \sin v \\ \frac{3}{2} \sin u \end{pmatrix}, \qquad \begin{matrix} u \in (\frac{-\pi}{2}, \frac{\pi}{2}) \\ v \in (-\pi, \pi), \end{matrix} \tag{1}$$

where u and v are the parameters of the surface.

Let us turn our attention to a robot hand equipped with four fingers. Each finger has three degrees of freedom. The relation between a finger and the object is illustrated in Figure 2. The frame K_p denotes the palm frame and it is the inertial frame in the manipulation system. The object frame K_o is fixed to the object. Without loss of generality, we assume that the origin of the object frame K_o coincides with the origin of the palm frame K_p. Let the vector ω_o denote the angular velocity of the object frame relative to the palm frame, as seen from the palm frame. Similarly, let v_o denote the linear velocity of the object frame relative to the palm frame, as seen from the palm frame. The frames K_{f_i}, $i = 1, \ldots, 4$, are attached to the fingertips. Let the homogeneous transformation between the palm frame and the base frames of the finger be given by

$$T_{p0_{f1}} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_{p0_{f2}} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{p0_{f3}} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_{p0_{f4}} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In this case, the fingers divide the workspace into quadrants. This kind of arrangement is beneficial because the four fingers work in different quadrants of the workspace. The initial contact points are determined by the intersection of the object and the lines $x = \pm y$ in the plane $z = 0$, as shown in

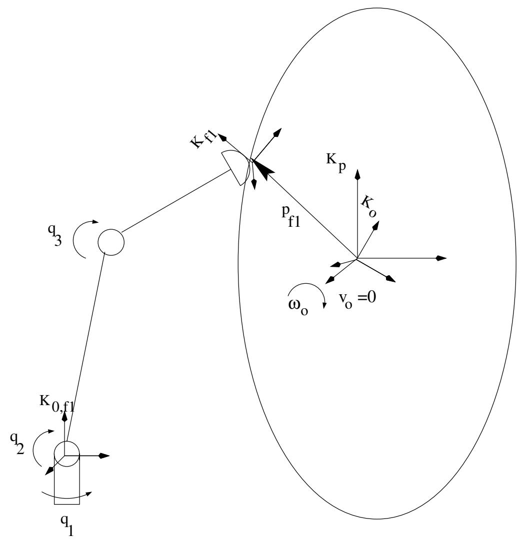

Fig. 2. The connection between a finger and the object.

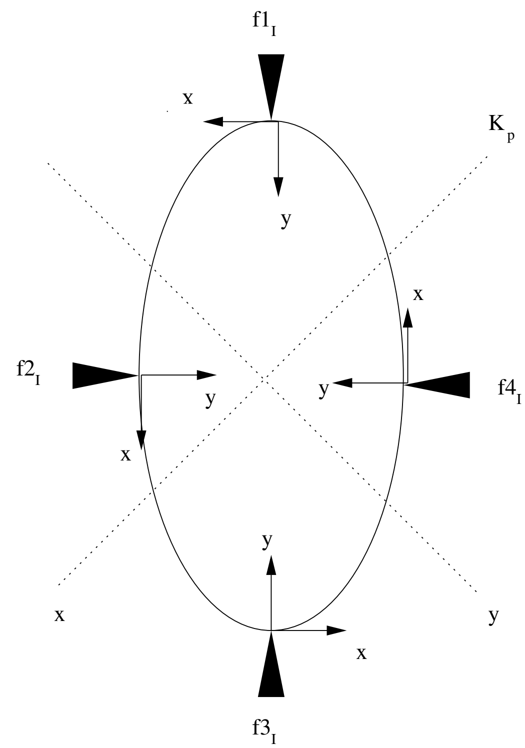

Fig. 3. The quadrants of the workspace.

Figure 1 (Figure 3). Considering local manipulation planning where the object and fingertip motion are sufficiently small, the quadrants ensure separated workspaces avoiding collision of the fingertips. In the case of global manipulation planning, the collision avoidance problem requires additional strategies. They will be discussed in Sections 5 and 6 in more detail. Their philosophy consists of inserting extra finger relocation back onto the lines $x = \pm y$ in the plane $z = 0$ if the fingertip is about to leave its own workspace (stratified manipulation) or one may carry out this finger relocation after a particular manipulation phase (see semi-stratified manipulation).

We suppose piecewise constant contact points or, more exactly, the contact model does not allow the fingertip to slide or roll on the object but one can carry out a fingertip relocation. In other words, during object motion, the fingertips in contact with the object keep their positions relative to the object frame K_o. Contact points with the above properties can be realized by the contact point with friction model (Salisbury and Mason 1985). Using this assumption, one describes the kinematics in the palm frame. Since the methods in this paper concern the kinematic problem, let J_{pf_i} denote the Jacobian matrix of the ith finger. The fingertip velocity of the ith finger is described by a reduced Jacobian matrix containing only those rows from J_{pf_i} which are related to the linear velocities. For the ith finger, the reduced Jacobian matrix that establishes a connection between the joint variables and the fingertip velocity (the linear velocity of the origin of K_f) in the palm frame K_p is given by

$$v_{pf_i} = J^v_{pf_i} \dot{\bar{q}}_i \tag{2}$$

where $J^v_{pf_i}$ has 3 rows and 3 columns. For example, a robotic hand equipped with the above homogeneous transformations claims that the Jacobian matrix of the first finger is given by

$$\begin{aligned}
J^v_{pf_1}(1,1) &= \sin(\bar{q}_1(1))(\sin(\bar{q}_1(2) - \bar{q}_1(3)) \\
&+ \sin(\bar{q}_1(2))) \\
J^v_{pf_1}(1,2) &= -\cos(\bar{q}_1(1))\cos(\bar{q}_1(2) \\
&- \bar{q}_1(3))\,(1 + \cos(\bar{q}_1(3))) \\
&+ \cos(\bar{q}_1(1))\sin(\bar{q}_1(2) - \bar{q}_1(3))\sin(\bar{q}_1(3)) \\
J^v_{pf_1}(1,3) &= -\cos(\bar{q}_1(1))\cos(\bar{q}_1(2) - \bar{q}_1(3)) \\
J^v_{pf_1}(2,1) &= -\cos(\bar{q}_1(1))(\sin(\bar{q}_1(2) - \bar{q}_1(3)) \\
&+ \sin(\bar{q}_1(2))) \\
J^v_{pf_1}(2,2) &= -\sin(\bar{q}_1(1))\cos(\bar{q}_1(2) \\
&- \bar{q}_1(3))\,(1 + \cos(\bar{q}_1(3))) \\
&+ \sin(\bar{q}_1(1))\sin(\bar{q}_1(2) - \bar{q}_1(3))\sin(\bar{q}_1(3)) \\
J^v_{pf_1}(2,3) &= -\sin(\bar{q}_1(1))\cos(\bar{q}_1(2) - \bar{q}_1(3)) \\
J^v_{pf_1}(3,1) &= 0 \\
J^v_{pf_1}(3,2) &= -\sin(\bar{q}_1(2) - \bar{q}_1(3))\,(1 + \cos(\bar{q}_1(3))) \\
&- \cos(\bar{q}_1(2) - \bar{q}_1(3))\sin(\bar{q}_1(3)) \\
J^v_{pf_1}(3,3) &= -\sin(\bar{q}_1(2) - \bar{q}_1(3)),
\end{aligned} \tag{3}$$

where $\bar{q}_1(j)$ is the jth joint variable of the first finger. Taking into account all the contact points we get the form

$$v_{pf} = \begin{pmatrix} v_{pf_1} \\ \vdots \\ v_{pf_4} \end{pmatrix} = \underbrace{\begin{bmatrix} J^v_{pf_1} & & \\ & \ddots & \\ & & J^v_{pf_4} \end{bmatrix}}_{J^v_{pf}} \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_4 \end{pmatrix} = J^v_{pf}\dot{q}. \tag{4}$$

The discussion of singularities is not a subject of this paper, hence one may insert the methods below into a global manipulation planning where singularities, collision avoidance, and force closure are investigated in a higher level (Vass et al. 1999). On the other hand, if v_o denotes the linear velocity and ω_o denotes the angular velocity of the object frame relative to the palm frame (as seen in the palm frame), then the fingertip velocities can be determined in an alternative way, namely,

$$\begin{pmatrix} v_{pf_i} \\ \omega_{pf_i} \end{pmatrix} = \begin{bmatrix} I & [-p_{pf_i}\times] \\ 0 & I \end{bmatrix} \begin{pmatrix} v_o \\ \omega_o \end{pmatrix}. \tag{5}$$

Introducing the notation

$$J_{po_i}(p_{pf_i}) = \begin{bmatrix} I & [-p_{pf_i}\times] \\ 0 & I \end{bmatrix} \tag{6}$$

and assuming that all the fingers are in contact, one extends the eq. (5) to all linear and angular velocities as

$$\begin{pmatrix} v_{pf_1} \\ \omega_{pf_1} \\ \vdots \\ v_{pf_4} \\ \omega_{pf_4} \end{pmatrix} = \underbrace{\begin{bmatrix} J_{po_1}(p_{pf_1}) \\ \vdots \\ J_{po_4}(p_{pf_4}) \end{bmatrix}}_{J_{po}(p_{pf})} \begin{pmatrix} v_o \\ \omega_o \end{pmatrix}, \tag{7}$$

where $p_{pf} = (p^T_{pf_1}, \dots, p^T_{pf_4})^T$. For clarity, we write $J_{po_1}(p_{pf_i})$ as J_{po_1} and $J_{po}(p_{pf})$ as J_{po}, respectively. To express the fingertip velocities, one should eliminate the rows belonging to angular velocities ω_{pf_i} from J_{po_i}, $i = 1, \dots, 4$, in eq. (7). It implies a reduced matrix J^v_{po} instead of J_{po} resulting fingertip velocities expressed in the palm frame as

$$\begin{pmatrix} v_{pf_1} \\ \vdots \\ v_{pf_4} \end{pmatrix} = \underbrace{\begin{bmatrix} I & [-p_{pf_1}\times] \\ & \vdots \\ I & [-p_{pf_4}\times] \end{bmatrix}}_{J^v_{po}} \begin{pmatrix} v_o \\ \omega_o \end{pmatrix}. \tag{8}$$

From eqs. (4) and (8) we can write

$$\begin{pmatrix} v_{pf_1} \\ \vdots \\ v_{pf_4} \end{pmatrix} = \underbrace{\begin{bmatrix} I & [-p_{pf_1}\times] \\ & \vdots \\ I & [-p_{pf_4}\times] \end{bmatrix}}_{J^v_{po}} \begin{pmatrix} v_o \\ \omega_o \end{pmatrix} = \underbrace{\begin{bmatrix} J^v_{pf_1} & & \\ & \ddots & \\ & & J^v_{pf_4} \end{bmatrix}}_{J^v_{pf}} \begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_4 \end{pmatrix}. \tag{9}$$

From here, the relation between joint variables and object motion can be obtained as:

$$\begin{pmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_4 \end{pmatrix} = \left(J^v_{pf}\right)^{-1} J^v_{po} \begin{pmatrix} v_o \\ \omega_o \end{pmatrix}. \tag{10}$$

Equation (10) is used below (Sections 4 and 5).

3. Stratified MP

Smooth MPA (see the Appendix) does not work on a nonlinear system having discontinuously changing equations of motion. However, stratified MP (Goodwine 1998) extends the results of smooth MPA and overcomes the difficulty. In this section, a brief outline of the method is given.

DEFINITION 1. A set $\aleph \subset \mathbb{R}^n$ defined by union of smooth manifolds (i.e., strata) is said to be a *regularly stratified set*.

DEFINITION 2. The system is *stratified* if its configuration space is defined by regularly stratified sets.

In the strata, the system is represented by different, smooth nonlinear systems. The main problem is to develop an MPA that combines the different systems from different strata into a unified approach.

Let $S_0 \equiv M$ be the whole configuration space where there is no constraint. Let the stratum $S_i \subset S_0$, $i > 0$, be a codimension one submanifold where the system is subjected to a kinematic constraint. Roughly speaking, this stratum corresponds to dimension $n-1$ manifold in the configuration space. Let $S_{ij} = S_i \cap S_j$, where system is subjected to the two constraints presented on S_i and S_j. In general, a stratum where a few constraints may appear is denoted by $S_I = S_{i_1 i_2 \dots i_k} = S_{i_1} \cap S_{i_2} \cap \cdots S_{i_k}$ where $I = i_1 i_2 \dots i_k$ is a multi-index (Isidori 1996).

DEFINITION 3. The stratum with lowest dimension is said to be the *bottom stratum*.

DEFINITION 4. A stratum is called the *lower stratum* if its dimension is lower than the dimension of the other one. The *higher stratum* is defined vice versa.

EXAMPLE 1. (Strata dimensions.) Take a smooth object manipulation system equipped with 4 fingers where each finger has 3 degrees of freedom. Suppose that all the contact points are fixed to the object, i.e., there is neither sliding nor rolling. Denote the linear and angular velocity vectors by v_o and ω_o, respectively. Furthermore, let v_{f_i} denote the ith fingertip velocity. One can easily see that the tangent space of the whole configuration space S_0 is spanned by the vector $v_o \oplus \omega_o \oplus v_{f_1} \oplus \cdots \oplus v_{f_4}$ having dimension 18.

Suppose all the fingers $1, \ldots, 4$ are in contact with the object (i.e., they are glued to the object) so the system is subjected to 4×3 velocity constraints (4 vector constraints). One can easily check that the degrees of the freedom of the system are provided by the orientation and the position of the object. It defines a system that moves on the stratum S_{1234} having dimension 6. The stratum S_{1234} is the bottom stratum (with the most constraints and the lower dimension). The tangent space is spanned by $v_o \oplus \omega_o$ and the equation of motion can be described as

$$\begin{aligned} S_{1234} \quad : \dot{x} &= \begin{pmatrix} v_o \\ \omega_o \end{pmatrix} \qquad (11) \\ &= \quad f_1(x)u_1^{S_{1234}} + \ldots + f_6(x)u_6^{S_{1234}} = F_{1234}(x)u. \end{aligned}$$

Note that one can choose only 6 independent inputs because the desired object motion demands all the fingertips to move together with the object. These fingertips provide six degrees of freedom in accordance to the tangent vector $v_o \oplus \omega_o$.

Now, assume that the finger 1 breaks off contact with the object and moves freely in the workspace. Meanwhile, the other three fingers keep contact with the object. The new situation leads to a new stratum S_{234} with equation of motion

$$\begin{aligned} S_{234} \ : \ \dot{x} &= \begin{pmatrix} v_o \\ \omega_o \\ v_{f_1} \end{pmatrix} \qquad (12) \\ &= f_1(x)u_1^{S_{234}} + \quad \ldots \quad + f_9(x)u_9^{S_{234}} = F_{234}(x)u. \end{aligned}$$

Note that we have 9 inputs because the desired object motion demands that fingertips 2, 3 and 4 move together with the object. These fingertips provide six degrees of freedom. Finger 1 can move in the free space, rendering three additional degrees of freedom. The scenario is similar when another finger breaks off the contact instead of finger 1.

To study stratified MP, one can define a more general notion of the controllability and find conditions when the stratified system is controllable in this general sense.

DEFINITION 5. Take a system with state vector x and input vector u. Consider an open set $V \subseteq M$. Let $R^V(x_0, T)$ be the set of states which can be reached up to time T, i.e., the set of states x such that there exists a control $u(t)$, $0 \leq t \leq T$ that steers the system from $x(0) = x_I$ to $x(T) = x_F$ while $x(t) \in V$ for $0 \leq t \leq T$.

DEFINITION 6. Let

$$R^V(x_0, \leq T) = \bigcup_{0 \leq \tau \leq T} R^V(x_0, \tau) \qquad (13)$$

be the set of all states reachable from x_0 up to time T.

DEFINITION 7. Consider a system with a stratified configuration manifold and a collection of strata,

$$\{S_{I_1}, S_{I_2}, \ldots, S_{I_m}\}.$$

The system is *small time locally stratified controllable* if $R^V(x_0, \leq T)$ contains a neighborhood of x_0 in $\{S_{I_1} \cup S_{I_2} \cup \cdots \cup S_{I_m}\}$ and $T > 0$.

An important result from Goodwine (1998) gives a sufficient condition for the stratified controllability, as stated in the next theorem.

THEOREM 1. (Goodwine) Let $T_{x_0}M$ be the tangent space of M at x_0 and let $\bar{\Delta}_{S_j} \mid_{x_0}$ denote the involutive closures of a distribution spanned by the vector fields of a stratum S_j in x_0. If there exists a nested sequence of strata $x_0 \in S_p \subset S_{p-1} \subset \cdots \subset S_1 \subset S_0$, such that the involutive closures of distributions (of strata) fulfill $\sum_{j=0}^{p} \bar{\Delta}_{S_j} \mid_{x_0} = T_{x_0}M$, then the system is locally stratified controllable from x_0.

The control methods presented for smooth systems show a major challenge in the general MPP because different strata are described by different equations of motion. The systems defined on the separated strata are typically not controllable. The idea of stratified control is to define a common state space where all the vector fields can be considered from all the strata. This usually is associated with the bottom stratum because the typical initial and final configurations lie in this stratum. The following example assists in demonstrating the essential point of this approach.

EXAMPLE 2. (Stratified control concept.) Consider a finger gaiting system with two fingers whose configuration space is sketched by Figure 4. Using the convention of the notation for strata, S_0 symbolizes the total configuration space, and S_1 stands for the stratum where finger 1 is in contact with the object. Similarly, S_2 represents the stratum where finger 2 contacts the object. The most important stratum is the bottom stratum S_{12} where both fingers touch the object. Let the initial and the desired final points lie in the bottom stratum (in accordance with a typical manipulation problem). If one uses the fixed contact points model then the system will not be controllable in the bottom stratum S_{12}. In other words, one cannot solve a general manipulation task on S_{12} since every fingertip must be fixed to the surface in this stratum. However, the whole system can be stratified controllable and manipulatable if one puts into use the vector fields from the higher strata allowing the system to move along an extra direction (in higher stratum) as is illustrated in Figure 4. This results in the physical epiphenomenon of finger gaiting. The manipulation

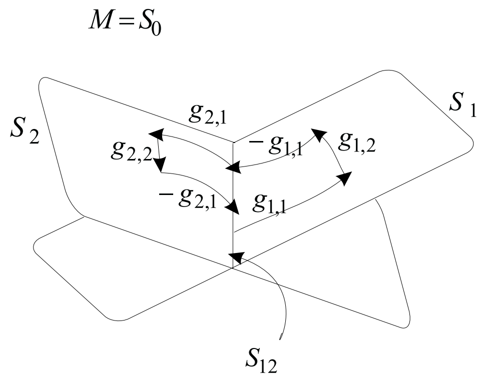

Fig. 4. Flow sequences in the stratified configuration space.

process is composed of a sequence of flows where vector field $g_{1,1}$ moves the system off from S_{12} onto S_1 (finger 2 disconnects the object), vector field $g_{2,1}$ moves the system off from S_{12} onto S_2 (finger 1 disconnects the object), $g_{1,2}$ is defined on stratum S_1, and $g_{2,2}$ is defined on stratum S_2.

DEFINITION 8. A vector field is said to be a *moving off* vector field if the existence of a contact between the finger and the object depends on it. In other words, a moving off vector field switches between strata.

DEFINITION 9. A vector field is said to be a *moving on* vector field at a given configuration x_0 if it does not leave the stratum where x_0 is defined.

PROPOSITION 1. (Goodwine) If the moving on vector fields commute with moving off vector fields (i.e., by definition, their Lie brackets are zero) then the flow sequence composed of them can be rearranged. The new order makes a simplification possible by eliminating some flows from the sequence.

COROLLARY 1. Switching between higher and lower strata is possible if the vector fields which lift off/on a finger from/to the object are decoupled from all vector fields defined on the substratum and higher strata (in other words, their Lie brackets are zero).

The benefit of the above statements is highlighted in the following example.

EXAMPLE 3. (Flow rearrangement) To illustrate the proposition, consider the stratified system of Example 2. The $g_{1,2}$ is a moving on vector field in S_1, $g_{2,2}$ is a moving on vector field in S_2. The $g_{1,1}$ and $g_{2,2}$ are the moving off vector fields of the system (they switch between the bottom stratum S_{12} and the higher strata S_1 or S_2). Let x_I and x_F be in the bottom stratum. Then the flow sequence

$$\begin{aligned} x_F \;=\; & \underbrace{\Phi^{t_6}_{g_{2,1}}}_{S_{12}\leftarrow S_2} \circ \underbrace{\Phi^{t_5}_{g_{2,1}}}_{\text{on } S_2} \circ \underbrace{\Phi^{t_4}_{g_{2,1}}}_{S_2\leftarrow S_{12}} \circ \underbrace{\Phi^{t_3}_{-g_{1,1}}}_{S_{12}\leftarrow S_1} \\ & \circ \underbrace{\Phi^{t_2}_{g_{1,2}}}_{\text{on } S_1} \circ \underbrace{\Phi^{t_1}_{g_{1,1}}}_{S_1\leftarrow S_{12}} (x_I) \end{aligned} \tag{14}$$

can solve the MP problem. If the condition of the proposition is satisfied, the moving on vector fields commute with the moving off vector fields, i.e.,

$$[g_{1,1}, g_{1,2}] = 0 \quad \text{and} \quad [g_{2,1}, g_{2,2}] = 0.$$

This means that $g_{1,1}$ and $g_{1,2}$ can be interchanged and so do $g_{2,1}$ and $g_{2,2}$. Then, this modifies the flow sequence (14) to

$$x_F = \Phi^{t_5}_{g_{2,1}} \circ \Phi^{t_6}_{g_{2,1}} \circ \Phi^{t_4}_{g_{2,1}} \circ \Phi^{t_2}_{g_{1,2}} \circ \Phi^{t_3}_{-g_{1,1}} \circ \Phi^{t_1}_{g_{1,1}}(x_I). \tag{15}$$

Assuming that $t_1 = t_3$ and $t_4 = t_6$, the sequence is simplified to

$$x_F = \underbrace{\Phi^{t_5}_{g_{2,1}} \circ \Phi^{t_2}_{g_{1,2}}}_{\text{on } S_{12}}(x_I), \tag{16}$$

where $g_{1,2}$ is defined on S_1 and $g_{2,2}$ is defined on S_2. However, one may evaluate them on the bottom stratum and then the flow sequence (16) will lead to the same result. In fact, if the $g_{1,2}$ and $g_{2,2}$ are in the tangent space of S_{12} then the flow sequence will remain in the bottom stratum.

COROLLARY 2. If the condition in Corollary 1 is satisfied, i.e., flow rearrangement is possible, then one can create a fictitious smooth system (defined later as the bottom stratified system) as a compound system of subsystems from different strata. This system is the starting point of stratified MPA.

Corollary 2 establishes a sort of connection between stratified and smooth controllability, based on Proposition 1. Exploiting the idea of flow rearrangement with the simplification shown in Example 3, the stratified motion planning algorithm is summarized as follows.

ALGORITHM 1. [Stratified Motion Planning (Goodwine 1998)]

Step 1. Determine the *multiple stratified system*. The equations of motion in the strata are

$$\begin{aligned} S_0 \;:\; \dot{x} &= g_{0,1}u^{0,1} + \cdots + g_{0,n_0}u^{0,n_0} \\ S_1 \;:\; \dot{x} &= g_{1,1}u^{1,1} + \cdots + g_{1,n_1}u^{1,n_1} \\ &\;\;\vdots \\ S_I \;:\; \dot{x} &= g_{I,1}u^{I,1} + \cdots + g_{I,n_I}u^{I,n_I}. \end{aligned}$$

Step 2. Create the *bottom stratified system*. It is clear from Theorem 1 and Proposition 1 that the stratified motion planning algorithm requires a special system on the bottom stratum (as a common space). Note, this system is also a fictitious system but without any connection to another fictitious system discussed by smooth motion planning (see the Appendix). In order to obtain this special (i.e., bottom stratified) system, one has to create a union set of vector fields in all strata and define with them a system

$$\begin{aligned} \dot{x} &= g_{0,1}u^{0,1} + \cdots + g_{0,n_0}u^{0,n_0} \\ &+ g_{1,1}\mid_{S_b} u^{1,1} + \cdots + g_{1,n_1}\mid_{S_b} u^{1,n_1} \\ &\vdots \\ &+ g_{I,1}\mid_{S_b} u^{I,1} + \cdots + g_{I,n_I}\mid_{S_b} u^{I,n_I}, \end{aligned} \quad (17)$$

where the notation $|_{S_b}$ refers to vector fields taking part in the bottom stratified system, however, they are defined originally not in this stratum. It should consist of all the moving on vector fields that commute with the moving off vector fields (i.e., the vector fields that disconnect a finger from the object). Note that a vector field is S_b means that one "cuts" some dimensions from the original vector field.

Step 3. Create the *bottom stratified extended system*

$$\begin{aligned} \dot{x} &= g_{0,1}u^{0,1} + \cdots + g_{0,n_0}u^{0,n_0} \\ &+ g_{1,1}\mid_{S_b} u^{1,1} + \cdots + g_{1,n_1}\mid_{S_b} u^{1,n_1} \\ &\vdots \\ &+ g_{I,1}\mid_{S_b} u^{I,1} + \cdots + g_{I,n_I}\mid_{S_b} u^{I,n_I} \\ &+ \textit{Lie brackets} \end{aligned} \quad (18)$$

including the vector fields of bottom stratified system and the Lie brackets among them. This system is the extension of bottom stratified system according to the way mentioned by smooth MPA in the Appendix.

Step 4. Solve the smooth MP on the bottom stratified extended system. The algorithm of the smooth MP on the bottom stratified extended system also solves indirectly (indirectly because it operates only with moving on vector fields) the stratified MPP. The result is a sequence of flows along the moving on vector fields specified by the following three sequences:

a. Sequence of moving on vector fields. A flow in the sequence is defined along the corresponding vector field in this sequence of moving on vector fields.

b. Sequence of time. The element of sequence defines how much time is needed to move along a vector field.

c. Sequence of inputs. Actually, the smooth motion planning algorithm (see the Appendix) sets the absolute value of the inputs to 1. However, we need to define a sequence that defines the sign of the inputs.

Step 5. Complete the solution for the stratified MPP. If two neighboring flows in the sequence are defined in different strata, one has to insert moving off vector fields between them in order to switch between their strata. The extra flows along the moving off vector fields have to assure that the set of satisfying constraints changes in accordance with the change of strata. It is possible on the base of Proposition 1 and Corollary 1.

4. The Description of the Manipulation Problem as a Stratified System

In this section, we intend to show that the manipulation system of Section 2 may be considered as a stratified system. Another goal is to describe the manipulation task in terms of stratification. Our intention is to provide more exact equations for strata that reflect the stratification. To study the problem in such a way, one should define the subsystems in the strata. Example 2 showed that the idea of finger gaiting fits the stratification discussion well. Following this idea, strata can be defined with different subsystems where each subsystem is subjected to a set of constraints depending on the fingers being in contact with the object. For the sake of simpler formulation, we adopt a treatment in the palm frame.

If all the fingers are in fixed contact with the object while the object is moving, the subsystem

$$\Sigma_{f,S_{1234}}: \quad \begin{pmatrix} v_o \\ \omega_o \\ \dot{p}_{pf_1} \\ \vdots \\ \dot{p}_{pf_4} \end{pmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \\ I & [-p_{pf_1}\times] \\ \vdots & \vdots \\ I & [-p_{pf_4}\times] \end{bmatrix} \begin{pmatrix} v_o^d \\ \omega_o^d \end{pmatrix} \quad (19)$$

realizes a possible manipulation phase. Here, $p_{pf_1}, \ldots, p_{pf_4}$ represent the fingertip positions. Indeed, $\Sigma_{f,S_{1234}}$ defines the system in the bottom stratum S_{1234}. The v_o^d and ω_o^d realize the desired linear and angular velocities of the object, however, they are also the (fictitious) inputs of $\Sigma_{f,S_{1234}}$. The physical inputs of the kinematic model are the derivatives of the joint variables $\bar{q}_i$. The transformation (10) makes a conversion between the fictitious and real inputs:

$$\begin{pmatrix} \dot{\bar{q}}_1 \\ \vdots \\ \dot{\bar{q}}_4 \end{pmatrix} = \left(J_{pf}^v\right)^{-1} J_{po}^v \begin{pmatrix} v_o^d \\ \omega_o^d \end{pmatrix}. \quad (20)$$

The use of fictitious inputs has the benefit that it leaves (19) in the form $\dot{x} = F(x)u$ instead of the more general $\dot{x} = F(x, u)$.

Recall that stratified MP in Section 3 was developed for a kinematic system given in the form $\dot{x} = F(x)u$.

Consider now a subsystem in a higher stratum. For instance, if all the fingers except finger 1 are in contact with the object, it assigns a stratum S_{234}. The equations of motion on this stratum yields $\Sigma_{f,S_{234}}$:

$$\begin{pmatrix} v_o \\ \omega_o \\ \dot{p}_{pf_1} \\ \vdots \\ \dot{p}_{pf_4} \end{pmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ I & [-p_{pf_2}\times] & 0 \\ I & [-p_{pf_3}\times] & 0 \\ I & [-p_{pf_4}\times] & 0 \end{bmatrix} \begin{pmatrix} v_o^d \\ \omega_o^d \\ \dot{p}_{pf_1}^d \end{pmatrix}, \tag{21}$$

where $\dot{p}^d_{pf_1}$ is the desired (unconstrained) fingertip velocity of the first finger. One may derive the connection between the fictitious input $((v_o^d)^T,\ (\omega_o^d)^T,\ (\dot{p}^d_{pf1})^T)^T$ and joint variables similarly to (20). The only difference is that the first finger is not subjected to a constraint now, hence

$$\begin{pmatrix} \dot{q}_1 \\ \hline \dot{q}_2 \\ \vdots \\ \dot{q}_4 \end{pmatrix} = \left[\begin{array}{c|c} 0 & {J^v_{pf_1}}^{-1} \\ \hline {J^v_{pf\{234\}}}^{-1} J^v_{po\{234\}} & 0 \end{array}\right] \begin{pmatrix} v_o^d \\ \omega_o^d \\ \hline \dot{p}^d_{pf_1} \end{pmatrix}, \tag{22}$$

where

$$J^v_{po\{234\}} = \begin{bmatrix} I & [-p_{pf_2}\times] \\ & \vdots \\ I & [-p_{pf_4}\times] \end{bmatrix} \tag{23}$$

and

$$J^v_{pf\{234\}} = \begin{bmatrix} J^v_{pf_2} & & \\ & \ddots & \\ & & J^v_{pf_4} \end{bmatrix}. \tag{24}$$

The equations of motion and the conversions due to other higher strata have similar forms, i.e., for instance, $\Sigma_{f,S_{134}}$:

$$\begin{pmatrix} v_o \\ \omega_o \\ \dot{p}_{pf_1} \\ \vdots \\ \dot{p}_{pf_4} \end{pmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ I & [-p_{pf_1}\times] & 0 \\ 0 & 0 & I \\ I & [-p_{pf_3}\times] & 0 \\ I & [-p_{pf_4}\times] & 0 \end{bmatrix} \begin{pmatrix} v_o^d \\ \omega_o^d \\ \dot{p}_{pf_2}^d \end{pmatrix} \tag{25}$$

and so on. The stratified system defined above will be denoted by Σ_f. Recall now that one may accomplish the stratified MPA if it satisfies the conditions of Proposition 1 and Corollary 1. As one of our contributions in this paper, we show that this condition is not satisfied in Σ_f.

PROPOSITION 1. The system Σ_f does not make possible switching between any two strata.

Proof. Consider the vector fields of the systems defined by (19) and (21). Define two groups of the vector fields by the last 3 vector fields of $\Sigma_{f,S_{1234}}$ and $\Sigma_{f,S_{134}}$

$$F_{on} = [f_1\ f_2\ f_3] = \begin{bmatrix} 0 \\ 0 \\ I \\ [-p_{pf_1}\times] \\ \vdots \\ [-p_{pf_4}\times] \end{bmatrix} \tag{26}$$

$$G_{off} = [g_1\ g_2\ g_3] = \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

(the size of each block is 3 by 3). By the definition of the moving on and moving off vector fields, the vector fields $f_1, \ldots, f_3$ are moving on vector fields, the vector fields $g_1, \ldots, g_3$ are moving off vector fields. One can find moving on vector fields from F_{on} to each moving off vector field from G_{off} such that the Lie brackets of the two vector fields are not zero. For example:

$$\begin{aligned} [f_2, g_1] &= [f_3, g_2] = [f_1, g_3] \\ &= \begin{pmatrix} 0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \end{pmatrix}^T \neq 0. \end{aligned}$$

Furthermore, these equations still hold if we define G_{off} as the last 3 vector fields of any higher strata. It means that based on Corollary 1, the stratified MP algorithm cannot switch between arbitrary two strata in the system Σ_f.

The switching problem arising in Σ_f motivated us to develop a new approach that fits well to stratified manipulation planning. This method is devised in Section 5. Additional motivation to elaborate a new technique is that the actual stratum identification requires us to check the contacts in Σ_f due to a finger. It needs tedious numerical calculation because the distances between the object and the fingertips do not appear directly in the state vector.

5. Fitted Stratified Manipulation Planning

This section concerns one of the two other main contributions of this paper. A new approach of stratified manipulation is proposed which is an extension of the above method. More precisely, our primary goal is to provide a description and a manipulation technique for the system outlined in Section 2 that permits the use of stratified MP in accordance with Corollary 1.

The idea is that fingertip position is described by a vector $p^o_{pf_i} = (u_i\ v_i\ z_i)^T$, where u_i and v_i describes the projection of the fingertip position onto the object along the normal vector of the surface (Harmati et al. 2001). The u_i and v_i are given as

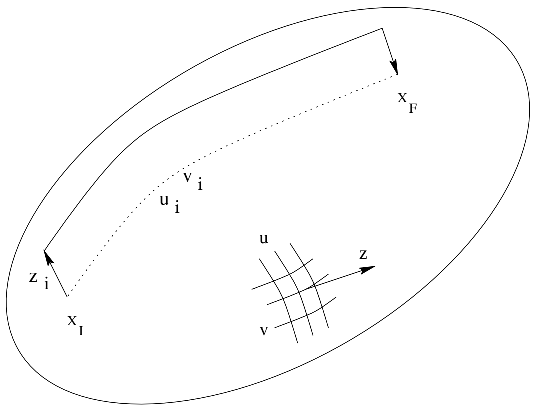

Fig. 5. Trajectory by finger relocation and its orthogonal projection onto the object.

the explicit parameters of the object surface. The z_i denotes the distance between the ith fingertip and the object (Figure 5). The superscript "o" refers to the object frame whose coordinates are defined by explicit parameters of the object. This description clearly defines moving on and moving off vector fields in the equations of motion. Such a description fits well to the stratified MP, hence we solve the MPP using these representations. However, this representation is a fictitious (called fitted) system in the sense that the real physical parameters (fingertip positions and joint variables) should be obtained via transformation. The method below includes a stratified MP on a fictitious system and transformations between the fictitious and the real physical systems.

Using the convention in notation above, the stratified manipulation system in Section 2 implies the following equation of motions on the fitted system.

In the bottom stratum, all the fingers are in contact with the object, i.e., the degrees of freedom come only from the object motion:

$$\Sigma_{fitted,S_{1234}} \ : \quad \begin{pmatrix} v_o \\ \omega_o \end{pmatrix} = I_6 \begin{pmatrix} v_o^d \\ \omega_o^d \end{pmatrix} \tag{28}$$

$\dot{v}_i = \dot{u}_i = z_i = 0, i = 1, \ldots, 4,$

where I_6 is the identity matrix with dimension 6. Similarly to (19), the v_o^d and ω_o^d realize the desired linear and angular velocities of the object and they are also the (fictitious) inputs of $\Sigma_{f,S1234}$. Recall that the fingertips are given in the object frame now with different coordinates, hence the transformation (10) between the fictitious and physical input (joint variables) should be modified. Let

$$p_{pf_i} = \text{object2palm}(Egg(u, v),\, p_{pf_i}^o) \tag{29}$$

be the function that transforms the object coordinates $p_{pf_i}^o$ to the palm coordinates p_{pf_i} if the object is of egg shape. Then, the physical inputs represented by the joint variables are obtained by

$$\begin{pmatrix} \dot{\bar{q}}_1 \\ \vdots \\ \dot{\bar{q}}_4 \end{pmatrix} = \tag{30}$$

$$\left(J_{pf}^v\right)^{-1} J_{po}^v(\text{object2palm}(Egg(u, v),\, p_{pf_i}^o)) \begin{pmatrix} v_o^d \\ \omega_o^d \end{pmatrix},$$

where $p_{pf}^o = ((p_{pf_1}^o)^T, \ldots, (p_{pf_4}^o)^T)^T$. As can be seen, the above equation emphasizes that J_{po}^v depends on p_{pf}^o and not on p_{pf} as in (6).

Let us turn our attention now to higher strata $S_{j_1 j_2 j_3}$ where the multi-index $j_1 j_2 j_3$ denotes fingers having contact with the object. The equations of motion on a higher stratum are given by

$\Sigma_{fitted,S_{j_1 j_2 j_3}}$:

$$\begin{pmatrix} v_o \\ \omega_o \\ \dot{u}_i \\ \dot{v}_i \\ \dot{z}_i \end{pmatrix} = \begin{bmatrix} I_6 & 0 \\ 0 & I_3 \end{bmatrix} \begin{pmatrix} v_o^d \\ \omega_o^d \\ \dot{u}_i^d \\ \dot{v}_i^d \\ \dot{z}_i^d \end{pmatrix} \tag{31}$$

$\dot{v}_{j_k} = \dot{u}_{j_k} = z_{j_k} = 0, \quad j_1 j_2 j_3 \in I_4,\ j_k \neq i, k = 1, \ldots, 3$ for any $i = 1, \ldots, 4$.

The transformation between the inputs of (31) and the joint variables is similar to (22). For instance, it leads on S_{234} to the transformation:

$$\begin{pmatrix} \dot{\bar{q}}_1 \\ \hline \dot{\bar{q}}_2 \\ \vdots \\ \dot{\bar{q}}_4 \end{pmatrix} = \left[\begin{array}{c|c} 0 & {J_{pf_1}^v}^{-1} \\ \hline {J_{pf_{\{234\}}}^v}^{-1} J_{po_{\{234\}}}^v & 0 \end{array} \right] \begin{pmatrix} v_o^d \\ \omega_o^d \\ \hline \dot{p}_{pf1}^{o,d} \end{pmatrix} \tag{32}$$

where $\dot{p}_{pf1}^{o,d}$ is the desired velocity of the first fingertip described by $(\dot{u}_1, \dot{v}_1, \dot{z}_1)^T$. Note, that $J_{po_{\{234\}}}^v(p_{pf_{234}}) = J_{po_{\{234\}}}^v(\text{object2palm}(Egg(u, v), p_{pf_{234}}^o))$ in accordance with (29).

Again, the superscript d in inputs refers implicitly to the fact that the derivatives of the state can be influenced directly by the desired values of the velocities as the inputs of the fictitious system Σ_{fitted}. The u_i, v_i in (31) determine the projection of the unconstrained fingertip motion on the object surface. The next proposition highlights the main benefit of our approach.

PROPOSITION 2. The fitted stratified model Σ_{fitted} makes the switching between two strata possible.

Proof. In order to justify the statement, we show that all the Lie brackets between a moving on and a moving off vector field are zero, i.e., they commute. At first, we show that the vector fields related to the variables z_i^d (e.g., last column in the matrix in (31)) are moving off vector fields and all the other vector fields are moving on vector fields. For this, one can see that the inputs z_i^d act only on the state variables z_i. However, the actual equations of motion (and the actual stratum) depend on the (not necessary only one) i for which $z_i = 0$. In other words, z_i^d determines the current stratum and at the same time, does not influence the state vector in the bottom stratum. In accordance with the definition of the moving off vector fields (see Definition 8), the vector fields related to the inputs z_i^d are moving off vector fields.

Furthermore, let e_i denote the unit vector

$$e_i = (0, \dots, 0, \underbrace{1}_{i^{th}\ pos.}, 0, \dots 0)^T \in \mathbb{R}^{18}. \tag{33}$$

Each vector field of system Σ_{fitted} in the whole configuration space is equal to e_i. It is an elementary result from the differential geometry that the Lie brackets $[e_i, e_j] = 0$. It holds for arbitrary two vector fields and it holds also for the Lie brackets defined between any moving on and any moving off vector fields. Based on Corollary 1, one can switch between two arbitrary strata in the system Σ_{fitted}.

COROLLARY 3. The stratified control algorithm can be applied to Σ_{fitted} by creating the bottom stratified (extended) system.

Let us see now how the fitted stratified manipulation planning works on Σ_{fitted}. The final goal in our manipulation planning is to obtain the joint variables of the fingers that steers the object into a desired position and orientation. At the final stage, the fingertips should reach desired contact points on the surface, as well. The stratified manipulation planning involves a stratified MPP in the configuration space. The equation of strata are given by (28) and (31). These subsystems define a bottom stratified fitted system that satisfies Corollary 1 and forms

$$\begin{pmatrix} v_o \\ \omega_o \\ \dot{u}_1 \\ \dot{v}_1 \\ \vdots \\ \dot{u}_4 \\ \dot{v}_4 \end{pmatrix} = \begin{bmatrix} I_6 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 \\ 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & I_2 & 0 \\ 0 & 0 & 0 & 0 & I_2 \end{bmatrix} \begin{pmatrix} v_o^d \\ \omega_o^d \\ \dot{u}_1^d \\ \dot{v}_1^d \\ \vdots \\ \dot{u}_4^d \\ \dot{v}_4^d \end{pmatrix}. \tag{34}$$

It is known from Section 3 that this bottom stratified (fitted) system is the foundation of the stratified MP used in our manipulation planning. In fact, the stratified motion planning on Σ_{fitted} can be reduced to a smooth MP (see the Appendix) on the bottom stratified fitted system (34). The vector fields of (34) are the moving on vector fields of Σ_{fitted}. Since the system is stratified, it is required to insert moving off vector fields where two flows of moving on vector fields from different strata meet at a point on the resulted trajectory. Smooth MP on (34) makes sense since the desired object position, orientations and fingertip positions are available. The only remaining problem is that the inputs of Σ_{fitted} are fictitious and not real. It means that one can accomplish the trajectory of stratified motion planning only if the fictitious inputs can be replaced with the joint variables. As has been mentioned, this is possible by using the transformations (30), (32) and so on.

The main steps of the method are illustrated in Figure 6 and are summarized as follows.

ALGORITHM 2. [Fitted stratified manipulation]

Step 1. Planning the motion for the object and the fingertips. The reference fingertips needs to be available in the object frame.

Step 2. Create the fitted system Σ_{fitted} (see (28) and(31)).

Step 3. Create the bottom stratified fitted system (34).

Step 4. Stratified motion planning (Algorithm 1) on fitted system. If the time schedule is important, one may use the considerations in Harmati et al. (2000a,b).

Step 5. Performing the finger relocations according to the insertion of moving off vector fields in Step 4.

Step 6. Transform the motion trajectory from the state space of the fitted system into the state space of the real system. One may use the geometric and kinematic transformations such as (30), (32). As a result, we obtain the joint variables (and their derivatives) for the desired manipulation.

Observe now some features generated by this approach. The main complication appeared in earlier stratified MPA (Goodwine 1999) when the Hall coordinates (see the Appendix) are computed. It proceeds via evaluation of recursive integrations. If the vector fields of the bottom stratified system have symbolically complex vector fields, the integrand becomes complicated. However, this is not the case for the fitted stratified system because the vector fields do not contain symbolic variables. It makes the computations very easy, almost entirely numerical.

As a consequence of simple vector fields, one can easily show the property of (stratified) controllability on the proposed fitted system with considerably fewer computation than in the earlier stratified approaches since all kind of computation with the constant vector fields are elementary in the differential geometry.

An additional advantage of the method in comparison with Goodwine (1999) is that one may directly interpret the resulting trajectory of the system since the fictitious inputs in (28), (31) etc. are strongly related to the object parameters. One can also see that the creation of an *extended* bottom stratified fitted system is unnecessary because the vector fields of the bottom stratified fitted system span the entire tangent space in a given configuration. In our example, the dimension of the

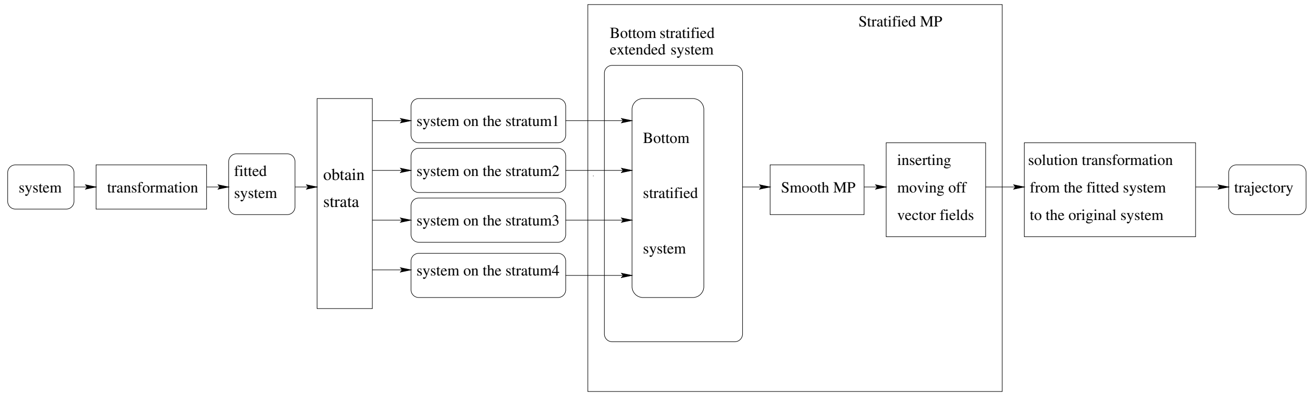

Fig. 6. The main steps of fitted stratified manipulation planning.

system (34) equals $3 + 3 + 4 \times 2 = 14$. Both the linear and angular velocity of the object take 3 coordinates. Additionally, every fingertip is described by 3 parameters, of which 2 determine the projection of the fingertip onto the object surface expressed by the exact parameters of the object. The third parameter does not take place in the bottom stratified fitted system but it is fixed to the moving off vector fields that lift the finger on and off the surface. Observe that every input in (34) affects only one state variable, i.e., the bottom stratified system is decoupled. It implies that the configuration space can be divided into subspaces where the strata assign disjoint subspaces (see the vector fields of (34). It poses the possibility of a stratified manipulation/motion planning where the trajectory may evolve locally on a submanifold whose dimension is greater than one. This means that the projection of fingertips may follow a special trajectory on the object surface. (Recall that the actual stratified MP relies on a smooth MP where the system trajectory evolves along sequence of flows, i.e., it is locally restricted to one dimensional submanifolds. It means that arbitrary fingertip trajectory cannot be prescribed, only a final state can be reached.) We intend to study this possibility in future work.

The technique introduced also possesses a main drawback. Namely, we need to execute extra transformations between the fictitious and real inputs. It needs to evaluate nonlinear transformations like (30) and (32) at each configuration. This means that the joint variables are not obtained directly but are derived.

Additionally, the method expects some reference data (fingertip positions) not in the palm frame but in the object (body) frame.

One more restriction appears when stratified manipulation is employed. Namely, the workspace was divided into four quadrants and it is assumed that all the fingers work in their own quadrant assuring collision avoidance. Unfortunately, stratified manipulation does not necessarily keep the fingers directly in their own workspace for arbitrary manipulation. If the reference fingertips are prescribed in their own workspaces, fitted stratified manipulation implies a dextrous manipulation. It can be done by choosing a desired final point close enough to the initial configuration. However, the most important requirement is that the object should reach the desired position and orientation, hence the fingertip positions play a relatively secondary role. In this special case, the problem should be overcome in the phase when the reference trajectory is generated. More exactly, if it turns out that a finger is about to leave its permitted workspace (quadrant) during the desired object motion, then one should insert an artifically generated finger relocation that pulls back the finger into its region, for example onto the lines $x = \pm y$ in the plane $z = 0$. Of course, it means that the reference fingertip positions must be modified. The semi-stratified manipulation planning in the next section is devoted to handling this problem.

6. Fitted Semi-Stratified Manipulation

In this section, we propose a decomposed manipulation concept for an object with smooth surfaces that combines fitted stratified motion planning with unconstrained motion planning. In this context, unconstrained motion means that a finger can move in the free space between two points independently of the object. The motivation to use unconstrained MP beside stratified manipulation may be useful for more reasons. One of them is to give a greater degree of freedom for finger relocations in manipulation planning. Second, it may be desired to dispose of one part of the complex computations appearing for instance in computation of Hall basis. Additionally, we would like to keep the fingers in their own workspaces independent of the object orientation.

If one considers the fitted stratified manipulation in Section 5, then a final configuration can be obtained as a sequence of 14 connected flows,

$$
\begin{aligned}
x_F &= \Phi(v^{o,d}_{pf_4}(2), t_1) \circ \Phi(v^{o,d}_{pf_4}(1), t_1) \circ \cdots \\
&\circ \Phi(v^{o,d}_{pf_1}(2), t_1) \circ \Phi(v^{o,d}_{pf_1}(1), t_1) \\
&\circ \Phi(\omega^{o,d}_{o}(3), t_1) \circ \cdots \circ \Phi(\omega^{o,d}_{o}(1), t_1) \qquad (35)\\
&\circ \Phi(v^{o,d}_{o}(3), t_1) \circ \cdots \circ \Phi(v^{o,d}_{o}(1), t_1)(x_I),
\end{aligned}
$$

where the first argument of flow Φ denotes the active input, and the second one defines the length of its active time interval. In addition, $v^{o,d}_{pf1} \equiv \dot{p}^{o,d}_{pf1}$, $v^{o,d}_{pf1}(1) = \dot{u}^d_1$, $v^{o,d}_{pf1}(2) = \dot{v}^d_1$ and so on. The first 6 flows (i.e., the last 6 in writing order) manipulate the object and every 3 flows following it belong to fingers. Note that active input uniquely assigns a moving on vector field from Σ_{fitted}. If two neighboring flows are defined in different strata, then insertion of the appropriate moving off vector field is required. It is done by activating inputs $v^{o,d}_{pfi}(3)$. It may occur already in the object manipulation part realized by flows $\Phi(\omega^{o,d}_{o}(3), t_1), \ldots, \Phi(v^{o,d}_{o}(1), t_1)$ that the fingertip positions exceed their allocated workspace.

To avoid this phenomenon, we pursue semi-stratified manipulation planning through several steps. The main steps are outlined in Figure 7.

In order to keep the fingertips in their own quadrants assuring their collision avoidance, the reference fingertip positions should be chosen in a suitable way. The *subsegment generation* algorithm concerns this issue, as follows. It is assumed that the four initial fingertips on the object surface lie on the lines $x = y$ and $x = -y$ in the plane $z = 0$.

Let us follow a manipulation phase now. At first, one fixes the contact points to the object and moves the object along the reference object trajectory. Before one of the fingertips is about to leave its quadrant (workspace), we record the object and fingertip configurations. This configuration will be the desired final configuration for the point to point fitted stratified MP. This configuration is reached by a sequence of flows (35).

At this point, we insert four *unconstrained finger relocations* that "pull back" the fingertips into the plane $z = 0$ where $x = y$ and $x = -y$ (see Figure 1). The fingers are relocated in order, i.e., only one of them moves back into the contact $z = 0$, $x = y$ or $x = -y$ at a time, the other three remain on the surface. After this, the following finger is relocated onto another contact point and the other three ones remain in contact with the object. While the fingertip is being relocated, it breaks the contact, moves in the free space and establishes a new contact in the above mentioned location. It can be shown (Goodwine 1999) that this kind of motion of the fingertips is able to ensure force closure stability. As a matter of fact, the unconstrained finger relocations are new features in fitted semi-stratified MP since it did not appear in fitted stratified MP. The unconstrained finger relocations allow us to steer the free fingertip using $J^v_{pf_i}$. In spite of fitted stratified motion planning, it is allowed to be more than one active input, so that the relocated fingertip has 3 degrees of freedom in the space whereas fitted stratified MP only had one. This means that one can define strategies which keep the fingers in their workspaces avoiding the fingertip collisions. Two simple ones are the following:

- Finger relocation with straight line subsegments. The trajectory is composed of three straight line subsegments. Determine two points above the object starting from the initial and the desired final points, in the direction of the normal vector of the surface. This can be a part of the overall planner. The two points must be at a distance from the object such that the straight-line connecting the two points does not intersect the object. The two points with the initial and final points define three joined straight-line subsegments.

- Finger relocation with constant distance from the object. The difference from the previous approach is in the trajectory between the two points above the object. It is a curve evolving in parallel manner to the surface of the object.

The whole manipulation procedure consists of subsegments snapping the object in the state where the fingertips does not leave their workspaces yet. Figures 8–10 illustrate the steps of the fitted semi-stratified algorithm if the manipulation task is a rotation around the axis $[1\ 0\ 1]^T$. Figure 8 shows the evaluation of reference contact points during the object motion. It means that the points marked by an asterisk belonged to $x = y$ and $x = -y$ in the plane $z = 0$ at the times when the finger relocations were needed. In fact, leaving a workspace is not the only reason to relocate a finger. It is better to choose relatively small subsegments because the convergence of smooth MP (see the Appendix) is guaranteed only under a critical distance between the initial and final configurations. Theoretical results are not known in general for the critical distance. Some experimental attempts can be found in Lafferriere and Sussmann (1991) and Harmati et al. (2000b). Consider now only a subsegment. Then, Figure 9 depicts the configuration where the phase of fitted stratified MP steers the system. After this, the fingers are relocated back into the plane $z = 0$ on the lines $x = y$ and $x = -y$. They assign four contact points. Finger 1 is relocated first, then finger 2, and so on. One configuration is snapped and illustrated in Figure 10. When all the fingers are relocated, the initial point of the new manipulation phase arrives (marked also by asterisk in Figure 8). The algorithm is summarized as follows.

ALGORITHM 3. [Fitted semi-stratified MP]

Step 1. Move the object through the desired path.

Step 2. Snap the object from time to time with small enough time steps so that the fingertips do not leave their quadrants.

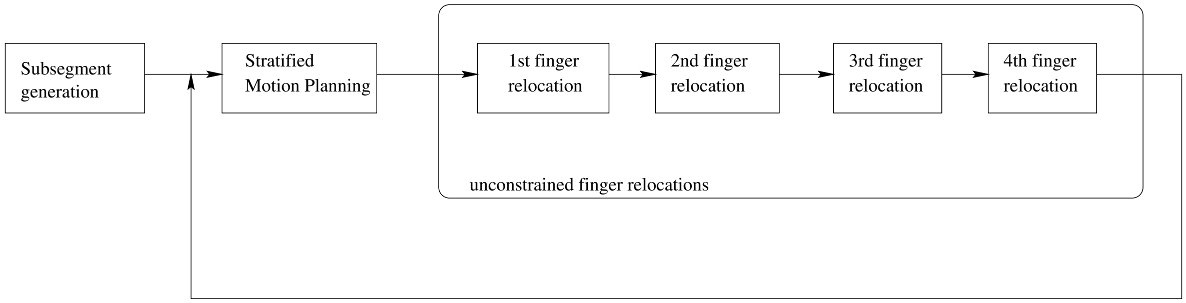

Fig. 7. The main steps of semi-stratified manipulation planning.

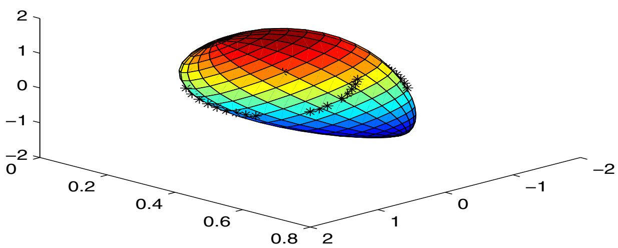

Fig. 8. The evolution of the planned contact points during the manipulation (axis z is directed upwards).

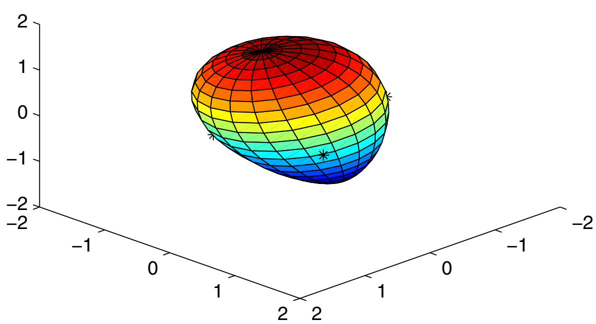

Fig. 9. The phase of fitted stratified manipulation (the object moves).

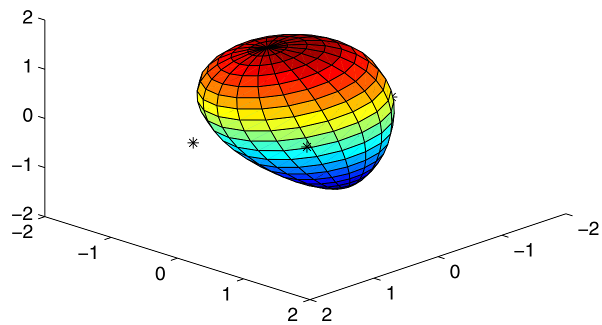

Fig. 10. The relocation of the first fingertip. (The phase of systematic finger relocation where the object does not move.)

Step 3. Consider two neighboring configurations. The first state represents the initial state $x_{o,I}$ of the object, the second one describes the final state $x_{o,F}$ of the object.

Step 4. Intersect the object in its initial state $x_{o,I}$ with the lines $x \pm y$ in the plane $z = 0$. Let the four contact points be denoted by $C_{i,initial}$, $i = 1, \ldots, 4$ (expressed in the object frame). The initial state: $x_I = x_{o,I} \oplus C_{1,initial} \oplus \cdots \oplus C_{4,initial}$.

Step 5. Define $x_F = x_{o,F} \oplus C_{1,inter} \oplus \cdots \oplus C_{4,inter}$ as the final state of the object manipulation phase. The intermediate hand configuration $C_{i,inter}$, $i = 1, \ldots, 4$ is prescribed by the manipulation task but in the simplest case (that is followed also in our treatment), one can choose $C_{i,inter} = C_{i,initial}$, $i = 1, \ldots, 4$ where the positions of the fingertips do not change relative to the object under the phase of object manipulation.

Step 6. Fitted stratified MP from x_I into x_F. If one chose $C_{i,inter} = C_{i,initial}$, then this phase is reduced only to an object manipulation and the fingertips do not change position on the object. (The phase of grasp adjustment is realized in the following steps.)

Step 7. Intersect the object in its final state $x_{o,F}$ with the lines $x \pm y$ in the plane $z = 0$. Let the four contact points be denoted by $C_{i,final}$, $i = 1, \dots, 4$.

Step 8. Relocate the finger 1 from $C_{1,inter}$ into $C_{1,final}$. This motion planning is carried out in a higher strata (in S_{234} where the motion of finger 1 is free). The actual hand configuration:
$x_{final1} = [C^T_{1,final}, C^T_{2,inter} \dots C^T_{4,inter}]^T$. In this and the following cases, only the free finger carries out a motion, the other ones stay in their state.

Step 9. Relocate finger 2 from $C_{2,int}$ into $C_{2,final}$. This motion planning is carried out in a higher stratum (in S_{134} where the motion of finger 2 is free). The actual hand configuration:
$x_{final2} = [C^T_{1,final}, C^T_{2,final} \dots C^T_{4,inter}]^T$.

Step 10. Relocate finger 3 from $C_{3,int}$ into $C_{3,final}$. This motion planning is carried out in a higher stratum (in S_{124} where the motion of finger 3 is free). The actual hand configuration:
$x_{final3} = [C^T_{1,final} \dots C^T_{3,final}, C^T_{4,inter}]^T$.

Step 11. Relocate finger 4 from $C_{4,int}$ into $C_{4,final}$. This motion planning is carried out in a higher stratum (in S_{123} where the motion of finger 4 is free). The actual hand configuration:
$x_{final4} = [C^T_{1,final} \dots C^T_{4,final}]^T$.

Step 12. Repeat the algorithm for the next subsegment.

It is worth concluding that fitted semi-stratified manipulation with its more global considerations (finger relocation strategy) belongs rather to a slightly higher level in the manipulation than the fitted stratified approach. It comes from the insertion of an extra phase consisting of four unconstrained finger relocations. These unconstrained finger relocations were separated from the fitted stratified manipulation. They substitute systematically one part (namely the finger relocation part) of the fitted stratified approach. As a result, one may ensure force closure stability, collision avoidance. However, one should choose the reference contact points in a restricted way (finger relocation when fingers are about to leave their workspaces). It results that the fitted semi-stratified manipulation accomplishes rather an object manipulation than a dextrous manipulation.

7. Simulation Results

We have proposed two MP algorithms for the object manipulation problem where the object is of smooth surface. The simulation results highlight the typical features. Let us consider a manipulation task consisting of object reorientation. Let the manipulation system be due to Section 2. Let the desired object motion be a rotation around the axis $[\ 1\ 1\ 1\ ]^T$ while the position p_x, p_y, p_z of the object frame does not change. Meanwhile, it is desired the final fingertip contacts to be in the plane $z = 0$ where the lines $x = \pm y$ intersect the object. It is a point to point MPP to fingers but a trajectory tracking problem for the object. Since the stratified and semi-stratified manipulation planning is able to handle only with point to point MPP, we should reduce the problem. To do so, we divide the object trajectory into connecting subsegments where the ends of the subsegments should lie on the reference trajectory generated by the prescribed rotation around axis $[\ 1\ 1\ 1\ ]^T$.

Let us turn our attention to the fitted stratified manipulation at first. As was discussed above, this method is not restricted in the reference fingertip positions. Theoretically, any fingertips can be reached at the end of the manipulation phase but, because of this, one should check if the fingers do not leave their allowed workspaces (the quadrants). Hence we assume that the manipulation task does not require the fingertips to leave their quadrant while they travel to their desired final state.

We prescribe now the desired fingertip positions in the following way. The desired final object configuration is determined in each subsegment from the desired object trajectory by snapping. At this state, we intersect the object with the $x = \pm y$ lines in the plane $z = 0$. It points out four fingertip positions. The object configuration and these 4 points give the desired final state for the fitted manipulation in this subsegment. Now, we carry out the next part of the desired object rotation phase. It will determine the desired final object states and the desired final fingertip position in the next subsegment and so on. Leading the object along the reference trajectory hypothetically, one can obtain the initial and final points to the fitted stratified manipulation in each subsegment. (The initial state is always the final state of the previous section.) The reference final fingertips are defined similarly, by means of a designed trajectory between initial and final state. The simulation results are illustrated in Figures 11–17. Introducing the notation $\omega_o = [\dot{\phi}_x\ \dot{\phi}_y\ \dot{\phi}_z]^T$ for the components of the angular velocity of the object, Figure 11 simulates the evolution of the orientation of the object during the manipulation.

In accordance with the philosophy of stratified control, only one fictitious input is active at a given time instant as seen in Figure 11. As a consequence, the object and fingers move along one vector field at any time. Since the fitted system is very simple, the nature of the motion is easily interpretable. More exactly, the object motion consists of a sequence of the rotation around the axis x, y and z which are performed after each other. The initial and final stages of this part of process are illustrated in Figures 13 and 14. As known from the discussion, one has to obtain the real input via transformations (30) and (32). The real inputs are the joint variables depicted due to finger 1 in Figure 12. Although only one fictitious input is active at a given time, it is not necessarily true for real inputs (i.e., for joint variables). One fictitious input is derived

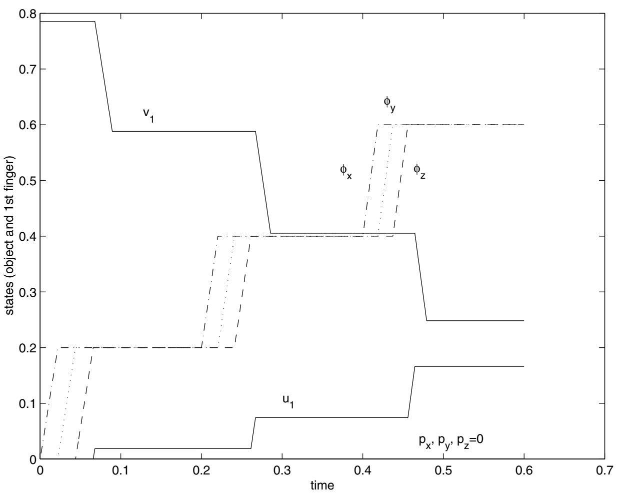

Fig. 11. The states related to the object orientation and the first fingertip position.

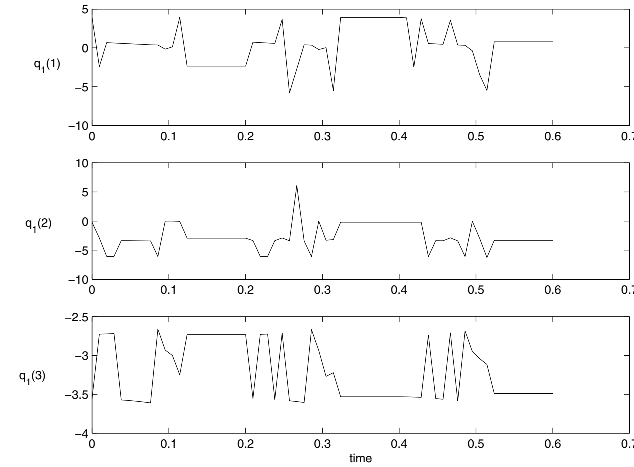

Fig. 12. The real inputs belonging to the joint variables of finger 1.

namely from more than one real input via a transformation. Comparing Figures 11 and 12, one observes that the most dynamic parts in the joint variables belong to finger relocation. The reason is that moving off vector fields are inserted at this time interval disturbing the smoothness of the characteristics and performing finger relocations. The finger relocations are carried out along distinct directions above the object. In the beginning, the finger arises from the object in the direction of the normal vector of the surface. Maintaining distance, the fingertip moves above the object. This motion consists of a motion along the explicit parameters of the object. It is very important that the finger moves along only one parameter at a given time instant. Furthermore, one can also see easily from the structure of the fitted system and the algorithm of the stratified control that the finger relocations are decoupled. While a finger is being relocated, the joint variables of other fingers do not change (see constant intervals in Figure 12). The process of two finger relocations during the manipulation is illustrated in Figures 15 and 16 (in fact, the other two fingers are also relocated in the same way). They snapped a state when one of the fingertips is being relocated, i.e., it is not in contact with the object. After the finger relocations, the manipulation is followed by a new local manipulation task in the next subsegment as before (Figures 17 and 18). The advantage of the demonstrated fitted stratified manipulation is simpler symbolic computation. As a matter of fact, the symbolic computation related mainly to Hall coordinates (see the Appendix) is reduced to almost pure numerical computations due to the simple vector fields. The fitted stratified MP enables us to interpret the fingertip motions directly from the result of its MPA and it is not needed to decode them from the joint variables.

It belongs to future work how to perform a finger relocation generally at arbitrary manipulation planning, when one finger is about to leave its own workspace. Although the quadrants as workspaces provide intuitively a reasonable structure to establish a grasp with force closure stability, the method does not ensure force closure stability automatically. It is required to choose reference contact points on the surface in an appropriate way. It is one of the reasons that motivated us to devise the fitted semi-stratified manipulation.

The simulation result of the manipulation using fitted semi-stratified manipulation leads to a similar simulation result as the fitted stratified MP. The difference between them is found only in finger relocations and the way of choosing the reference points. The semi-stratified motion planning is devoted primarily only to object manipulation because the fingertips are not chosen in an arbitrary way. For reference states, we adopt the procedure of the fitted stratified approach excepting one difference. The method has two main parts. The desired final states for semi-stratified manipulation in a subsegment during the object manipulation phase is given by the object motion with fixed contact points to the surface. Then the actual fingertip configurations will be the initial state of a second phase, namely, of unconstrained finger relocations. The prescribed final states for the unconstrained finger relocations are the contact points in the lines $x = \pm y$, $z = 0$ as before at the fitted stratified manipulation. So the fingertips are pulled back into $z = 0$. The semi-stratified approach sets apart a systematic finger relocation task from the object manipulation, ensuring collision avoidance and force closure stability (it can be seen similarly to Goodwine (1999)). At the same time, the fingertips cannot be relocated to an arbitrary place on the surface. The object motion implied by the semi-stratified approach coincides with the pure stratified approach (Fig-

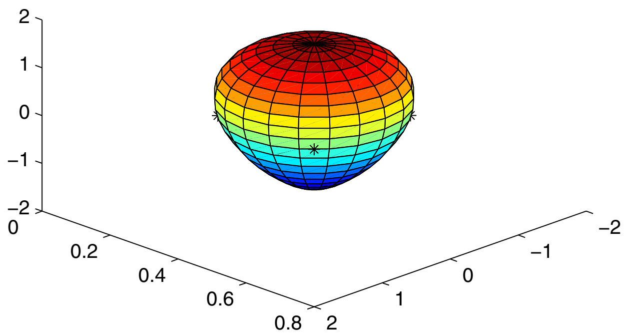

Fig. 13. Snap 1. The initial configuration.

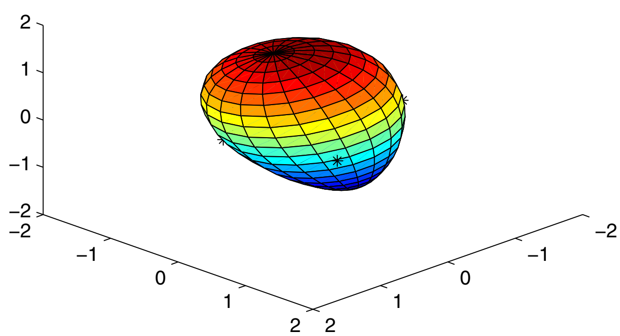

Fig. 14. Snap 2. Object manipulation phase in the bottom stratum.

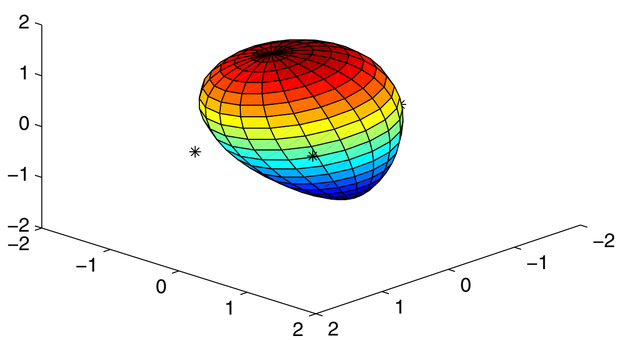

Fig. 15. Snap 3. Finger relocation in the higher stratum.

ures 13–18). However, the motion of the fingertips during the relocations is not necessarily the same. Semi-stratified control gives a greater freedom in choosing the path of the fingers when they are not in contact with the object. At the same time, this technique is not as unified as the (fitted) stratified MP.

In addition, one needs to solve an extra computational task in "free" space for relocating the fingers from a surface point

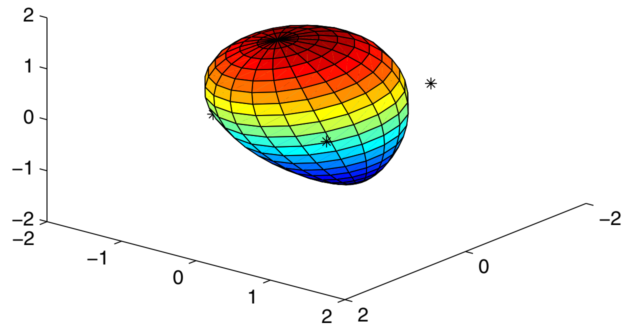

Fig. 16. Snap 4. Another finger relocation in another stratum.

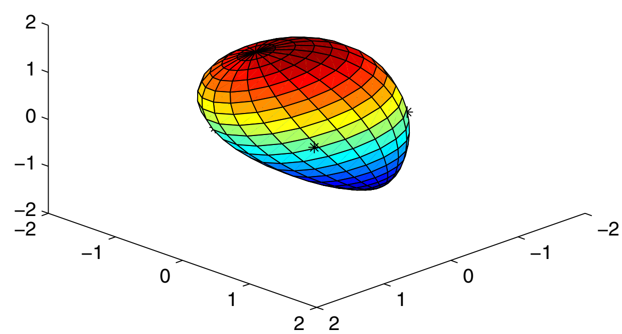

Fig. 17. Snap 5. The final configuration of fitted stratified manipulation in the first subsegment.

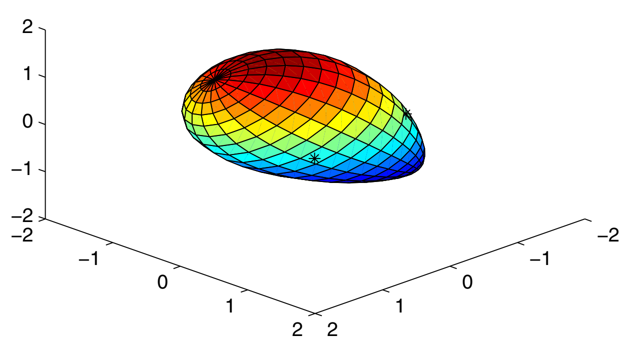

Fig. 18. Snap 6. The final configuration in the last subsegment.

to another one. It is also worth remarking that fitted semi-stratified manipulation can be applied specifically only for the manipulation problem posed in Section 2.

The software for fitted stratified and fitted semi-stratified manipulations has been implemented in MATLAB. Its program frame realizes smooth and stratified MPA. The implementation also handles the unconstrained finger relocation problem. The unconstrained finger relocation algorithm uses the simple approach summarized before, i.e., it puts 3 straight line trajectories together (detaching object, traveling above object, attaching object). The simulation is based on MATLAB's Control and Symbolic Toolbox. The software package is also able to assign time scaling to manipulation (Harmati et al. 1999, 2000b) that have been built in the basic stratified MP method (Goodwine 1998). The software does not handle the singular configurations.

8. Conclusions

This paper has proposed two manipulation methods based on the fitted model. Both of them rely on stratified motion planning and aim to reach a final configuration through the end points of connected subsegments composing the desired trajectory. The fitted system provides simple control vector fields. It reduces the complexity of the differential geometric computations playing an important role for motion planning algorithms based on symbolic treatments.

The method using stratified MP on a fitted system shows advantages in comparison to the earlier stratified motion planning, such as avoiding the hard symbolic computation, easily interpretable trajectory. It also assists in reducing the symbolic steps almost to pure numerical computations. The features are based on the special parameterization of the fitted system. Furthermore, it is easy to check a property of the fitted model such as the stratified controllability. The method cannot keep the fingertips automatically in their regions since we do not know anything about the trajectory between the initial and final points in a subsegment. It results that the realization needs extra functions on the higher level which check fingerip positions on-line and modify the specifications in a subsegment, if it is necessary. It assists in avoiding collisions. If the specifications of the manipulation task guarantees collision avoidance then the technique is able to perform an dextrous manipulation in that (fingertip and object) region, i.e., both the desired object position, orientation and the fingertip position become reachable. In return, one has to accomplish an extra transformation between the original and the fitted systems. This transformation related to Jacobian matrices of the fingers is to provide the physical control signals represented by the joint variables. Although the transformations require additional steps, their usage become useful because they make it possible to reduce the most complex parts of the symbolic computations. One drawback of the method appears in the issue of grasp stability. It is not guaranteed during the manipulation unless one takes into account higher level considerations such as the generation of appropriate reference fingertip trajectories.

The elimination of this drawback is one of the main motivations to fitted semi-stratified manipulation. The method differs at two main points from the above mentioned stratified version. First, it requires suitably chosen reference contact points. Second, it extends the fitted stratified approach with unconstrained finger relocations. The technique which provides appropriate contact points also restricts manipulation. Namely, dextrous manipulation becomes rather only object manipulation. However, it ensures collision avoidance and grasp stability at the same time. The unconstrained finger relocations in the semi-stratified approach realize a second phase in the manipulation after the fitted stratified manipulation is executed. It accomplishes a finger relocation algorithm with a greater freedom than the stratified approach. The finger relocations necessitate extra computations but they increase the degree of freedom in the fingertip motions. Namely, a finger can be relocated along an arbitrary path above the object while a fitted stratified approach should move the fingertip along only one certain direction above the object at a time.

The two methods result in the same object motions during the simulation but the fingertips followed different trajectories. The paper demonstrated the algorithms via a smooth object manipulation example using a four fingered robotic hand. The fitted concept may allow us to divide the configuration space of the bottom stratum into slices (subspace decompositions) where one can hopefully increase the freedom using more than one active input at any time. It motivates our future work to devise new methods toward a certain class of trajectory tracking. Our another intention is to extend the method for object manipulation where the object surface is not smooth but has edges.

Appendix

In the appendix, we give some basic definitions of differential geometry and provide a brief summary of smooth motion planning (Lafferriere and Sussmann 1991).

DEFINITION 10. (Control vector fields) Consider the nonlinear system given by

$$\Sigma \ : \dot{x} = u_1 g_1(x) + \ldots + u_m g_m(x) = G(x)u, \qquad x \in \mathbb{R}^n. \tag{A1}$$

The *control vector fields* of the system are defined by the vector fields $g_1, \ldots, g_m$.

In this paper the control vector fields play an important role because the proposed motion planning algorithms utilize them. The most important operation is the Lie bracket. By definition, the Lie bracket of two vector fields g_1 and g_2 is described by

$$[g_1(x), g_2(x)] = \frac{\partial g_2(x)}{\partial x} g_1(x) - \frac{\partial g_1(x)}{\partial x} g_2(x). \quad \text{(A2)}$$

The Lie brackets for a small time ϵ can be also interpreted by the sequence of flows via the Campbell-Baker-Hausdorff formula:

$$\Phi^{\epsilon}_{-g_2} \circ \Phi^{\epsilon}_{-g_1} \circ \Phi^{\epsilon}_{g_2} \circ \Phi^{\epsilon}_{g_1}(x) = \epsilon^2 [g_1, g_2](x) + \mathcal{O}(\epsilon^3). \quad \text{(A3)}$$

In the most cases, one adopts only the second order approximation. The Lie brackets have a crucial importance because they define "new directions" in which the system may move. They are also linearly independent of the vector fields they are derived from. Lie brackets satisfy the skew symmetry and the Jacobi identity supporting to define a Lie algebra.

DEFINITION 11. A vector space V (over $\mathbb{R}$) is a *Lie algebra* if there exist a bilinear operator $V \times V \rightarrow V$ denoted $[\cdot, \cdot]$, satisfying

1. Skew symmetry: $[g_2, g_1] = -[g_1, g_2]$
2. Jacobi identity:
   $[g_1, [g_2, g_3]] + [g_2, [g_3, g_1]] + [g_3, [g_1, g_2]] = 0$

where $g_1, g_2, g_3 \in V$.

DEFINITION 12. The Lie algebra generated by the control vector fields of the system is called the *control Lie algebra*.

To control a nonlinear system, it is important to find a basis for the control Lie algebra which points out all the possible directions for the trajectory in the configuration space. It is not elementary to generate this kind of basis because of the skew symmetry and Jacobi's identity. The Hall basis can be considered as a possible way.

DEFINITION 13. (Hall basis.) Given a set of vector fields $\{g_1, \ldots, g_m\}$, define the *degree of Lie product* (bracket) as

$$l(g_i) = 1 \quad i = 1, \ldots, m$$
$$l([X, Y]) = l(X) + l(Y), \quad \text{(A4)}$$

where X and Y may be Lie products, as well. A *Hall basis* is an ordered set of Lie products $H = \{B_i\}$ satisfying:

1. $g_i \in H, i = 1, \ldots, m$
2. If $l(B_i) < l(B_j)$, then $B_i < B_j$
3. $[B_i, B_j] \in H$ if and only if
   a) $B_i, B_j \in H$ and $B_i < B_j$, and
   b) either $B_j = g_k$ for some k or $B_j = [B_l, B_r]$ with $B_l, B_r \in H$ and $B_l \leq B_i$.

The Hall basis play a central role in the smooth MP achieved by sequence of flows. Before the exact mathematical treatment, it is worth discussing an overview of the idea. We want to steer a system (e.g., finger/object system) from an initial state x_I to a desired state x_F in the n dimensional configuration space.

If the control vector fields span the whole configuration space then it is a simple task since the vector fields support movement in any direction in the configuration space (planning the straight-line segment between x_I and x_F is the simplest one). This implies the need of n independent control vector fields.

However, in general, the system has less input controls than n (and at the same time the system may still be controllable). The key point is that extra possible moving directions are acquired if we move the system not only along the control vector fields but along their Lie brackets. Extending the system with these new directions and associating fictitious inputs to them, one can span the whole configuration space and solve the problem for this fictitious system.

MPP for the fictitious (i.e., extended) system is solved as a sequence of flows along the control vector fields (Figure 19). This specification implies restrictions on the solution for the MPP because the task is not so easy comparing with the case where one could move in any direction determined by the combination of n independent vector fields. In spite of the restrictions, the benefit of the procedure arises from the fact that it makes the transformation between the original and extended systems possible.

The only remaining problem is to replace these extra directions and control inputs with the original ones. If the system moves along only one of the all control vector fields at a given time then the substitution is not a difficult problem because, by definition, the extra directions (defined by Lie brackets) and their inputs can be substituted with appropriate original flow sequences.

Using this "flow sequence" philosophy, one can reach x_F exactly, if the system is nilpotent because, only in this case, each flow along any recursive Lie bracket can be developed exactly as a finite combination of the control vector fields.

A more detailed discussion now follows. Consider a smooth nonlinear system with m inputs which has no drift, i.e., $\Sigma\ : \dot{x} = u_1 f_1(x) + \ldots + u_m f_m(x) = F(x)u, x \in \mathbb{R}^n$. Assume that the vector fields f_i are real analytic and the system Σ is controllable.

DEFINITION 14. *Nilpotent Lie algebra* with order k is defined by Lie algebra L of the vector fields f_i where all the Lie brackets $[f_{j_1}, [f_{j_2}, \ldots, [f_{j_k}, f_{j_{k+1}}] \ldots]], j_i = 1, \ldots, m$ equal zero.

DEFINITION 15. The system Σ is said to be *nilpotent* if its control Lie algebra $L(f)$ is nilpotent.

If the system is nilpotent, each exponential of Lie brackets can be developed exactly as a finite combination of the control vector fields: such an operation can be done by using the Campbell-Baker-Hausdorff formula. The algorithm of the smooth MP outlined previously consists of the following steps.

ALGORITHM 4. (Smooth MP (Lafferriere and Sussmann 1991))

Step 1. System extension. Extend the system Σ to

$$\begin{aligned} \Sigma_e : \dot{x} &= v_1 f_1(x) + \ldots + v_m f_m(x) \\ &+ v_{m+1} f_{m+1}(x) + \ldots + v_r f_r(x) \\ &= F_e(x)v, \end{aligned} \quad (36)$$

where $span\{f_{m+1}(x), \ldots, f_r(x)\} = \mathbb{R}^n$ and vector fields $f_{m+1}, \ldots, f_r$ are defined by the elements of the Hall basis as higher order Lie brackets of the f_i, $i = 1, \ldots, m$.

In this step, we created a system (from the original one) whose vector fields support moving in any direction in the whole configuration space because we have at least n independent vector fields. Moving along a vector field can be realized by the corresponding fictitious input v. In general, some vector fields in the original and extended systems are the same, so their corresponding inputs will coincide as well.

Step 2. Steering the extended system Σ_e along any arbitrary path. Attain a control v that steers the extended system Σ_e from x_I to x_F. Since the vector fields of Σ_e span the whole configuration space $\mathbb{R}^n$, it is easy to plan the fictitious input v to the prescribed path. In general, one prescribes the simplest path, i.e., a straight-line trajectory segment in the configuration space between x_I and x_F. Based on this, one can obtain the fictitious control v from the equation $\dot{x} = F_e(x)v$.

Step 3. Solving the MPP for the extended system Σ_e as a sequence of flows. In the previous step, the fictitious control v solved the MPP for the extended system according to an arbitrary desired trajectory between x_I and x_F. In this step, we derive a special solution (from the fictitious control v) as a sequence of flows along the control vector fields of the extended system (the type of this solution is very important for the stratified MP). The step is broken into two parts:

I. Search the solution of the MPP for Σ_e as a sequence of flows. In other words, one has to solve the formal differential equation (Sussmann 1992)

$$\Sigma_{fe} : \quad (A6)$$
$$\dot{S}(t) = S(t)(v_1(t) f_1 + \ldots + v_m(t) f_m + v_{m+1}(t) f_{m+1} + \ldots + v_r(t) f_r)$$

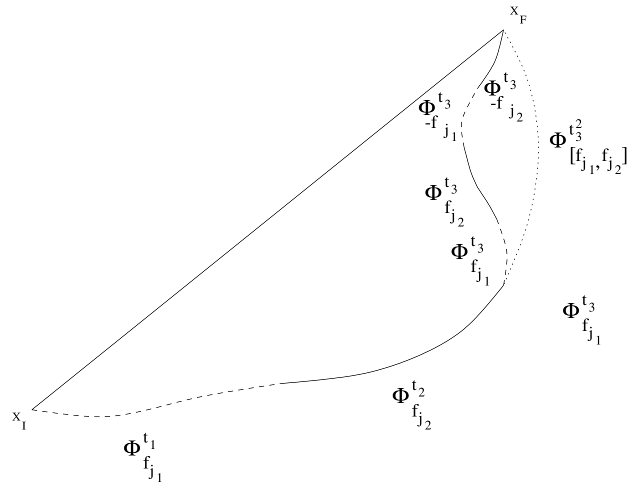

Fig. 19. Approximation of Lie bracket in a flow sequence.

on the Lie group in the form

$$S = e^{\tilde{h}_1 B_1} \cdots e^{\tilde{h}_{z-1} B_{z-1}} e^{\tilde{h}_z B_z} \quad (A7)$$

where B_i are the Hall basis, $\tilde{h}_i$ are the forward Hall coordinates. The notations in (A6) follow a special convention discussed briefly in Sussmann (1992). The solution S also solves the MPP between x_I and x_F. However, one cannot plan a desired trajectory between the two points. At first, the system moves for a time $|\tilde{h}_1|$ along the vector fields which is symbolized by the Hall base B_1 with $|v_1| = 1$ (the sign of v_1 is determined by the sign of $\tilde{h}_1$ and $v_i = 0$, $i \neq 1$). After this, the system moves along the next vector field represented by B_2 for a time $\tilde{h}_2$ while $|v_2| = 1$ and $v_i = 0$, $i \neq 2$ and so on up to the zth Hall base and coordinate. As a result, one gains a sequence of flows wthat leads eventually the system from x_I to x_F. Note that the differential equation (A6) is formal so the solution for the Hall coordinates will be formal as well. To obtain an exact solution where eq. (A7) has numerical Hall coordinates, we need to follow the next step.

II. Compute the Hall coordinates. If the degree of $B_i = 1$ for $1 \leq i \leq z$ then the corresponding forward Hall coordinate is given by

$$\tilde{h}_i(t) = \int_0^t v_i(s) ds. \quad (A8)$$

For the other case, B_i can be given in the recursive form $B_i = [B_j, B_k]$ determining the corresponding forward Hall coordinate by evaluating the recursive formal integrals

$$\tilde{h}_i(t) = \int_0^t (-v_j(s)\tilde{h}_k(s) + v_i(s))ds. \qquad \text{(A9)}$$

Step 4. Obtain the control u from the Hall coordinates. Actually, if the degree of a Hall base is more than one, then one should substitute the corresponding system motion with extra flow sequences along the original control vector fields. For example, a single Lie bracket substitution can be seen in Figure 19. Since the solution of MPP for the extended system Σ_e uses the vector fields $f_1, \ldots, f_r$ separately (i.e., there is only one active vector field at a time), we can substitute separately the flows along $f_{m+1}, \ldots, f_r$ with a sequence of flows consisting only of $f_1, \ldots, f_m$. For this substitution, one can use the Campbell-Baker-Hausdorff formula to each flow whose vector field is constructed by Lie brackets, i.e.,

$$\Phi^{\epsilon^2}_{[f_{i_1}, f_{i_2}]}(x_I) \approx \Phi^{\epsilon}_{-f_{i_2}} \circ \Phi^{\epsilon}_{-f_{i_1}} \circ \Phi^{\epsilon}_{f_{i_2}} \circ \Phi^{\epsilon}_{f_{i_1}}(x_I), \qquad \text{(A10)}$$

where, in general, $\Phi^{\epsilon}_{f_i}$ denotes the flow along a vector field f_i for a small time ϵ. In our application, f_{i_1} and f_{i_1} are represented by (any) two control vector fields of the original system Σ belonging to u_i's.

Following this procedure, every flow in the sequence will be defined *along precisely one* of the control vector fields of the original system.

REMARK 1. If the system is not nilpotent then the solution is only an approximation.

Nomenclature

$\bar{q}_i$ = the vector of joint variables belonging to the ith finger.

u_i, v_i = the coordinates of the projection of ith fingertip position onto the surface expressed by the exact parameters of the surface.

z_i = the distance of the ith fingertip from the surface.

u_i^d, v_i^d = fictitious inputs. The desired coordinates of the projection of ith fingertip position onto the surface expressed by the exact parameters of the surface.

z_i^d = fictitious input. The distance of the ith fingertip from the surface.

p_{pf_i} = the ith fingertip position in the palm frame.

p_{pf} = the vector of all the fingertip positions $p_{pf} = (p_{pf_1}, \ldots, p_{pf_4})^T$.

$p^o_{pf_i}$ = the ith fingertip position expressed by the object coordinates: $p^o_{pf_i} = (u_i, v_i, z_i)^T$.

$p^{o,d}_{pf_i}$ = the desired ith fingertip position expressed by the object coordinates: $p^{o,d}_{pf_i} = (u_i^d, v_i^d, z_i^d)^T$.

v_{pf_i} = the velocity of the ith fingertip as seen in the palm frame.

ω_o = the angular velocity of the object as seen in the palm frame.

v_o = the linear velocity of the object as seen in the palm frame.

ω_o^d = fictitious input. The desired (reference) angular velocity of the object as seen in the palm frame.

v_o^d = fictitious input. The desired (reference) linear velocity of the object as seen in the palm frame.

J_{pf_i} = the Jacobian matrix of the ith finger.

J_{pf} = hypermatrix whose diagonal consists of the Jacobian matrices of the fingers.

$J^v_{pf_i}$ = the reduced Jacobian matrix of the ith finger containing only the rows belonging to linear velocities.

J^v_{pf} = hypermatrix whose diagonal consists of the reduced Jacobian matrices (in order).

$J^v_{pf\{234\}}$ = hypermatrix whose diagonal consists of the reduced Jacobian matrices of fingers 2, 3 and 4.

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