

Additional Problems: Chapter Twelve: Probability Analysis

12S.1

To attract customers, Megalithic Bank offers a combined investment/gambling plan. You deposit your money, and a year later, the bank manager rolls a six-sided die. If a 1 comes up, you get back your money plus 10%. If a 2 comes up, you get back your money plus 20%, and so on, up to 5. But if he rolls a 6, he pays you no interest and confiscates your principal. What is the expected rate of return on deposits made under this scheme?

*12S.2

You are a merchant in 13-th century Baghdad. You invest 400 000 gold pieces in a fleet of four ships, which you plan to send out on year-long trading voyages to remote islands of the Indian ocean. Each ship costs 50 000 gold pieces to build. Every time a ship prepares to set out on a voyage, a further 50 000 gold pieces are needed to crew, outfit and provision it. You send the ships off at the beginning of the year, knowing that for each ship, there is a 25% chance that it will never return, a 25% chance that it will return empty at the end of the year, and a 50% chance that it will return laden with merchandise that you can sell for 250 000 gold pieces. What is your expected rate of return on your initial investment?

12S.3

You have a chance of being a contestant on a popular Japanese TV quiz show. The rules of the show are as follows: you are asked a series of questions. If you get the first question right, you get ¥100 000 and are allowed to continue. If you get the n -th question right, $1 \leq n < 10$, you get ¥100 000 times 2^{n-1} for that question, and you are allowed to continue. If you get the tenth question (worth ¥102 400 000) right, you get the ¥102 400 000, but you cannot continue further. If you get any question wrong, you are allowed to keep all the money you have won up to that time, but you do not get any additional money, and you have to stop playing. (If you do not speak Japanese, each question is quickly and accurately translated for you.)

Having watched the previous few contestants, you estimate that you have a 50% chance of getting the right answer to each question. (The questions don't get harder as you continue.) How much would it be rational for you to pay for the privilege of being a contestant on the show?

How much would it be rational to pay to be a contestant on the quiz show if there were no upper limit on the payouts, that is, if you were allowed to continue indefinitely as long as you kept getting the right answers?

***12S.4**

A communications company is looking for a launcher for a new communications satellite. They have collected data on the success rates of five national space programs -- the Russian, US, European, Japanese and Chinese -- and have quotes for launch costs from each agency. If the satellite is destroyed or lost on launch, insurance will cover the cost, but there will be a twelve-month delay before a replacement satellite can be ready. Some of the national space programs also have a habit of delaying commercial launches when a military payload needs to be put up. Once the satellite is launched, it will start generating revenue at a rate of \$100 000/month. The communications company has a pre-tax MARR of 2% per month, compounded monthly. Which agency should it choose?

Probability of Delay							
Agency		Launch Cost (\$000 000)	On Time	3 months	6 months	9 months	12 months
	US	1.20	0.5	0.20	0.10	0.05	0.15
	Russia	0.90	0.5	0.10	0.00	0.00	0.40
	PRC	0.75	0.4	0.15	0.10	0.05	0.30
	Europe	1.00	0.7	0.00	0.00	0.00	0.30
	Japan	1.50	0.9	0.05	0.00	0.00	0.05

12S.5

Your company produces a high-energy-density battery. Unfortunately, due to imperfections in the manufacturing process, an average of one battery in every ten thousand is defective. A defective battery doesn't just fail to work, it disintegrates messily, oozing a corrosive fluid. A second company uses your batteries to power portable communications devices. They buy your batteries in lots of 2 000/year, placing a repeat order every year. If they ever get a defective battery, they will never buy from you again. (Though they will not sue you to recover damages.) You could ensure they don't get defective batteries by putting each battery you produce through a complex test, but this will be expensive. If you make a profit of \$5.00 on every battery sold, and if your pre-tax MARR is 25%, what is the most it is worth spending to test a single battery? (This question is asking you to calculate the “expected value of perfect information”, a topic which is not covered in the text. Nevertheless, you know enough to be able to solve it.)

(Hint: if your answer is not between 0 and \$5.00, you've made a mistake.)

***12S.6**

(This problem is not, strictly speaking, about economics. It is designed to give insight into how the Monte Carlo method works by applying the method to a problem whose solution we know in advance.)

We are going to use the Monte Carlo method to estimate the value of π , the transcendental number which represents, among other things, the ratio of the area of a circle to the square of its radius. We will drop points at random into a square, and count how many of those points fall within the unit circle inscribed in the square.

To solve this problem, you will need either to write a short program in a programming language of your choice, or to make use of commercially available Monte Carlo software (see 'Net Value 12.1' in the text). If you write your own program, you will need to call a function that generates random numbers (See 'Close-up 12.2' in the text.)

Here's what the program or package should do:

Generate a pair of random numbers, each in the range $(-1, 1)$. You can think of these numbers as being the (x,y) coordinates of a random point in a square centered on the origin, with sides two units long. Add one to the total number of points (call this total ' I ').

See if $x^2 + y^2 < 1$. This test checks to see if the point is inside the circle, radius one, with centre at the origin. If the point passes this test, add one to the number of points that fall inside the circle (call this total ' J ').

If the total number of points has reached a pre-set limit, ' N ', say, calculate the ratio $4J/I$. This is your estimate of the value of π . N should be at least 100.

Carry on with this process up to a limit of MN , where M is 10, 100, or 1 000. Plot the value of your estimate of π against the total number of points $N, 2N, \dots MN$, and see how the accuracy of the estimate improves.

This exercise should give you an idea of how many iterations of the Monte Carlo method you need to get a desired accuracy in your estimate. Note that once you've set up the problem, it is no more difficult to do a million iterations than to do a hundred. (Though on the other hand, there's no point in trying to get an answer more accurate than the accuracy of your initial assumptions.)

12S.7

Your company is developing a palm-top computer. You and your partners in the business are arguing about when to put it on the market. One possibility is to rush the development and get it out next January. This will involve some technical compromises: you'll have to use an existing display, which will give you a heavy product. Market research indicates that this will be marginally acceptable as a palm-top; sales are expected to be between 40 000 and 80 000 units per year over the next four years. The variable costs of production are expected to be about \$5.00/unit. Selling price/unit is expected to be about \$15, plus or minus 50 cents.

You are working on an improved display unit which will be much lighter. If this works, you can expect sales of 50 000 to 100 000 units within the first year of sales (after which the market will be saturated, so sales in subsequent years will be in the 40-80 000 range). Variable costs of production are again expected to be about \$5.00, and the selling price/unit will be between \$20 and \$25. But there are significant technical problems to be solved; if you concentrate all your development efforts in this area, it will be between 18 and 36 months before you can have the product ready for market.

The fixed operating costs of your company are \$30 000/month. You will need to seek additional capital to survive until you have a product on the market. Depending on what rate you have to pay for capital, your pre-tax MARR may be anywhere between 20% and 30%.

Using the Monte Carlo method, or otherwise, write a brief report explaining what strategy you should follow, and why. Document and justify your assumptions carefully. In particular, you should explain

what study period you are using.