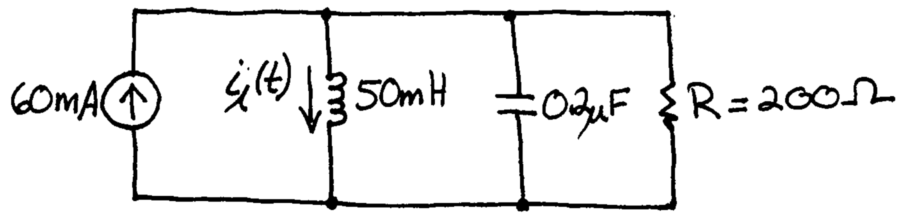
P 8.24



$i_L(0) = -45mA$, $V_C(0) = 15V$, $\frac{di_L(0)}{dt} = \frac{V}{L} = \frac{15}{50m} = 300 A/s$

$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\mu)} = 12{,}500 \text{ rad/s}$, $\alpha^2 = 1.5625 \times 10^8 (\text{rad/s})^2$

$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50m(0.2\mu)}} = 1 \times 10^4 \text{ rad/s}$, $w_0^2 = 1 \times 10^8 (\text{rad/s})^2$

$\alpha^2 > w_0^2$ $\therefore$ overdamped

$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -12500 + \sqrt{12500^2 - (1 \times 10^4)^2} = -5 \times 10^3 \text{ rad/s}$

$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2} = -12500 - \sqrt{12500^2 - (1 \times 10^4)^2} = -20 \times 10^3 \text{ rad/s}$

$i(t) = I_S + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$i_L(0) = I_S + A_1 + A_2$

$-45m = 60m + A_1 + A_2$

$\frac{di_L(0)}{dt} = S_1 A_1 + S_2 A_2$

$300 = -5000 A_1 - 20000 A_2$

$\Rightarrow A_1 = -4A_2 - 60m$

$-45m = 60m - 4A_2 - 60m + A_2$

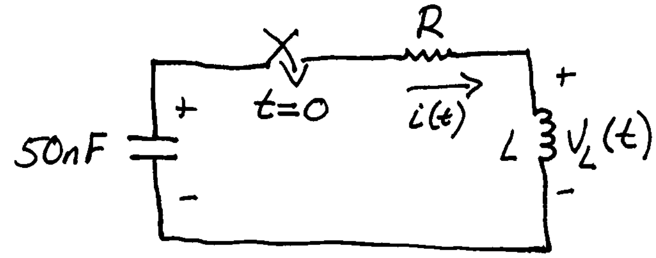
$-45m = -3A_2$

$A_2 = 15m$, $A_1 = -4(15m) - 60m$

$= -120m$

$i_L(t) = 60 - 120e^{-5000t} + 15e^{-20{,}000t}$ mA

P 8.35



- initial energy in capacitor is $90\mu J$

$S_1 = -1000\,s^{-1}$

$S_2 = -4000\,s^{-1}$

a) $s^2 + \frac{Rs}{L} + \frac{1}{LC} = 0$

$\frac{Rs}{L} = -s^2 - \frac{1}{LC}$

$R = -sL - \frac{1}{sC}$

$= 1000L - \frac{1}{(-1000)(50n)}$

$= 1000L + 20\times10^3$

$R = 4000L - \frac{1}{(-4000)(50n)}$

$= 4000L + 5\times10^3$

$1000L + 20\times10^3 = 4000L + 5\times10^3$

$-3000L = -15\times10^3$

$L = 5H \Rightarrow R = 1000(5) + 20\times10^3$

$= 25k\Omega$

b) $i_L(0) = 0$ since current cannot change abruptly in an inductor

$i_L(t) = i(t)$

$\frac{di(0)}{dt} = \frac{V_0}{L}$

initial energy in the capacitor is $90\mu J$

$w = \frac{1}{2}cv^2$

$\therefore$ initial voltage is $V_0 = \sqrt{\frac{2w}{c}} = \sqrt{\frac{2(90\mu)}{50n}} = 60V$

$\Rightarrow \frac{di(0)}{dt} = \frac{V_0}{L} = \frac{60}{5} = 12A/s$

c) $\alpha = \dfrac{R}{2L} = \dfrac{25k}{2(5)} = 2500 \text{ rad/s}$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5(50n)}} = 2000 \text{ rad/s}$$

$\omega_0^2 < \alpha^2 \therefore$ overdamped $\Rightarrow i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$$i(0) = A_1 + A_2$$
$$0 = A_1 + A_2$$
$$A_1 = -A_2$$

$$\frac{di(0)}{dt} = s_1 A_1 + s_2 A_2$$
$$12 = -1000A_1 - 4000A_2$$
$$12 = 1000A_2 - 400A_2$$
$$12 = -3000A_2$$
$$A_2 = -4\times10^{-3},\ A_1 = 4\times10^{-3}$$

$$i(t) = 4e^{-100t} - 4e^{-4000t} \text{ mA}$$

d) $\dfrac{di(t)}{dt} = -4e^{-1000t} + 16e^{-4000t} = 0$

$$16e^{-4000t} = 4e^{-1000t}$$
$$4 = e^{3000t}$$
$$\ln(4) = 3000t$$
$$t = 462.098\times10^{-6}\text{s}$$

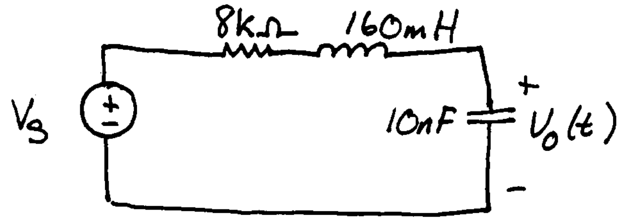
e) $i_{max} = i(462.098\mu s)$

$$= 4\times10^{-3}e^{-1000(462.098\mu)} - 4\times10^{-3}e^{-4000(462.098\mu)}$$
$$= 1.889881575\times10^{-3}\text{A}$$
$$= 1.89 \text{mA}$$

f) $V_L(t) = L\dfrac{di(t)}{dt}$

$$= 5\left(-4e^{-1000t} + 16e^{-4000t}\right)$$
$$= -20e^{-1000t} + 80e^{-4000t} \text{ V}$$

P8.45



$t \leq 0,\ V_s = 50V$

$t \geq 0,\ V_s = 250V$

$$\alpha = \frac{R}{2L} = \frac{8k}{2(160m)} = 25\times10^3 \text{ rad/s}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 25\times10^3$$

$\alpha^2 = \omega_o^2$ ∴ critically damped

$$V_o(t) = V_f + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

$$V_o(0) = 50V$$

$$V_o(t=\infty) = V_f = 250V$$

$$i = \frac{C\,dV_o(0)}{dt} = 0$$

$$V_o(0) = V_f + D_2$$

$$50 = 250 + D_2$$

$$D_2 = -200V$$

$$\frac{dV_o(t)}{dt} = D_1 e^{-\alpha t} - \alpha D_1 t e^{-\alpha t} - \alpha D_2 e^{-\alpha t}$$

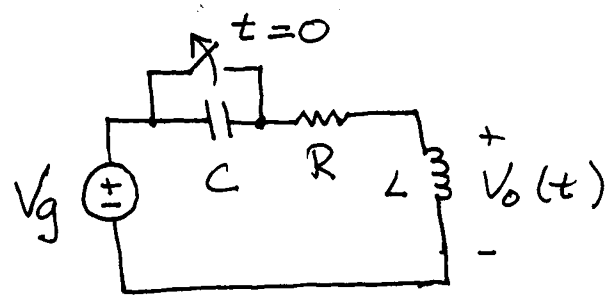
$$\frac{dV_o(0)}{dt} = D_1 - \alpha D_2$$

$$0 = D_1 - 25\times10^3(-200)$$

$$D_1 = -5\times10^6$$

$$V_o(t) = 250 - 5\times10^6 t e^{-25\times10^3 t} - 200 e^{-25\times10^3 t}\ V$$

P8.49



a) underdamped $\Rightarrow$ $\omega_o^2 > \alpha^2$

$$i(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$$

$$V_o(t) = \frac{L\,di(t)}{dt}$$

$$= L\left(B_1(-\alpha)e^{-\alpha t}\cos\omega_d t + B_1 e^{-\alpha t}(-\sin\omega_d t)\omega_d \right.$$
$$\left. + B_2(-\alpha)e^{-\alpha t}\sin\omega_d t + B_2 e^{-\alpha t}(\cos\omega_d t)\omega_d\right)$$

$$= L\left((-\alpha B_1 + \omega_d B_2)e^{-\alpha t}\cos\omega_d t + (-\omega_d B_1 - \alpha B_2)e^{-\alpha t}\sin\omega_d t\right)$$

- immediately after the switch is opened

$$i(0) = \frac{V_g}{R}\,A \qquad i(0) = B_1 = \frac{V_g}{R}$$

$$\frac{L\,di(0)}{dt} = V_o(0) = 0 \Rightarrow \frac{di(0)}{dt} = 0$$

$$\frac{di(0)}{dt} = -\alpha B_1 + \omega_d B_2$$

$$0 = -\alpha\frac{V_g}{R} + \omega_d B_2$$

$$B_2 = \frac{\alpha V_g}{\omega_d R}$$

$$i(t) = \frac{V_g}{R} e^{-\alpha t} \cos\omega_d t + \frac{\alpha V_g}{\omega_d R} e^{-\alpha t} \sin\omega_d t$$

$$V_o(t) = L\left(\left(-\alpha \frac{V_g}{R} + \cancel{\omega_d} \frac{\alpha V_g}{\cancel{\omega_d} R}\right) e^{-\alpha t} \cos\omega_d t\right.$$

$$\left. + \left(-\omega_d \frac{V_g}{R} - \alpha \frac{\alpha V_g}{\omega_d R}\right) e^{-\alpha t} \sin\omega_d t\right)$$

$$= -\frac{L V_g}{R}\left(\omega_d + \frac{\alpha^2}{\omega_d}\right) e^{-\alpha t} \sin\omega_d t$$

b) $$\frac{dV_o(t)}{dt} = -\frac{L V_g}{R}\left(\omega_d + \frac{\alpha^2}{\omega_d}\right)\left(-\alpha e^{-\alpha t} \sin\omega_d t + \omega_d e^{-\alpha t} \cos\omega_d t\right)$$

$$0 = -\frac{L V_g}{R}\left(\omega_d + \frac{\alpha^2}{\omega_d}\right)\left(-\alpha e^{-\alpha t} \sin\omega_d t + \omega_d e^{-\alpha t} \cos\omega_d t\right)$$

$$\alpha e^{-\alpha t} \sin\omega_d t = \omega_d e^{-\alpha t} \cos\omega_d t$$

$$\frac{\sin\omega_d t}{\cos\omega_d t} = \frac{\omega_d}{\alpha}$$

$$\omega_d t = \tan^{-1}\left(\omega_d/\alpha\right)$$

$$t = 1/\omega_d \tan^{-1}\left(\omega_d/\alpha\right)$$

P8.50

$R = 4800\Omega$, $L = 64mH$, $C = 4nF$, $V_g = -72V$

a) $V_o(t) = -\frac{LV_g}{R}\left(\omega_d + \frac{\alpha^2}{\omega_d}\right) e^{-\alpha t} \sin\omega_d t$

$$\alpha = \frac{R}{2L} = \frac{4.8k}{2(64m)} = 3.75 \times 10^3\ s^{-1} = 37.5ks^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{64m(4n)}} = 62.5 \times 10^3\ rad/s = 62.5\ krad/s$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(62.5\times10^3)^2 - (37.5\times10^3)^2} = 5\times10^4\ rad/s$$

$$V_o(t) = -\frac{6m(-72)}{4.8k}\left(5\times10^4 + \frac{(37.5\times10^3)^2}{5\times10^4}\right)e^{-37.5\times10^3 t}\sin(5\times10^4 t)$$

$$= 75e^{-37.5\times10^3 t}\sin(5\times10^4 t)V$$

b) $$t_{max} = \frac{\tan^{-1}(\omega_d/\alpha)}{\omega_d} = \frac{1}{5\times10^4}\tan^{-1}\left(\frac{5\times10^4}{37.5\times10^3}\right)$$

$$= 18.54590436\times10^{-6}s = 18.55\mu s$$

c) $V_{max} = V_o(t_{max}) =$

$$= 75e^{-37.5\times10^3(18.55\mu)}\sin(5\times10^4(18.55\mu))$$

$$= 29.93035398V$$

d) $V_o(t) = 601.08e^{-3750t}\sin(6.24\times10^4 t)V$

$t_{max} = 24.22\mu s$

$V_{max} = 547.92V$