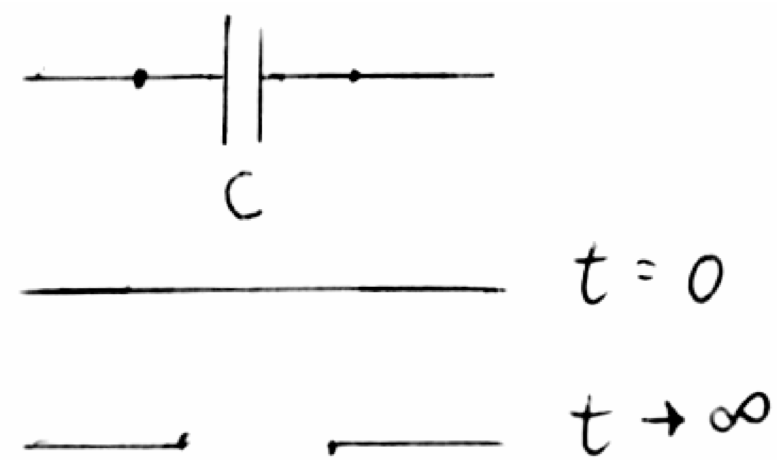
Capacitors Initial and Final Response to a "Step Function"

- Inductors and Capacitors react differently to a Voltage step
- Just after the step Capacitors act as a short if uncharged

$$I_C(t) = C\frac{dV}{dt}$$

- If charged Capacitor acts as an voltage source
- As time goes to infinity change in voltage goes to zero
- Then C act as an open (become fully charge or discharged)
- Thus can find initial and final conditions of circuit
- Use KVL with on circuit with these two models of the C’s

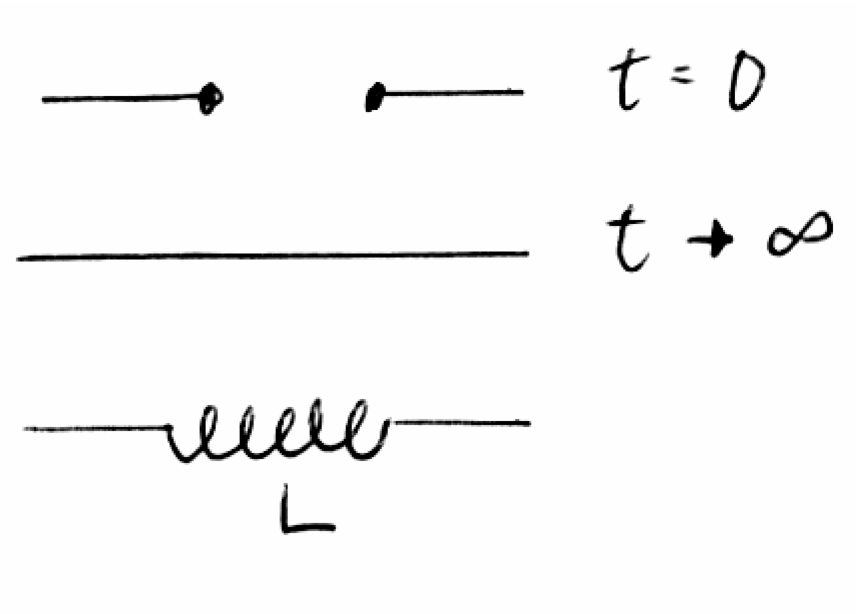


Inductor Initial and Final Response to a "Step Function"

- Inductors react differently to a voltage step
- Just after the step inductors act as opens
- Reason: opposes sudden change in current

$$v_L(t) = L\frac{di}{dt}$$

- As time goes to infinityInductors act as shorts
- Thus no more current change in final state
- Thus can find initial and final conditions of circuit
- Use KVL with on circuit with these two models of the L’s



General Solution Method of First Order Circuits

- General first order solution to a sudden change

(1) Use Kirchoff's laws for circuit equation

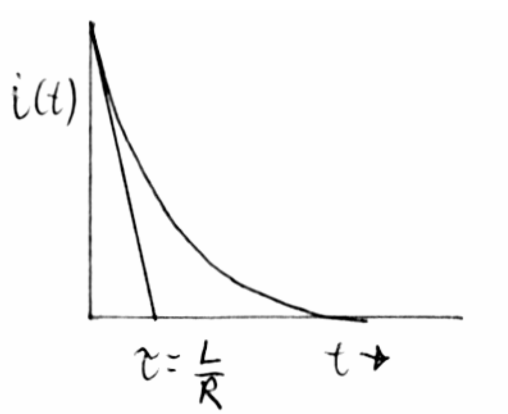
(2) Manipulate to get I or V in terms of derivates in time

(3) Generate the "Differential Equation Form"
- May need to differentiate to obtain

(4) Solve the Differential equation: 2 methods:
(4a) Integration method: get in integral form
- Integrate to get solution: eg RC circuit

(4b) Solution substitution method: assume a solution
- For step change assume exponential (or exp to constant)
- Solve for time constant

(5) Use initial or final conditions for constants of integration



Resistor Inductor Circuits

- Consider an inductor in series with a resistor.
- Called an LR circuit
- At time zero switch is opened

(1) Writing down KVL

Recall that the voltage across the inductor is

$$v_L(t) = L\frac{di}{dt}$$

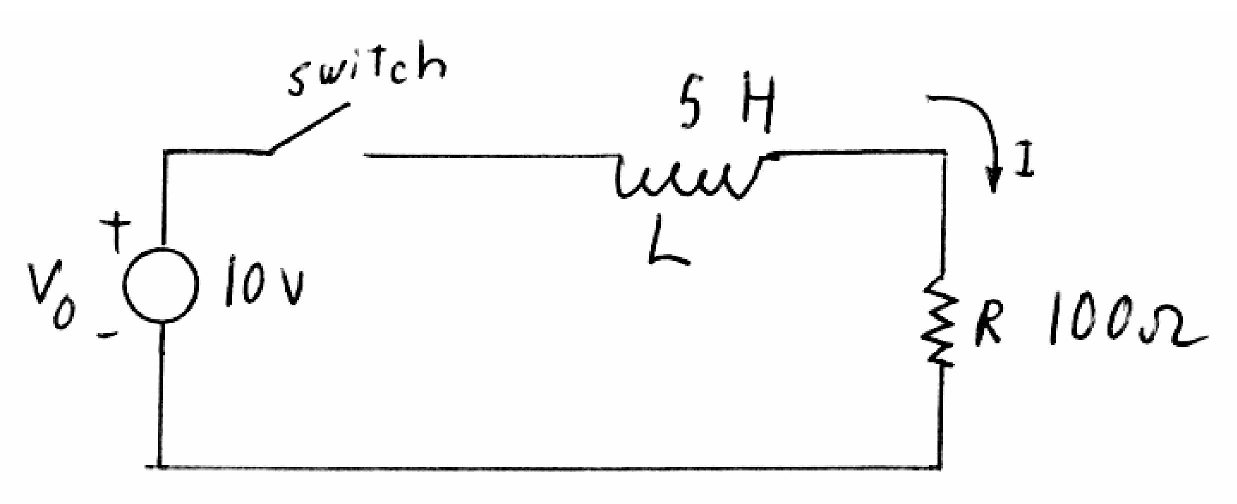
- Thus

$$V_0 - L\frac{di(t)}{dt} - i(t)R = 0$$

(2) Thus no differentiating is needed for RL circuits to get the DE

(3) Then the differential equation is

$$V_0 = L\frac{di(t)}{dt} - i(t)R$$



Resistor Inductor Circuit with Initial Current

- Starts with inductor in series with a resistor and a voltage source
- At time zero switch shorts out voltage source

(1) Writing down KVL

- Recalling

$$v_L(t) = L\frac{di}{dt}$$

- There is no voltage source after short switch is closed
- Inductor now acts as a voltage source supplying current to R thus

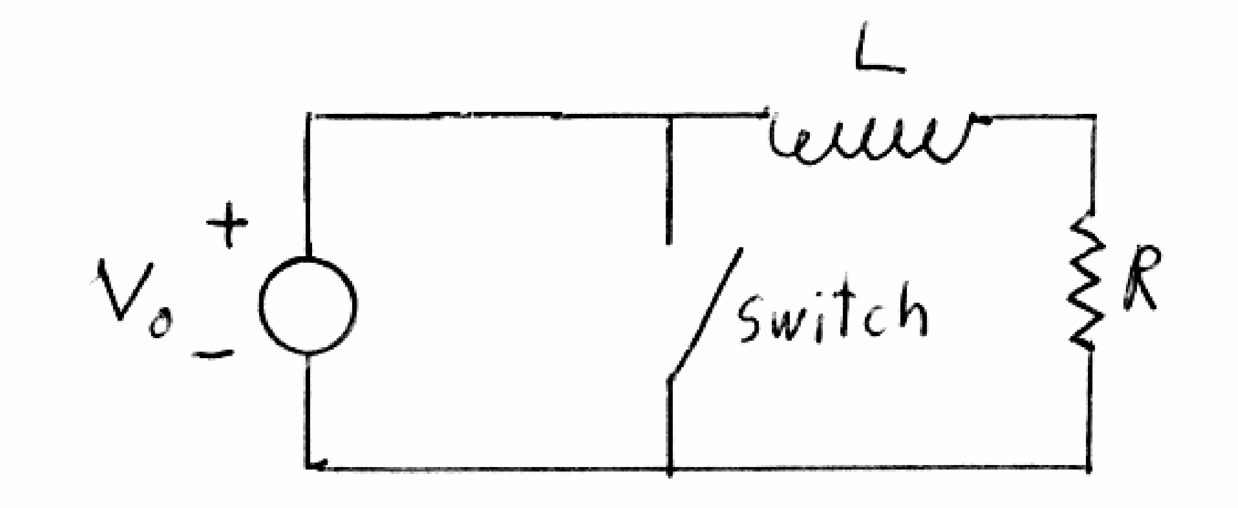
$$0 = L\frac{di(t)}{dt} - i(t)R$$

- Current declines as resistor consumes energy from inductor
-

(2) Thus no differentiating in KVL for RL circuits

(3) Thus using KVL V_L must equal V_R

$$0 = L\frac{di(t)}{dt} - i(t)R$$



Solving Resistor Inductor Circuits

4(b) To solve assume an exponential type decay of the current

$$i(t) = I_0 \exp(st)$$

Where I_0 = the initial current at t=0+

s = inverse of the time constant of the exp

- Then substituting into the differential equation

$$0 = L\frac{di(t)}{dt} - i(t)R = L\frac{d}{dt}I_0 \exp(st) - I_0 R \exp(st)$$

$$0 = LsI_0 \exp(st) - I_0 R \exp(st)$$

- Then divide out the exponential and I_0 terms
- This results in the "Characteristic Equation"

$$0 = Ls + R$$

- The time constant becomes for RL circuits

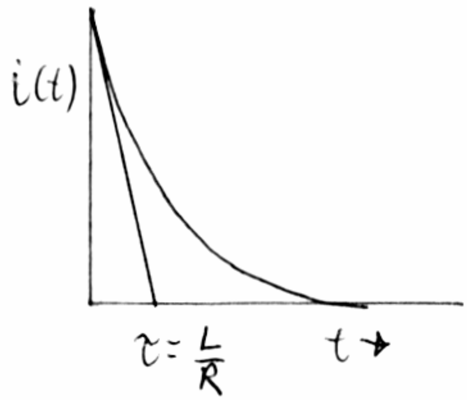
$$\tau = -\frac{1}{s} = \frac{L}{R}$$

(5) Solving for the initial conditions: at time t=0

- At t=0- inductor acts as a short (assuming V applied for long time)
- From KVL initial current in L must be set by V_0 and R

$$i(t=0) = I_0 = \frac{V_0}{R}$$

- Thus have both initial current and known exponential decay



Example Inductor Decay up to Current

- Consider a 10 V source is suddenly placed in series with RL
- L=5 H inductor and R=100 Ω resistor

(3) Writing the KVL then

- Because I is increasing both L and R oppose current change

$$V_0 = L\frac{di(t)}{dt} + i(t)R$$

- Assuming an exponential type solution then assume:

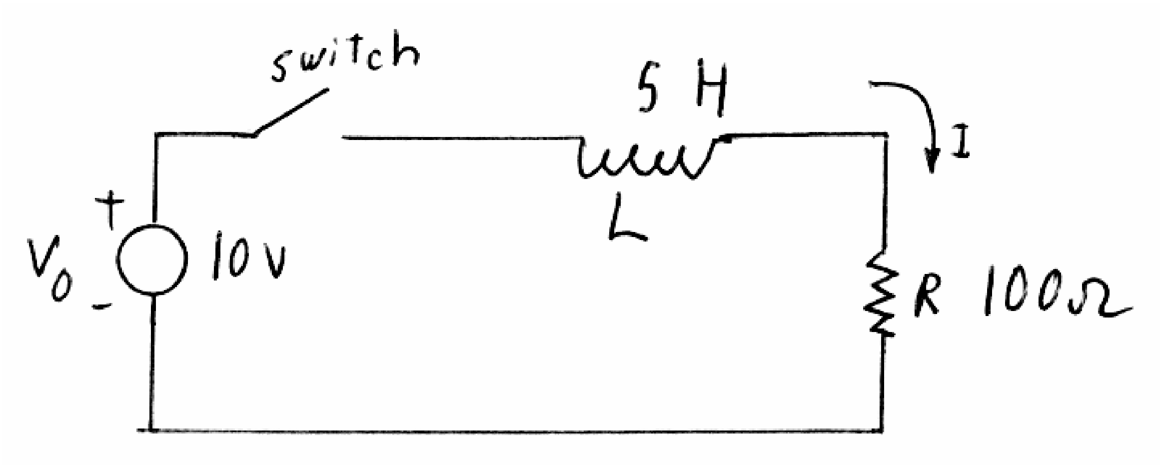
$$i(t) = I_0\,[1 - exp(st)]$$

- Why - know that must go to a steady state current
- But be zero at time zero.

- Substituting this in gives

$$V_0 = -L\frac{di(t)}{dt} - i(t)R = L\frac{d}{dt}I_0\ exp(st) + I_0R\,[1 - exp(st)]$$

$$V_0 = -LsI_0\ exp(st) + I_0R\,[1 - exp(st)]$$



Example Inductor Decay up to Current Con'd

- When time goes to infinity the inductor acts as a short.

$$i(t \to \infty) = \frac{V_0}{R} = \frac{10}{100} = 10\ mA$$

- Thus the exponential must decay to zero as t $\to\infty$

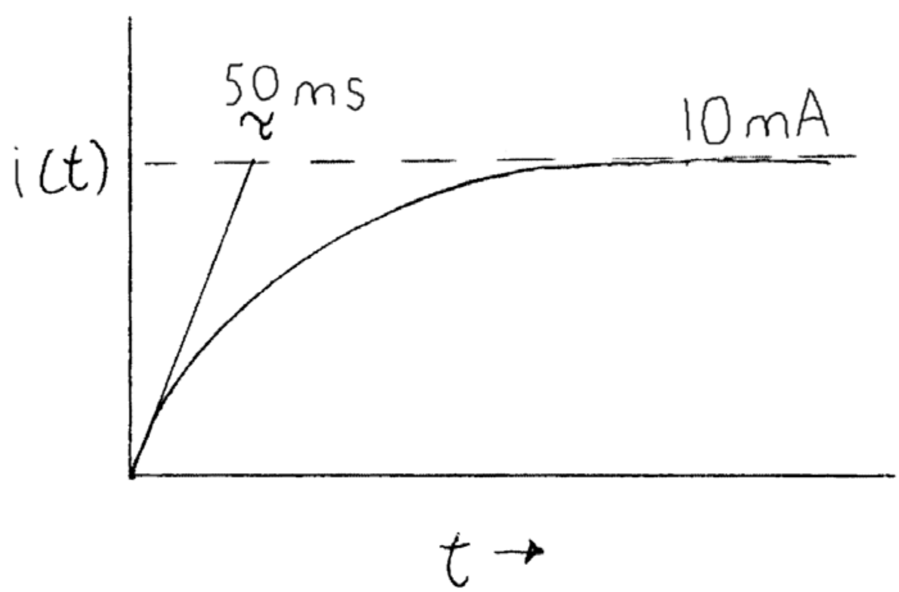
$$0 = Ls + R$$

$$s = -\frac{L}{R}$$

- The time constant is

$$\tau = \frac{1}{s} = \frac{L}{R} = \frac{5}{100} = 50\ m\sec$$

$$i(t) = 10\left[1 - exp\left(-\frac{t}{0.05}\right)\right]$$



Signal Processing Circuits: Signal Waveforms (EC 10.1)

- Time response analysis of circuit
- response to a time varying input signal
- Several common types of input signals
- Direct current (DC), or continuous: unvarying in time
- Step function: sudden change in DC level: only once
- Exponential decays or increase: only for some time

- Periodic signals: repeat with some period in time
- Pulsed: repeatitive change in DC values
- Sinusoidal: general Alternating Current signal
- Sawtooth (ramp): linear increase in time to max, then drop.

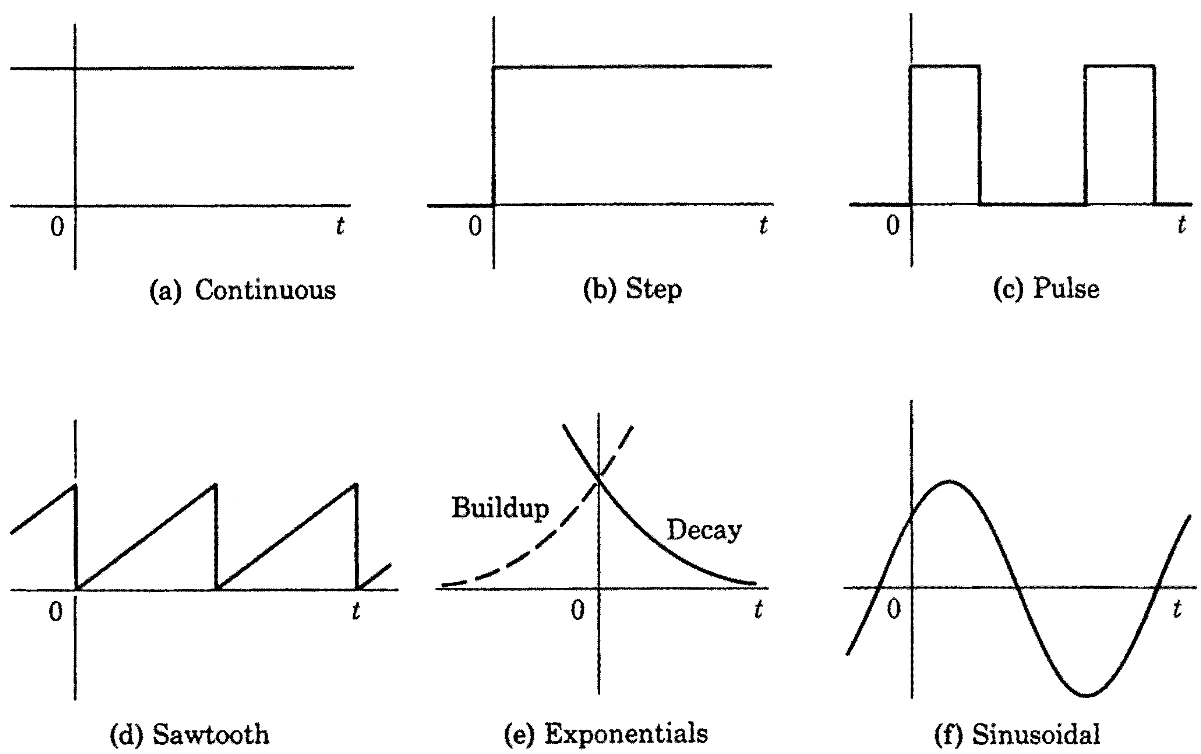


Figure 3.1 Common signal waveforms.

Differentiating Circuits (EC7)

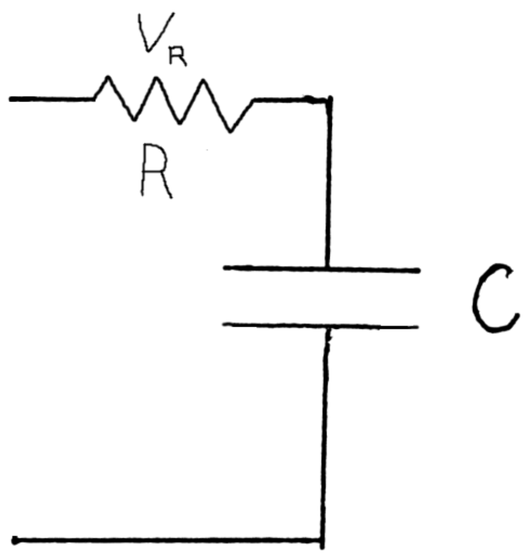
- A capacitor followed by a resistor
- If the RC time constant is short relative to period of any signal
- The capacitor dominates, changes the current to:

$$i(t) \approx C \frac{dV_{in}}{dt}$$

- This occurs independent of the waveform
- Thus the voltage across the resistor becomes

$$V_{out} = V_R = i(t)R$$

- Thus get the derivative of the signal



Integrating Circuits

- Resistor followed by as capacitor
- If the RC time constant is long relative to period
- The resistor dominates the voltage drop and

$$i(t) = \frac{V_{in}}{R}$$

- The voltage across the capacitor becomes

$$V_{out} = V_c = \frac{1}{RC} \int V_{in} \, dt$$

- This occurs independent of the waveform
- Thus get the integral of the signal