Capacitors Initial and Final Response to a "Step Function"

- Inductors and Capacitors react differently to a Voltage step
- Just after the step Capacitors act as a short if uncharged

\[ I_C(t) = C \frac{dV}{dt} \]

- If charged Capacitor acts as an voltage source
- As time goes to infinity change in voltage goes to zero
- Then C act as an open (become fully charge or discharged)
- Thus can find initial and final conditions of circuit
- Use KVL with on circuit with these two models of the C’s
Inductor Initial and Final Response to a "Step Function"

- Inductors react differently to a voltage step
- Just after the step inductors act as opens
- Reason: opposes sudden change in current

\[ v_L(t) = L \frac{di}{dt} \]

- As time goes to infinity, inductors act as shorts
- Thus no more current change in final state
- Thus can find initial and final conditions of circuit
- Use KVL with on circuit with these two models of the L’s

\[ \text{Diagram:} \]

\[ \text{Initial state: } t = 0 \]

\[ \text{Final state: } t \to \infty \]
General Solution Method of First Order Circuits

• General first order solution to a sudden change

(1) Use Kirchoff's laws for circuit equation

(2) Manipulate to get I or V in terms of derivates in time

(3) Generate the “Differential Equation Form”
• May need to differentiate to obtain

(4) Solve the Differential equation: 2 methods:
(4a) Integration method: get in integral form
• Integrate to get solution: eg RC circuit

(4b) Solution substitution method: assume a solution
• For step change assume exponential (or exp to constant)
• Solve for time constant

(5) Use initial or final conditions for constants of integration
Resistor Inductor Circuits

- Consider an inductor in series with a resistor.
- Called an LR circuit
- At time zero switch is opened

(1) Writing down KVL
Recall that the voltage across the inductor is

\[ v_L(t) = L \frac{di}{dt} \]

- Thus

\[ V_0 - L \frac{di(t)}{dt} - i(t)R = 0 \]

(2) Thus no differentiating is needed for RL circuits to get the DE

(3) Then the differential equation is

\[ V_0 = L \frac{di(t)}{dt} - i(t)R \]
Resistor Inductor Circuit with Initial Current

- Starts with inductor in series with a resistor and a voltage source
- At time zero switch shorts out voltage source

1) Writing down KVL
- Recalling

\[ v_L(t) = L \frac{di}{dt} \]

- There is no voltage source after short switch is closed
- Inductor now acts as a voltage source supplying current to R thus

\[ 0 = L \frac{di(t)}{dt} - i(t)R \]

- Current declines as resistor consumes energy from inductor

2) Thus no differentiating in KVL for RL circuits

3) Thus using KVL \( V_L \) must equal \( V_R \)

\[ 0 = L \frac{di(t)}{dt} - i(t)R \]
Solving Resistor Inductor Circuits

4(b) To solve assume an exponential type decay of the current

\[ i(t) = I_0 \exp(st) \]

Where \( I_0 \) = the initial current at \( t=0^+ \)
\( s = \) inverse of the time constant of the exp
- Then substituting into the differential equation

\[ 0 = L \frac{di(t)}{dt} - i(t)R = L \frac{d}{dt} I_0 \exp(st) - I_0R \exp(st) \]

\[ 0 = LsI_0 \exp(st) - I_0R \exp(st) \]
- Then divide out the exponential and \( I_0 \) terms
- This results in the "Characteristic Equation"

\[ 0 = Ls + R \]

- The time constant becomes for RL circuits

\[ \tau = -\frac{1}{s} = \frac{L}{R} \]

(5) Solving for the initial conditions: at time \( t=0 \)
- At \( t=0^+ \) inductor acts as a short (assuming \( V \) applied for long time)
- From KVL initial current in \( L \) must be set by \( V_0 \) and \( R \)

\[ i(t = 0) = I_0 = \frac{V_0}{R} \]
- Thus have both initial current and known exponential decay
Example Inductor Decay up to Current

- Consider a 10 V source is suddenly placed in series with RL
- \( L = 5 \) H inductor and \( R = 100 \) Ω resistor

(3) Writing the KVL then
- Because I is increasing both L and R oppose current change

\[
V_0 = L \frac{di(t)}{dt} + i(t)R
\]

- Assuming an exponential type solution then assume:

\[
i(t) = I_0 [1 - \exp(st)]
\]

- Why - know that must go to a steady state current
- But be zero at time zero.

- Substituting this in gives

\[
V_0 = -L \frac{di(t)}{dt} - i(t)R = L \frac{d}{dt} I_0 \exp(st) + I_0 R [I - \exp(st)]
\]

\[
V_0 = -LsI_0 \exp(st) + I_0 R [1 - \exp(st)]
\]
Example Inductor Decay up to Current Con’d

- When time goes to infinity the inductor acts as a short.

\[
i(t \to \infty) = \frac{V_0}{R} = \frac{10}{100} = 10 \text{ mA}
\]

- Thus the exponential must decay to zero as \( t \to \infty \)

\[
0 = Ls + R
\]

\[
s = -\frac{L}{R}
\]

- The time constant is

\[
\tau = \frac{1}{s} = \frac{L}{R} = \frac{5}{100} = 50 \text{ m sec}
\]

\[
i(t) = 10 \left[ 1 - \exp\left(-\frac{t}{0.05}\right) \right]
\]
Signal Processing Circuits: Signal Waveforms (EC 10.1)

- Time response analysis of circuit
- Response to a time varying input signal
- Several common types of input signals
- Direct current (DC), or continuous: unvarying in time
- Step function: sudden change in DC level: only once
- Exponential decays or increase: only for some time

- Periodic signals: repeat with some period in time
- Pulsed: repetitive change in DC values
- Sinusoidal: general Alternating Current signal
- Sawtooth (ramp): linear increase in time to max, then drop.

![Waveforms](https://via.placeholder.com/150)

**Figure 3.1** Common signal waveforms.
Differentiating Circuits (EC7)

- A capacitor followed by a resistor
- If the RC time constant is short relative to period of any signal
- The capacitor dominates, changes the current to:

\[ i(t) \approx C \frac{dV_{in}}{dt} \]

- This occurs independent of the waveform
- Thus the voltage across the resistor becomes

\[ V_{out} = V_R = i(t)R \]

- Thus get the derivative of the signal
Integrating Circuits

- Resistor followed by a capacitor
- If the RC time constant is long relative to period
- The resistor dominates the voltage drop and
  \[ i(t) = \frac{V_{in}}{R} \]

- The voltage across the capacitor becomes
  \[ V_{out} = V_c = \frac{1}{RC} \int V_{in} \, dt \]

- This occurs independent of the waveform
- Thus get the integral of the signal